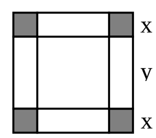
7^{th} Grade *Algebra & Geometry* Main Lesson
Lesson Plan Outline

Day #1

- Ask students: "What do you think algebra is?"
- The essence of algebra is:
  - It is the universal language of mathematics. Everywhere in the world, algebra is basically the same.
  - The solving of puzzles called equations.
- The Greeks didn't have algebra as we know it today. It was developed in the Arab world in the 800's.
- Introductory Problem. Give an example of how algebra can used be used to solve a difficult math question. The purpose here is to give students an impression of the power of algebra. **They should not understand the algebra given**, but rather be curious and excited for the multi-year algebra journey that they just beginning, and appreciate that algebra allows us to solve difficult math puzzles quite easily.
  Here are two possibilities:
  - Option #1: *An example from high school algebra.* Question: Jeff spent a total of \$159 at a store on books, food and clothes. He spent 4 times as much money on clothes as he did on books, and \$6 less on books than on food. How much money did Jeff spend on clothes?
    Solution: $B = F - 6$
    $C = 4B \rightarrow C = 4(F-6)$
    $B + F + C = 159$
    $(F-6) + F + 4(F-6) = 159$
    $F - 6 + F + 4F - 24 = 159$
    $6F - 30 = 159$
    $6F = 159 + 30$
    $6F = 189$
    $F = 31.5; \quad B = 25.5; \quad C = 102$
    Therefore, Jeff spent \$102 on clothes.
  - Option #2: *An example from (12^{th} grade) calculus*: Question: A box with an open top is to be made from a square piece of cardboard measuring 20 inches on each side, by cutting off four squares from the corners and then folding the sides up. What are the dimensions of such a box that has the largest possible volume?



    - Solution: Using calculus, the solution is written in terms of algebra and looks like this:

$$v = y^2x \rightarrow x = \tfrac{1}{2}(20-y) \rightarrow v = \tfrac{1}{2}y^2(20-y) \rightarrow v = 10y^2 - \tfrac{1}{2}y^3$$
$$\rightarrow \frac{dv}{dy} = 20y - \frac{3}{2}y^2 \rightarrow 0 = y(20 - \frac{3}{2}y) \rightarrow y = 13\tfrac{1}{3}"; x = 3\tfrac{1}{3}"$$

- Geometry: Give outline of Greek mathematics:
  - Thales (ca.600 b.c.) was the first to come up with any simple proofs.
    - The Theorem of Thales: A triangle inscribed inside a semicircle is a right triangle.
  - Pythagoras (ca.540 b.c.)
    - Developed a whole philosophy largely based upon mathematical relationships. The Pythagorean Theorem (to be done toward the end of this main lesson) is the first substantial proof.
  - Euclid (ca.300 b.c.) gathered all the math known at his time into one amazing book, *The Elements*, that served as *the* math textbook for more than 2000 years.
  - Archimedes (ca.250 b.c.) was most brilliant mind of ancient times. Discovered many important mathematical theorems and principles in physics.
- HW: Rough draft essay on Greek mathematics.

Day #2

- Have class try to add the numbers from 1 to 100. Then tell the story of Carl Freidrich Gauss (1777-1855). He was one of the greatest mathematicians ever. When Carl was 9 or 10 years old, his teacher (Herr Büttner) gave the class (in a poor school in Braunschweig, Germany) the assignment to sum all the numbers from 1 to 100 (i.e. 1+2+3+4+5+…+100) in order to keep the students busy. Carl did the problem in his head almost immediately, wrote the answer on his slate, handed it in, and then sat with his hands folded as the rest of the students worked diligently, and the teacher looked at him scornfully. When the teacher finally went through the stack of slates, Carl was the only one to have the correct answer**:** 5050. Carl realized that he could add the numbers in pairs**:** 1+100, and then 2+99, and then 3+98, etc. He saw that this sequence really consisted of 50 pairs of numbers, each pair adding to 101. He then simply multiplied 50 times 101 to get 5050.
  - Ask the class to try and come up with his formula for homework.
- Do car rental formula:

Example: Nifty Car Rental charges $35 per day and 9¢ per mile. What would be the cost (before tax) for a car that is rented for one week and driven a total of 550 miles?

Solution: The formula here is**:** $C = 35 \cdot D + 0.09 \cdot M$

where C is the cost, D is the number days, and M is the number of miles.

Putting 7 into D, and 550 into M, we get**:**

$C = 35 \cdot 7 + 0.09 \cdot 550 \rightarrow C = 245 + 49.5$ giving us a final answer of <u>$294.50</u>.

- <u>Geometry:</u> In main lesson book: Geometric division. (See MS Curriculum Book)

Day #3

- Review story of Gauss and give his formula.
- Introduce Galileo's Law of Falling Bodies: $D = 16 \cdot T^2$
  - D is the distance in feet, and T is the time in seconds.
  - The formula gives the distance traveled by a dropped object (assuming no air resistance).

Example: A rock is dropped out of a plane. How far does it fall after 10 seconds?
Solution: We put 10 into the formula, and get $D = 16 \cdot 10^2$. The *Order of Operations* says that we must first square 10 (which is 100), then multiply by 16 to get a final answer of <u>1600 feet</u>.

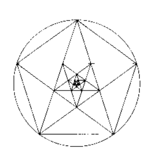
- Groupwork: Practice doing problems that use formulas (car rental, Gauss's, Galileo's).
- End with question: Can you have less than nothing?
- <u>Geometry:</u> Intro the idea of the Great Greek Geometric Game. (See MS Curriculum Book)

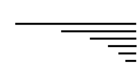
Day #4

- Review formula problems and put a formula page into main lesson book.
- Intro negative numbers. **<u>Don't do a number line or do the Death Valley example</u>**. A place that negative numbers appear is with money. Simply think of negative numbers in terms of money – having less than nothing: i.e. debt.
- <u>Geometry:</u> Briefly show how to construct a pentagon inside a circle, using first the guess and check method, and then the Euclidean, theoretically exact method. (Students will do this drawing tomorrow.)

Day #5

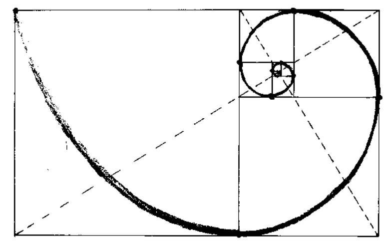
- Introduce idea of combining like terms: X's get combined with X's and constants with constants.
  - 7–4 is now seen as combining positive 7 with a negative 4.
- Intro idea of equation.
- Groupwork: Combining signed numbers (if stuck, think of a checking account) and combining like terms.
- Put page into book: "Signed Numbers".
- <u>Geometry:</u> Put nested pentagons and pentagrams into book.
  - Then copy (on the board only) the various length line segments in order of increasing size. End class by asking: what is the special relationship between these line segments?





Day #6

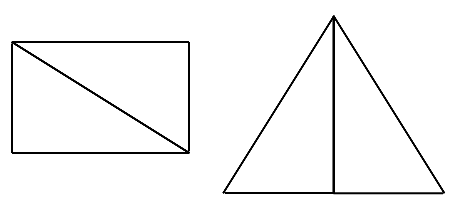
- Intro the idea of solving simple equations as trying to guess the value that it could be in order to make the equation work. **Example**: Solve $3X = 12$. We can see that if we put 4 into X then the equation works.
- Groupwork: Do Algebra Sheet#1
- Geometry: The special relationship between the line segments in the pentagon drawing (from above):
  - The ratio of any two consecutive line segments is about 1.618:1 (i.e., each line segment is about 1.618 times longer than the one before it.) This is the golden ratio.
  - The length of any line segment is equal to the sum of the lengths of the previous two line segments.
- Do the drawing of the *The Golden Rectangle and The Rectangle of Whirling Squares*
  - **Construction** (of the golden rectangle)**:** One way to construct a golden rectangle is to use the length of the side of a pentagon as the rectangle's height, and to use the length of that same pentagon's diagonal as the rectangle's base.



  - **Construction** (adding the Whirling Squares inside the Golden rectangle)**:** Construct a large golden rectangle and then draw a line that divides the rectangle into a square and a smaller golden rectangle. Draw a diagonal across the original (larger) rectangle, and a diagonal across the smaller rectangle, so that they intersect. Draw a line dividing the smaller rectangle into a square and another golden rectangle, and divide that rectangle, and every succeeding one in the same manner, so that the squares *spiral* in toward the intersection of the two diagonals. The students should draw the spiral freehand as shown in the drawing at the right.

Day #7

- The most important thing of the algebra main lesson block - Solving a puzzle with a scale:
  We use a scale to represent the equation $3x + 9 = 5x + 2$. First, we place 9 equal weights and 3 bags on the left side of the scale, where each bag is hiding the solution (3½ weights) inside it. We also place 2 weights and 5 bags (with each bag again hiding 3½ weights) on the right side of the scale. The scale should balance. Each bag represents the *variable* (or *unknown*). The students should be told that all the bags have the same number of weights inside them and that the goal is to solve the puzzle**:** How many weights are in each bag? Soon the students should come to realize that they can remove two weights from each side of the scale, and that they can remove three bags from each side. The scale remains balanced with 2 bags on one side and 7 weights on the other. They can then figure out that each bag must contain half of 7, or 3½ weights. Make sure that all the students really understand each step that was done in order to solve the puzzle.
- Introduce multiplying signed numbers. The two laws are:
  - A negative times a negative is always a positive.
  - A negative times a positive (or positive times a negative) is always a negative.
- Groupwork: Do Algebra Sheet#2
- Geometry: More with the Golden Rectangle:
  - With the golden rectangle, the ratio of the length to the width is Φ**:**1 (which is also the ratio of the diagonal to the side of a pentagon).
  - This is the only shape for a rectangle where you can cut off a square, and the remaining smaller rectangle will be similar to the original rectangle.
  - *Historical Importance.*
  - The Golden Rectangle was considered the most aesthetically pleasing proportions for a rectangle.
  - The Parthenon was built using golden rectangles.
  - If we take a golden rectangle, split it along its diagonal, and join the two resulting right triangles along their middle-sized sides, then we get the shape of the isosceles triangle that was used to build the Great Pyramid. (See drawing at right.)



- Put page in Main lesson book on the golden ratio.

Day #8

- Review the scale puzzle thoroughly.
- Dividing signed numbers: the laws are the same as for multiplication.
- Page in main lesson book: Arithmetic with signed numbers.
- Page in main lesson book: Solving a puzzle with a scale.
- History of Algebra and the Father of Algebra:
    - The roots of algebra go back to the Greeks, but it was the Arabs who developed the basis of algebra between 650 and 850ad.
    - In the early 800's, the Abbasid Empire, perhaps the largest empire in the world at that time, was under the rule of the caliph (king) Al-Ma'mun (809-833) who was very interested in mathematics and astronomy. He collected many of the classic works from the Greeks, Jews, Hindus, and other cultures from around the world. He then established his school, *The House of Wisdom* in Baghdad, and invited the greatest scholars in his empire to join it.
    - Mohammad ibn Musa al-Khwarizmi was one of the mathematicians who joined the House of Wisdom. He came from the city of Khiva in Amudarya, which was just south of the Aral Sea in what is now Uzbekistan.
    - Al-Khwarizmi wrote a book around 825 called *Hisab al-jabr wal-muqabala,* which roughly translated means "the science of equations". Little, if anything, from the book was original. What made the book so great was that it was a collection of all the algebra known at that time (especially from Greece and India), and it was written in a way that people could fairly easily understand. It was translated into Latin 300 years later and it made a big impact on the mathematicians of Europe. Today, we call al-Khwarizmi *the father of algebra*.
    - The book had none of the algebra notation that we take for granted today. It was written out in words, in paragraph form, like any ordinary book. Problems and their step-by-step solutions were "talked" about in normal written language. Most of our basic modern mathematical notation wasn't developed until the 1400's and 1500's. For instance, writing "+" to mean adding two numbers was first used in Germany in 1489. Negative numbers were not accepted (e.g., as solutions to equations) until the 1600's.
    - Algebra has developed into a powerful universal language that allows people to communicate complex mathematical thoughts in a simple and concise form.
- Groupwork: Do Algebra Sheet#3
- (No geometry.)

Day #9

- Show that the scale puzzle can be written as an equation, like this:

$$
\begin{aligned}
3x + 9 &= 5x + 2\\
\underline{\quad -2} &\underline{=\quad -2}\\
3x + 7 &= 5x\\
\underline{-3x\quad} &\underline{-3x}\\
7 &= 2x\\
\underline{\div 2} &\underline{\div 2}\\
3\tfrac{1}{2} &= x
\end{aligned}
$$

- An Equation is a Puzzle (just like the scale puzzle).
  The goal is to find a value (or values) that we can put into the equation in order to make the equation work, or balance. If we plug the solution into the equation, then both sides of the equation will have the same value, thereby showing that the solution works.
- Page in main lesson book: The Father of Algebra.
- Page in main lesson book: Solving one-step equations.
- Groupwork: Do Algebra Sheet#4
- Geometry: Introduce Theorems from Two Parallel Lines and a Transversal. (See MS Curriculum Book) Give simple proofs also.

Day #10

- Groupwork: Do Algebra Sheet#5
- Solve two more multi-step equations together as a class.
- Introduce *The Golden Rule of Equations*.
- Page in main lesson book: Solving multi-step equations.
- Geometry: Page in Book on Theorems from Two Parallel Lines and a Transversal.
- Students discover that the three angles in a triangle add to 180° by cutting out the angles.

Day #11

- Groupwork: Do Algebra Sheet#6.   **(Students start to solve multi-step equations in small groups.)**
- Page in main lesson book: The Golden Rule of Equations.
- Geometry: Do cutout Puzzle in groups of Pythagorean Theorem   (See MS Curriculum Book), but don't tell them what it is.
- Page in main lesson book: The *Half-Wheel Theorem* (including two proofs, see MS Curriculum Book). This theorem states that the angles in a triangle add to 180°, so named because the proof resembles a "half-wheel".

Day #12

- Groupwork: Do Algebra Sheet#7.   **(Students start to solve multi-step equations on their own.)**
- Page in main lesson book: Examples (four of them as chosen by the student) of multi-step equations.
- Geometry: Ask what yesterday's cutout puzzle showed.   They should (as a class) come to the statement: *With any right triangle, the area of the square of the hypotenuse is equal to the sum of the areas of the squares of the other two sides.* Also mention that the Pythagorean isn't just $a^2 + b^2 = c^2$; Pythagoras won't have recognized this! For him it was about the areas of squares.
- Page in main lesson book: Pythagorean Theorem.

Day #13

- Groupwork: Do Algebra Sheet#8
- Geometry: *Theorem of Morley*. The theorem states: "The six angle trisectors of any triangle meet to form an equilateral triangle." You need to trisect each of the original three angles of the triangle. Trisecting an angle with a compass and straightedge is impossible, so you need to use a protractor.

Day #14

Test! (On algebra only) It should be simple and build confidence!