Summary of Key Concepts

Percents Unit (8^{th} Grade)

- <u>Formulas, algebra, and understanding</u>. Once again, we want to minimize the use of formulas. This is a unit that emphasizes mathematical thinking perhaps more than any other unit this year. Most textbooks show how to do these problems by setting up an algebraic equation. We will learn how to do this next year. For now, mathematical thinking will be our most powerful tool.
- <u>Calculators</u>. Calculators will be given out halfway through this unit. There are two basic rules for calculator usage: (1) Only use it for problems that can't be done in your head; and (2) in the space provided (on a worksheet or test), write down whatever you put into your calculator.
- <u>Facts and flashcards</u>. Much of the work in this unit is dependent upon familiarity with basic percent-to-fraction conversions (such as ¾ = 75%). There are 15 of them in total – all of them found at the start of the first worksheet in this unit. Students should make flashcards for any facts with which they are not completely confident.
  Example: What is 87.5% of 240? This translates this into: "What is $^7/_8$ of 240?"
- <u>The art of rephrasing the question</u>. Often, we are given a word problem, and we need to extract a simpler question from it that only has to with numbers. *This skill is critically important in order to be successful in this unit.*
  Example: Betty paid 8% tax on the purchase of her bicycle. What does she have to pay in tax if the bike cost $370? We rephrase this as: "What is 8% of $370?"
  Example: Last year, Jeff had 25% fewer vacation days than Susan. How many vacation days did Susan have if Jeff had 24? We rephrase this as: "24 is 25% less than what?"
- <u>A Trap to Avoid</u>. Going from 1500 up to 2400 is a 60% increase. So what percentage decrease is it going from 2400 down to 1500? We might think that it is a 60% decrease, but it isn't. This is because each problem has a different starting point (base).
- <u>Terminology</u>. We have three terms: *base* (starting point), *percentage*, and *destination* (ending point). In general, we take the *percentage* of a *base* to end up at the *destination*.

Here is a list of most types of problems from this unit:

- <u>Normal Percent Problem</u>. We are given the base and a percentage; we need to find the destination.
  Example: What is 61% of 450?
  Solution: $0.61 \cdot 450 \to$ <u>**274.5**</u>
- <u>Increase/Decrease (Inc/Dec) Problem</u>. We are given the base and a percentage increase or decrease, and we need to find the destination.
  Example: What is 2300 increased by 30%?
  Solution: There are two methods. (1) 30% of 2300 is 690, then we add $690 + 2300 =$ <u>**2990**</u>; or (2) We rephrase the question as: "What is *130% of* 2300" $\to 1.3 \cdot 2300 =$ <u>**2990**</u>.
  Example: What is 70 decreased by 20%?
  Solution: There are two methods. (1) 20% of 70 is 14, then we subtract $70 - 14 =$ <u>**56**</u>; or (2) We rephrase the question as: "What is *80% of* 70?" $\to 08. \cdot 70 =$ <u>**56**</u>.
- <u>Finding the Percentage</u>. We are given the base and the destination and need to find the percentage.
  Example: 210 is what percent of 280?
  Solution: *Think fractions!* $\frac{210}{280}$, which reduces to ¾, which is <u>**75%**</u>

- Finding the Percentage combined with Inc/Dec. We are asked to find the percentage of increase or decrease.

  Example: 280 is what percent more than 210?

  Solution: There are two methods.

  (1) We rephrase this to ask: "The amount of increase is what percent of the starting point (base)?" The amount of increase (or difference) is 70; starting point is 280.
  Answer = $\frac{\text{difference}}{\text{start}} = \frac{70}{210}$: **<u>33⅓%</u>**

  (2) Here's an easier method: we instead ask: "280 is what percent of 210?", which is $\frac{280}{210} = 1.\overline{3}$ = 133⅓%. Since 280 is 133⅓% of 210, we can also say that it is **<u>33⅓%</u>** more than 210.

  Example: 210 is what percent less than 280?

  Solution: Again, there are two methods.

  (1) We rephrase this to ask: "The amount of decrease is what percent of the starting point (base)?" The amount of decrease (or difference) is 70; starting point is 280.
  Answer = $\frac{\text{difference}}{\text{start}} = \frac{70}{280}$: **<u>25%</u>**

  (2) Or, again, we instead ask: "210 is what percent of 280?", which is $\frac{210}{280} \rightarrow$ 75%.
  Since 210 is 75% of 280, we can also say that it is **<u>25%</u>** less than 280.

- Reverse Problem. We are given the percentage and the destination, but need to find the base (starting point).

  Example: 560 is 35% of what number?

  Solution: This is a "reverse" question, so we must *divide*. Answer: 560 ÷ 0.35 → **<u>1600</u>**

- Reverse Problem combined with Inc/Dec. This is the trickiest problem. It requires that we completely rephrase the question.

  Example: 247 is 30% greater than what number?

  Solution: We rephrase this to: "247 is 130% of what number?", which then becomes: "247 is 1.3 times what number?" It is a reverse problem, so we need to divide.
  247 ÷ 1.3 = **<u>190</u>**

  Example: 648 is 40% less than what number?

  Solution: Rephrase this to: "648 is 60% of what number?", which then becomes: "648 is 0.6 times what number?" It is a reverse problem, so we need to divide.
  648 ÷ 0.6 = **<u>1080</u>**

- Exponential Growth. Mostly, this is done for bank accounts or for populations. There is a table at the back of the workbook that makes our work much easier.

  The Exponential Growth Formula $P = P_0(1 + r)^t$

  where P_0 ("P sub zero") is the initial amount, r is the percentage growth rate as a decimal, t is the time (i.e., number of years), P is the end amount after t years.

  Example: What is the balance in a savings account after 20 years at 5% interest if the initial deposit was $300?

  Solution: We use the formula $P = P_0(1+r)^t$. Now, using 20 for the value t, and 1.05 for the value for (1+r), and see that the table tells us that $(1+r)^t$ is 2.6533. Then we get the final balance by multiplying that number by the value of P_0, which is 300. This gives us an answer of <u>$795.99</u>. Alternatively, we could have used a calculator.