New York Times Magazine July 23, 2014

Why Do Americans Stink at Math? By ELIZABETH GREEN

When Akihiko Takahashi was a junior in college in 1978, he was like most of the other students at his university in suburban Tokyo. He had a vague sense of wanting to accomplish something but no clue what that something should be. But that spring he met a man who would become his mentor, and this relationship set the course of his entire career.

Takeshi Matsuyama was an elementary-school teacher, but like a small number of instructors in Japan, he taught not just young children but also college students who wanted to become teachers. At the university-affiliated elementary school where Matsuyama taught, he turned his classroom into a kind of laboratory, concocting and trying out new teaching ideas. When Takahashi met him, Matsuyama was in the middle of his boldest experiment yet — revolutionizing the way students learned math by radically changing the way teachers taught it.

Instead of having students memorize and then practice endless lists of equations — which Takahashi remembered from his own days in school — Matsuyama taught his college students to encourage passionate discussions among children so they would come to uncover math's procedures, properties and proofs for themselves. One day, for example, the young students would derive the formula for finding the area of a rectangle; the next, they would use what they learned to do the same for parallelograms. Taught this new way, math itself seemed transformed. It was not dull misery but challenging, stimulating and even fun.

Takahashi quickly became a convert. He discovered that these ideas came from reformers in the United States, and he dedicated himself to learning to teach like an American. Over the next 12 years, as the Japanese educational system embraced this more vibrant approach to math, Takahashi taught first through sixth grade. Teaching, and thinking about teaching, was practically all he did. A quiet man with calm, smiling eyes, his passion for a new kind of math instruction could take his colleagues by surprise. "He looks very gentle and kind," Kazuyuki Shirai, a fellow math teacher, told me through a translator. "But when he starts talking about math, everything changes."

Takahashi was especially enthralled with an American group called the National Council of Teachers of Mathematics, or N.C.T.M., which published manifestoes throughout the 1980s, prescribing radical changes in the teaching of math. Spending late nights at school, Takahashi read every one. Like many professionals in Japan, teachers often said they did their work in the name of their mentor. It was as if Takahashi bore two influences: Matsuyama and the American reformers.

Takahashi, who is 58, became one of his country's leading math teachers, once attracting 1,000 observers to a public lesson. He participated in a classroom equivalent of "Iron Chef," the popular Japanese television show. But in 1991, when he got the opportunity to take a new job in America, teaching at a school run by the Japanese Education Ministry for expats in Chicago, he did not hesitate. With his wife, a graphic designer, he left his friends, family, colleagues — everything he knew — and moved to the United States, eager to be at the center of the new math.

As soon as he arrived, he started spending his days off visiting American schools. One of the first math classes he observed gave him such a jolt that he assumed there must have been some kind of mistake. The class looked exactly like his own memories of school. "I thought, Well, that's only this class," Takahashi said. But the next class looked like the first, and so did the next and the one after that. The Americans might have invented the world's best methods for teaching math to children, but it was difficult to find anyone actually using them.

It wasn't the first time that Americans had dreamed up a better way to teach math and then failed to implement it. The same pattern played out in the 1960s, when schools gripped by a post-Sputnik inferiority complex unveiled an ambitious "new math," only to find, a few years later, that nothing actually changed. In fact, efforts to introduce a better way of teaching math stretch back to the 1800s. The story is the same every time: a big, excited push, followed by mass confusion and then a return to conventional practices.

The trouble always starts when teachers are told to put innovative ideas into practice without much guidance on how to do it. In the hands of unprepared teachers, the reforms turn to nonsense, perplexing students more than helping them. One 1965 Peanuts cartoon depicts the young blond-haired Sally struggling to understand her new-math assignment: "Sets . . . one to one matching . . . equivalent sets . . . sets of one . . . sets of two . . . renaming two. . . .” After persisting for three valiant frames, she throws back her head and bursts into tears: “All I want to know is, how much is two and two?”

Today the frustrating descent from good intentions to tears is playing out once again, as states across the country carry out the latest wave of math reforms: the Common Core. A new set of academic standards developed to replace states’ individually designed learning goals, the Common Core math standards are like earlier math reforms, only further refined and more ambitious. Whereas previous movements found teachers haphazardly, through organizations like Takahashi’s beloved N.C.T.M. math-teacher group, the Common Core has a broader reach. A group of governors and education chiefs from 48 states initiated the writing of the standards, for both math and language arts, in 2009. The same year, the Obama administration encouraged the idea, making the adoption of rigorous “common standards” a criterion for receiving a portion of the more than $4 billion in Race to the Top grants. Forty-three states have adopted the standards.

The opportunity to change the way math is taught, as N.C.T.M. declared in its endorsement of the Common Core standards, is “unprecedented.” And yet, once again, the reforms have arrived without any good system for helping teachers learn to teach them. Responding to a recent survey by Education Week, teachers said they had typically spent fewer than four days in Common Core training, and that included training for the language-arts standards as well as the math.

Carefully taught, the assignments can help make math more concrete. Students don’t just memorize their times tables and addition facts but also understand how arithmetic works and how to apply it to real-life situations. But in practice, most teachers are unprepared and children are baffled, leaving parents furious. The comedian Louis C.K. parodied his daughters’ homework in an appearance on “The Late Show With David Letterman”: “It’s like, Bill has three goldfish. He buys two more. How many dogs live in London?”

The inadequate implementation can make math reforms seem like the most absurd form of policy change — one that creates a whole new problem to solve. Why try something we’ve failed at a half-dozen times before, only to watch it backfire? Just four years after the standards were first released, this argument has gained traction on both sides of the aisle. Since March, four Republican governors have opposed the standards. In New York, a Republican candidate is trying to establish another ballot line, called Stop Common Core, for the November gubernatorial election. On the left, meanwhile, teachers’ unions in Chicago and New York have opposed the reforms.

The fact that countries like Japan have implemented a similar approach with great success offers little consolation when the results here seem so dreadful. Americans might have written the new math, but maybe we simply aren’t suited to it. “By God,” wrote Erick Erickson, editor of the website RedState, in an anti-Common Core attack, is it such “a horrific idea that we might teach math the way math has always been taught.”

The new math of the ‘60s, the new new math of the ‘80s and today’s Common Core math all stem from the idea that the traditional way of teaching math simply does not work.

As a nation, we suffer from an ailment that John Allen Paulos, a Temple University math professor and an author, calls innumeracy — the mathematical equivalent of not being able to read. On national tests, nearly two-thirds of fourth graders and eighth graders are not proficient in math. More than half of fourth graders taking the 2013 National Assessment of Educational Progress could not accurately read the temperature on a neatly drawn thermometer. (They did not understand that each hash mark represented two degrees rather than one, leading many students to mistake 46 degrees for 43 degrees.) On the same multiple-choice test, three-quarters of fourth graders could not translate a simple word problem about a girl who sold 15 cups of lemonade on Saturday and twice as many on Sunday into the expression “15 + (2×15).” Even in Massachusetts, one of the country’s highest-performing states, math students are more than two years behind their counterparts in Shanghai.

Adulthood does not alleviate our quantitative deficiency. A 2012 study comparing 16-to-65-year-olds in 20 countries found that Americans rank in the bottom five in numeracy. On a scale of 1 to 5, 29 percent of them scored at Level 1 or below, meaning they could do basic arithmetic but not computations requiring two or more steps. One study that examined medical prescriptions gone awry found that 17 percent of errors were caused by math mistakes on the part of doctors or pharmacists. A survey found that three-quarters of doctors inaccurately estimated the rates of death and major complications associated with common medical procedures, even in their own specialty areas.

One of the most vivid arithmetic failings displayed by Americans occurred in the early 1980s, when the A&W restaurant chain released a new hamburger to rival the McDonald’s Quarter Pounder. With a third-pound of beef, the A&W burger had more meat than the Quarter Pounder; in taste tests, customers preferred A&W’s

burger. And it was less expensive. A lavish A&W television and radio marketing campaign cited these benefits. Yet instead of leaping at the great value, customers snubbed it.

Only when the company held customer focus groups did it become clear why. The Third Pounder presented the American public with a test in fractions. And we failed. Misunderstanding the value of one-third, customers believed they were being overcharged. Why, they asked the researchers, should they pay the same amount for a third of a pound of meat as they did for a quarter-pound of meat at McDonald's. The "4" in "¼," larger than the "3" in "⅓," led them astray.

But our innumeracy isn't inevitable. In the 1970s and the 1980s, cognitive scientists studied a population known as the unschooled, people with little or no formal education. Observing workers at a Baltimore dairy factory in the '80s, the psychologist Sylvia Scribner noted that even basic tasks required an extensive amount of math. For instance, many of the workers charged with loading quarts and gallons of milk into crates had no more than a sixth-grade education. But they were able to do math, in order to assemble their loads efficiently, that was "equivalent to shifting between different base systems of numbers." Throughout these mental calculations, errors were "virtually nonexistent." And yet when these workers were out sick and the dairy's better-educated office workers filled in for them, productivity declined.

The unschooled may have been more capable of complex math than people who were specifically taught it, but in the context of school, they were stymied by math they already knew. Studies of children in Brazil, who helped support their families by roaming the streets selling roasted peanuts and coconuts, showed that the children routinely solved complex problems in their heads to calculate a bill or make change. When cognitive scientists presented the children with the very same problem, however, this time with pen and paper, they stumbled. A 12-year-old boy who accurately computed the price of four coconuts at 35 cruzeiros each was later given the problem on paper. Incorrectly using the multiplication method he was taught in school, he came up with the wrong answer. Similarly, when Scribner gave her dairy workers tests using the language of math class, their scores averaged around 64 percent. The cognitive-science research suggested a startling cause of Americans' innumeracy: school.

Most American math classes follow the same pattern, a ritualistic series of steps so ingrained that one researcher termed it a cultural script. Some teachers call the pattern "I, We, You." After checking homework, teachers announce the day's topic, demonstrating a new procedure: "Today, I'm going to show you how to divide a three-digit number by a two-digit number" (I). Then they lead the class in trying out a sample problem: "Let's try out the steps for 242 ÷ 16" (We). Finally they let students work through similar problems on their own, usually by silently making their way through a work sheet: "Keep your eyes on your own paper!" (You).

By focusing only on procedures — "Draw a division house, put '242' on the inside and '16' on the outside, etc." — and not on what the procedures mean, "I, We, You" turns school math into a sort of arbitrary process wholly divorced from the real world of numbers. Students learn not math but, in the words of one math educator, answer-getting. Instead of trying to convey, say, the essence of what it means to subtract fractions, teachers tell students to draw butterflies and multiply along the diagonal wings, add the antennas and finally reduce and simplify as needed. The answer-getting strategies may serve them well for a class period of practice problems, but after a week, they forget. And students often can't figure out how to apply the strategy for a particular problem to new problems.

How could you teach math in school that mirrors the way children learn it in the world? That was the challenge Magdalene Lampert set for herself in the 1980s, when she began teaching elementary-school math in Cambridge, Mass. She grew up in Trenton, accompanying her father on his milk deliveries around town, solving the milk-related math problems he encountered. "Like, you know: If Mrs. Jones wants three quarts of this and Mrs. Smith, who lives next door, wants eight quarts, how many cases do you have to put on the truck?" Lampert, who is 67 years old, explained to me.

She knew there must be a way to tap into what students already understood and then build on it. In her classroom, she replaced "I, We, You" with a structure you might call "You, Y'all, We." Rather than starting each lesson by introducing the main idea to be learned that day, she assigned a single "problem of the day," designed to let students struggle toward it — first on their own (You), then in peer groups (Y'all) and finally as a whole class (We). The result was a process that replaced answer-getting with what Lampert called sense-making. By pushing students to talk about math, she invited them to share the misunderstandings most American students keep quiet until the test. In the process, she gave them an opportunity to realize, on their own, why their answers were wrong.

Lampert, who until recently was a professor of education at the University of Michigan in Ann Arbor, now works for the Boston Teacher Residency, a program serving Boston public schools, and the New Visions for Public Schools network in New York City, instructing educators on how to train teachers. In her book, “Teaching Problems and the Problems of Teaching,” Lampert tells the story of how one of her fifth-grade classes learned fractions. One day, a student made a “conjecture” that reflected a common misconception among children. The fraction 5 / 6, the student argued, goes on the same place on the number line as 5 / 12. For the rest of the class period, the student listened as a lineup of peers detailed all the reasons the two numbers couldn’t possibly be equivalent, even though they had the same numerator. A few days later, when Lampert gave a quiz on the topic (“Prove that 3 / 12 = 1 / 4 ,” for example), the student could confidently declare why: “Three sections of the 12 go into each fourth.”

Over the years, observers who have studied Lampert’s classroom have found that students learn an unusual amount of math. Rather than forgetting algorithms, they retain and even understand them. One boy who began fifth grade declaring math to be his worst subject ended it able to solve multiplication, long division and fraction problems, not to mention simple multivariable equations. It’s hard to look at Lampert’s results without concluding that with the help of a great teacher, even Americans can become the so-called math people we don’t think we are.

Among math reformers, Lampert’s work gained attention. Her research was cited in the same N.C.T.M. standards documents that Takahashi later pored over. She was featured in Time magazine in 1989 and was retained by the producers of “Sesame Street” to help create the show “Square One Television,” aimed at making math accessible to children. Yet as her ideas took off, she began to see a problem. In Japan, she was influencing teachers she had never met, by way of the N.C.T.M. standards. But where she lived, in America, teachers had few opportunities for learning the methods she developed.

American institutions charged with training teachers in new approaches to math have proved largely unable to do it. At most education schools, the professors with the research budgets and deanships have little interest in the science of teaching. Indeed, when Lampert attended Harvard’s Graduate School of Education in the 1970s, she could find only one listing in the entire course catalog that used the word “teaching” in its title. (Today only 19 out of 231 courses include it.) Methods courses, meanwhile, are usually taught by the lowest ranks of professors — chronically underpaid, overworked and, ultimately, ineffective.

Without the right training, most teachers do not understand math well enough to teach it the way Lampert does. “Remember,” Lampert says, “American teachers are only a subset of Americans.” As graduates of American schools, they are no more likely to display numeracy than the rest of us. “I’m just not a math person,” Lampert says her education students would say with an apologetic shrug.

Consequently, the most powerful influence on teachers is the one most beyond our control. The sociologist Dan Lortie calls the phenomenon the apprenticeship of observation. **Teachers learn to teach primarily by recalling their memories of having been taught, an average of 13,000 hours of instruction over a typical childhood. The apprenticeship of observation exacerbates what the education scholar Suzanne Wilson calls education reform’s double bind. The very people who embody the problem — teachers — are also the ones charged with solving it.**

Lampert witnessed the effects of the double bind in 1986, a year after California announced its intention to adopt “teaching for understanding,” a style of math instruction similar to Lampert’s. A team of researchers that included Lampert’s husband, David Cohen, traveled to California to see how the teachers were doing as they began to put the reforms into practice. But after studying three dozen classrooms over four years, they found the new teaching simply wasn’t happening. Some of the failure could be explained by active resistance. One teacher deliberately replaced a new textbook’s problem-solving pages with the old worksheets he was accustomed to using.

Much more common, though, were teachers who wanted to change, and were willing to work hard to do it, but didn’t know how. Cohen observed one teacher, for example, who claimed to have incited a “revolution” in her classroom. But on closer inspection, her classroom had changed but not in the way California reformers intended it to. Instead of focusing on mathematical ideas, she inserted new activities into the traditional “I, We You” framework. The supposedly cooperative learning groups she used to replace her rows of desks, for example, seemed in practice less a tool to encourage discussion than a means to dismiss the class for lunch (this group can line up first, now that group, etc.).

And how could she have known to do anything different? Her principal praised her efforts, holding them up as an example for others. Official math-reform training did not help, either. Sometimes trainers offered patently bad information — failing to clarify, for example, that even though teachers were to elicit wrong answers from students, they still needed, eventually, to get to correct ones. Textbooks, too, barely changed, despite publishers' claims to the contrary.

With the Common Core, teachers are once more being asked to unlearn an old approach and learn an entirely new one, essentially on their own. Training is still weak and infrequent, and principals — who are no more skilled at math than their teachers — remain unprepared to offer support. Textbooks, once again, have received only surface adjustments, despite the shiny Common Core labels that decorate their covers. "To have a vendor say their product is Common Core is close to meaningless," says Phil Daro, an author of the math standards.

Left to their own devices, teachers are once again trying to incorporate new ideas into old scripts, often botching them in the process. One especially nonsensical result stems from the Common Core's suggestion that students not just find answers but also "illustrate and explain the calculation by using equations, rectangular arrays, and/or area models." The idea of utilizing arrays of dots makes sense in the hands of a skilled teacher, who can use them to help a student understand how multiplication actually works. For example, a teacher trying to explain multiplication might ask a student to first draw three rows of dots with two dots in each row and then imagine what the picture would look like with three or four or five dots in each row. Guiding the student through the exercise, the teacher could help her see that each march up the times table (3x2, 3x3, 3x4) just means adding another dot per row. But if a teacher doesn't use the dots to illustrate bigger ideas, they become just another meaningless exercise. Instead of memorizing familiar steps, students now practice even stranger rituals, like drawing dots only to count them or breaking simple addition problems into complicated forms (62+26, for example, must become 60+2+20+6) without understanding why. This can make for even poorer math students. "In the hands of unprepared teachers," Lampert says, "alternative algorithms are worse than just teaching them standard algorithms."

No wonder parents and some mathematicians denigrate the reforms as "fuzzy math." In the warped way untrained teachers interpret them, they are fuzzy.

When Akihiko Takahashi arrived in America, he was surprised to find how rarely teachers discussed their teaching methods. A year after he got to Chicago, he went to a one-day conference of teachers and mathematicians and was perplexed by the fact that the gathering occurred only twice a year. In Japan, meetings between math-education professors and teachers happened as a matter of course, even before the new American ideas arrived. More distressing to Takahashi was that American teachers had almost no opportunities to watch one another teach.

In Japan, teachers had always depended on jugyokenkyu, which translates literally as "lesson study," a set of practices that Japanese teachers use to hone their craft. A teacher first plans lessons, then teaches in front of an audience of students and other teachers along with at least one university observer. Then the observers talk with the teacher about what has just taken place. Each public lesson poses a hypothesis, a new idea about how to help children learn. And each discussion offers a chance to determine whether it worked. Without jugyokenkyu, it was no wonder the American teachers' work fell short of the model set by their best thinkers. Withoutjugyokenyku, Takahashi never would have learned to teach at all. Neither, certainly, would the rest of Japan's teachers.

The best discussions were the most microscopic, minute-by-minute recollections of what had occurred, with commentary. If the students were struggling to represent their subtractions visually, why not help them by, say, arranging tile blocks in groups of 10, a teacher would suggest. Or after a geometry lesson, someone might note the inherent challenge for children in seeing angles as not just corners of a triangle but as quantities — a more difficult stretch than making the same mental step for area. By the end, the teachers had learned not just how to teach the material from that day but also about math and the shape of students' thoughts and how to mold them.

If teachers weren't able to observe the methods firsthand, they could find textbooks, written by the leading instructors and focusing on the idea of allowing students to work on a single problem each day. Lesson study helped the textbook writers home in on the most productive problems. For example, if you are trying to decide on the best problem to teach children to subtract a one-digit number from a two-digit number using borrowing, or regrouping, you have many choices: 11 minus 2, 18 minus 9, etc. Yet from all these options, five of the six textbook companies in Japan converged on the same exact problem, Toshiakira Fujii, a professor of math education at Tokyo Gakugei University, told me. They determined that 13 minus 9 was the best. Other problems, it turned out, were likely to lead students to discover only one solution method. With 12 minus 3, for instance, the natural approach for most students was to take away 2 and then 1 (the subtraction-subtraction method). Very few would take 3 from 10 and then add back 2 (the subtraction-addition method).

But Japanese teachers knew that students were best served by understanding both methods. They used 13 minus 9 because, faced with that particular problem, students were equally likely to employ subtraction-subtraction (take away 3 to get 10, and then subtract the remaining 6 to get 4) as they were to use subtraction-addition (break 13 into 10 and 3, and then take 9 from 10 and add the remaining 1 and 3 to get 4). A teacher leading the "We" part of the lesson, when students shared their strategies, could do so with full confidence that both methods would emerge.

By 1995, when American researchers videotaped eighth-grade classrooms in the United States and Japan, Japanese schools had overwhelmingly traded the old "I, We, You" script for "You, Y'all, We." (American schools, meanwhile didn't look much different than they did before the reforms.) Japanese students had changed too. Participating in class, they spoke more often than Americans and had more to say. In fact, when Takahashi came to Chicago initially, the first thing he noticed was how uncomfortably silent all the classrooms were. One teacher must have said, "Shh!" a hundred times, he said. Later, when he took American visitors on tours of Japanese schools, he had to warn them about the noise from children talking, arguing, shrieking about the best way to solve problems. The research showed that Japanese students initiated the method for solving a problem in 40 percent of the lessons; Americans initiated 9 percent of the time. Similarly, 96 percent of American students' work fell into the category of "practice," while Japanese students spent only 41 percent of their time practicing. Almost half of Japanese students' time was spent doing work that the researchers termed "invent/think." (American students spent less than 1 percent of their time on it.) Even the equipment in classrooms reflected the focus on getting students to think. Whereas American teachers all used overhead projectors, allowing them to focus students' attention on the teacher's rules and equations, rather than their own, in Japan, the preferred device was a blackboard, allowing students to track the evolution of everyone's ideas.

Japanese schools are far from perfect. Though lesson study is pervasive in elementary and middle school, it is less so in high school, where the emphasis is on cramming for college entrance exams. As is true in the United States, lower-income students in Japan have recently been falling behind their peers, and people there worry about staying competitive on international tests. Yet while the United States regularly hovers in the middle of the pack or below on these tests, Japan scores at the top. And other countries now inching ahead of Japan imitate the jugyokenkyu approach. Some, like China, do this by drawing on their own native jugyokenkyu-style traditions(zuanyan jiaocai, or "studying teaching materials intensively," Chinese teachers call it). Others, including Singapore, adopt lesson study as a deliberate matter of government policy. Finland, meanwhile, made the shift by carving out time for teachers to spend learning. There, as in Japan, teachers teach for 600 or fewer hours each school year, leaving them ample time to prepare, revise and learn. By contrast, American teachers spend nearly 1,100 hours with little feedback.

It could be tempting to dismiss Japan's success as a cultural novelty, an unreproducible result of an affluent, homogeneous, and math-positive society. Perhaps the Japanese are simply the "math people" Americans aren't. Yet when I visited Japan, every teacher I spoke to told me a story that sounded distinctly American. "I used to hate math," an elementary-school teacher named Shinichiro Kurita said through a translator. "I couldn't calculate. I was slow. I was always at the bottom of the ladder, wondering why I had to memorize these equations." Like Takahashi, when he went to college and saw his instructors teaching differently, "it was an enlightenment."

Learning to teach the new way himself was not easy. "I had so much trouble," Kurita said. "I had absolutely no idea how to do it." He listened carefully for what Japanese teachers call children's twitters — mumbled nuggets of inchoate thoughts that teachers can mold into the fully formed concept they are trying to teach. And he worked hard on bansho, the term Japanese teachers use to describe the art of blackboard writing that helps students visualize the flow of ideas from problem to solution to broader mathematical principles. But for all his efforts, he said, "the children didn't twitter, and I couldn't write on the blackboard." Yet Kurita didn't give up — and he had resources to help him persevere. He went to study sessions with other teachers, watched as many public lessons as he could and spent time with his old professors. Eventually, as he learned more, his students started to do the same. Today Kurita is the head of the math department at Setagaya Elementary School in Tokyo, the position once held by Takahashi's mentor, Matsuyama.

Of all the lessons Japan has to offer the United States, the most important might be the belief in patience and the possibility of change. Japan, after all, was able to shift a country full of teachers to a new approach. Telling me his story, Kurita quoted what he described as an old Japanese saying about perseverance: "Sit on a stone for three years to accomplish anything." Admittedly, a tenacious commitment to improvement seems to be part of the Japanese national heritage, showing up among teachers, autoworkers, sushi chefs and tea-ceremony masters. Yet for his part, Akihiko Takahashi extends his optimism even to a cause that can sometimes seem hopeless —

the United States. After the great disappointment of moving here in 1991, he made a decision his colleagues back in Japan thought was strange. He decided to stay and try to help American teachers embrace the innovative ideas that reformers like Magdalene Lampert pioneered.

Today Takahashi lives in Chicago and holds a full-time job in the education department at DePaul University. (He also has a special appointment at his alma mater in Japan, where he and his wife frequently visit.) When it comes to transforming teaching in America, Takahashi sees promise in individual American schools that have decided to embrace lesson study. Some do this deliberately, working with Takahashi to transform the way they teach math. Others have built versions of lesson study without using that name. Sometimes these efforts turn out to be duds. When carefully implemented, though, they show promise. In one experiment in which more than 200 American teachers took part in lesson study, student achievement rose, as did teachers' math knowledge — two rare accomplishments.

Training teachers in a new way of thinking will take time, and American parents will need to be patient. In Japan, the transition did not happen overnight. When Takahashi began teaching in the new style, parents initially complained about the young instructor experimenting on their children. But his early explorations were confined to just a few lessons, giving him a chance to learn what he was doing and to bring the parents along too. He began sending home a monthly newsletter summarizing what the students had done in class and why. By his third year, he was sending out the newsletter every day. If they were going to support their children, and support Takahashi, the parents needed to know the new math as well. And over time, they learned.

To cure our innumeracy, we will have to accept that the traditional approach we take to teaching math — the one that can be mind-numbing, but also comfortingly familiar — does not work. We will have to come to see math not as a list of rules to be memorized but as a way of looking at the world that really makes sense.

The other shift Americans will have to make extends beyond just math. Across all school subjects, teachers receive a pale imitation of the preparation, support and tools they need. And across all subjects, the neglect shows in students' work. In addition to misunderstanding math, American students also, on average, write weakly, read poorly, think unscientifically and grasp history only superficially. Examining nearly 3,000 teachers in six school districts, the Bill & Melinda Gates Foundation recently found that nearly two-thirds scored less than "proficient" in the areas of "intellectual challenge" and "classroom discourse." Odds-defying individual teachers can be found in every state, but the overall picture is of a profession struggling to make the best of an impossible hand.

Most policies aimed at improving teaching conceive of the job not as a craft that needs to be taught but as a natural-born talent that teachers either decide to muster or don't possess. Instead of acknowledging that changes like the new math are something teachers must learn over time, we mandate them as "standards" that teachers are expected to simply "adopt." We shouldn't be surprised, then, that their students don't improve.

Here, too, the Japanese experience is telling. The teachers I met in Tokyo had changed not just their ideas about math; they also changed their whole conception of what it means to be a teacher. "The term 'teaching' came to mean something totally different to me," a teacher named Hideto Hirayama told me through a translator. It was more sophisticated, more challenging — and more rewarding. "The moment that a child changes, the moment that he understands something, is amazing, and this transition happens right before your eyes," he said. "It seems like my heart stops every day."

Elizabeth Green, the chief executive of Chalkbeat, is the author of "Building a Better Teacher," to be published by W. W. Norton next month, from which this article is adapted.