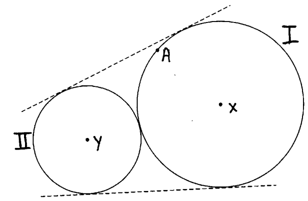
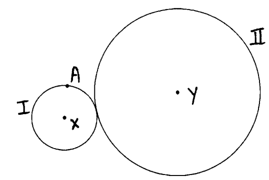
# The Three Tangent Circles Puzzle

There are six variations of this puzzle. In each case we start with two given tangent circles: Circle I (with center X) and Circle II (with center Y). The task is to find a compass and straightedge construction that locates the center, Z, of Circle III, such that Circle III is tangent to Circle I at point A as well as tangent to Circle II (at some unknown point, B). Further conditions are as follows:

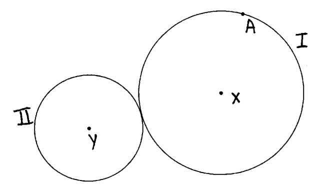
<u>Variation #1</u>: Circles I and II are *externally* tangent, point A is *inside* the double tangent line, and Circle I is *larger* than Circle II (as shown with the drawing at the right).



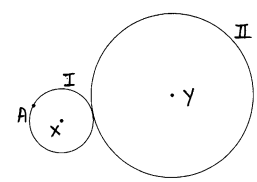

<u>Variation #2</u>: Circles I and II are *externally* tangent, point A is *inside* the double tangent line, and Circle I is *smaller* than Circle II (as shown with the drawing at the right).



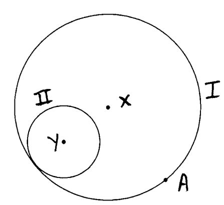

<u>Variation #3</u>: Circles I and II are *externally* tangent, point A is *outside* the double tangent line, and Circle I is *larger* than Circle II (as shown with the drawing at the right).



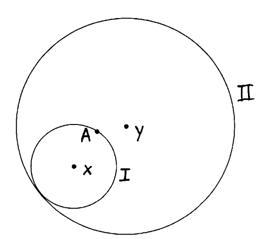

<u>Variation #4</u>: Circles I and II are *externally* tangent, point A is *outside* the double tangent line, and Circle I is *smaller* than Circle II (as shown with the drawing at the right).




<u>Variation #5</u>: Circles I and II are *internally* tangent, and Circle II is *inside* Circle I (as shown with the drawing at the right).




<u>Variation #6</u>: Circles I and II are *internally* tangent, and Circle I is *inside* Circle II (as shown with the drawing at the right).

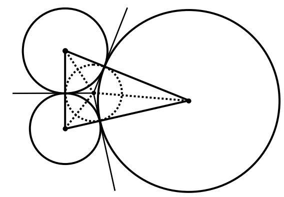


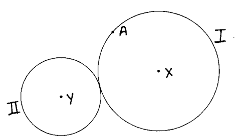
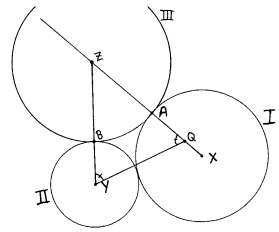
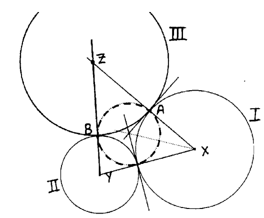
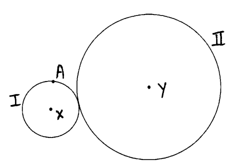
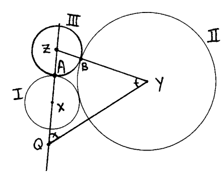
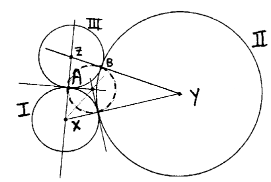
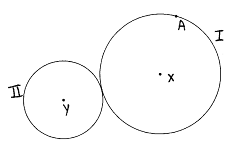
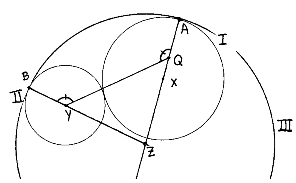
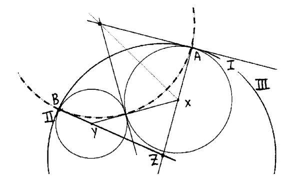
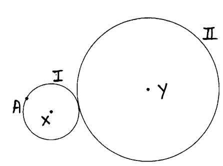
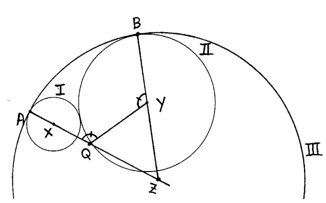
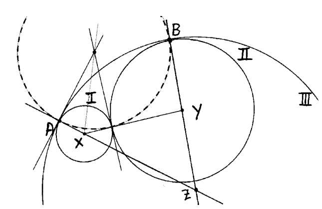
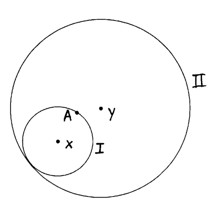
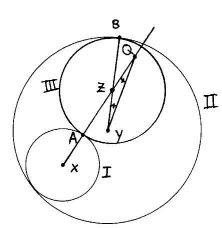
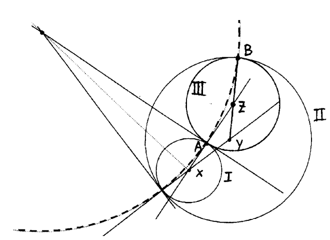
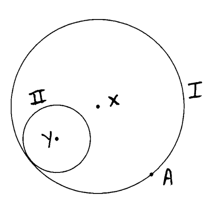
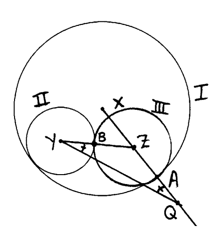
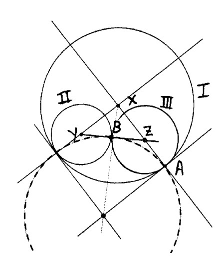

<u>Note</u>: Solutions are given on the next page.

# **Solutions** to the Three Tangent Circles Puzzle

<u>Method #1</u>: Draw AX. Locate point Q on line AX such that AQ equals the radius of Circle II. (You'll need to think carefully about whether Q should be placed toward X, or away from X.) Draw YQ. Copy $\angle AQY$ to $\angle QYB$. (Point B is the point of tangency between Circle II and Circle III.) Lines AX and BY intersect at point Z, the center of Circle III, which is tangent to Circles I and II. Note that $ZY \cong ZQ$ and $ZB \cong ZA$. This works for all six variations of the puzzle.

<u>Method #2</u>: This method makes use of the fact that if three circles are mutually tangent, then the in-circle of the triangle formed by the three centers passes through the three points of tangency (see drawing at right). The center of this in-circle is the meeting point of six lines: the triangle's three angle bisectors, and the three tangent lines (drawn at the circles' tangent points). Therefore we can find the desired third point of tangency by drawing two tangent lines (at right angles to Circle I's radius), and then drawing the triangle's in-circle (dotted in the drawings).



| | **Given Drawing** | **Method #1** | **Method #2** |
|---|---|---|---|
| **Variation #1** |  |  |  |
| **Variation #2** |  |  |  |
| **Variation #3** |  |  |  |
| **Variation #4** |  |  |  |
| **Variation #5** |  |  |  |
| **Variation #6** |  |  |  |