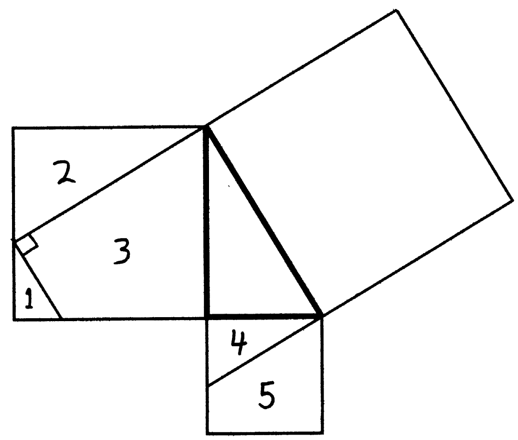
Six Proofs of the Pythagorean Theorem

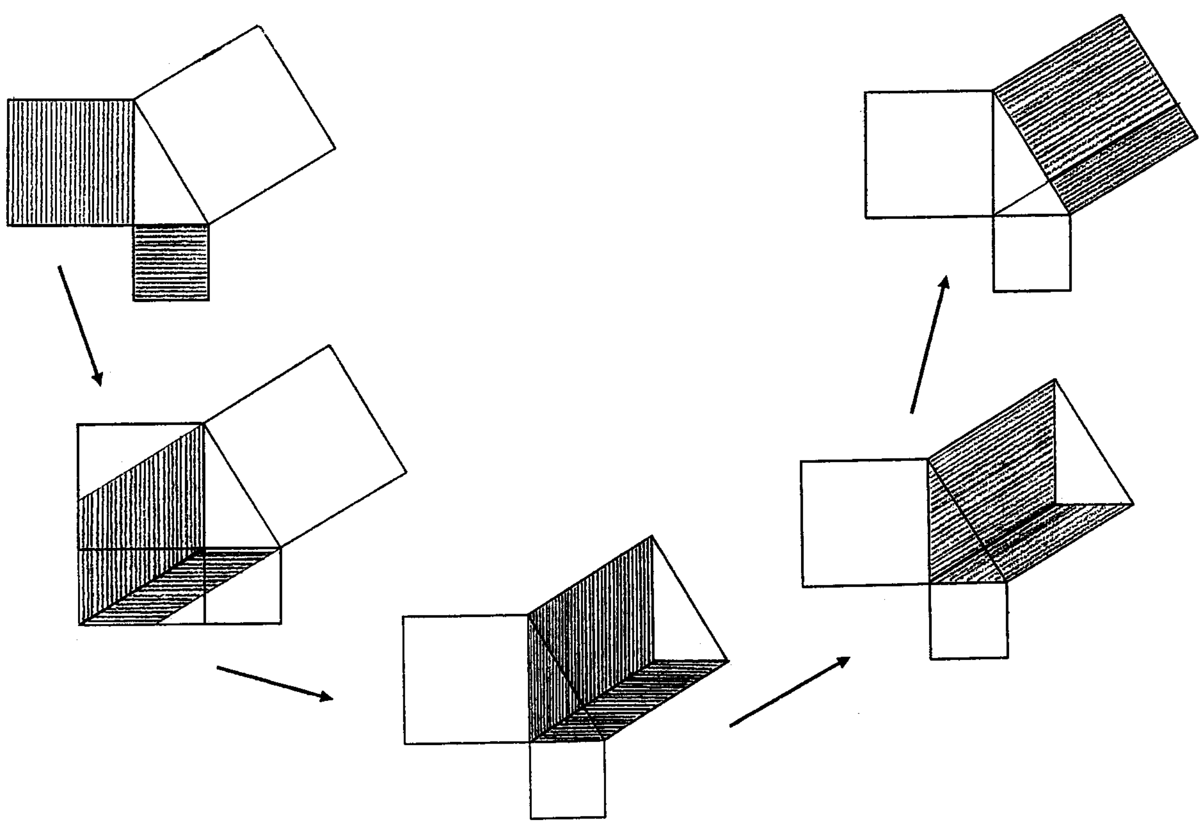
The idea here is to show that a proof doesn't have to be a two-column proof; to see that very different approaches can be taken to prove a given theorem; and to discuss the strengths and weaknesses of each proof.

1. *A paper cut-out puzzle proof*

If this was done in 7th grade (as it should be), then simply remind them of this "experiential" proof.
With any right scalene (non-isosceles) triangle, draw a square coming off each of the three sides. Extend two lines from the sides of the largest square so that they cut through the other two squares, in each case dividing the squares into a triangle and a trapezoid. Draw a line perpendicular to the line that divided the second largest square, by starting at the intersection point on the edge of that square. The second largest square has now been cut into three pieces: two triangles and a quadrilateral having two right angles. Cut the pieces (numbered 1 to 5 in the drawing) out of the two smaller squares. Place the 5 pieces on top of the large square so that they fit perfectly. This shows that the sum of the areas of the two smaller squares equals the area of the largest square.

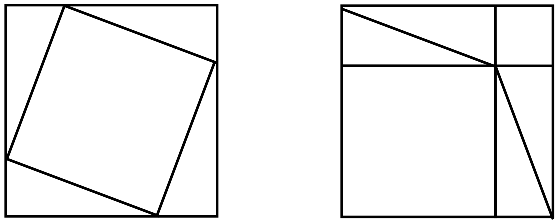


2. *Baravalle's proof using the Shear and Stretch.* (This should have been covered in 8th grade)

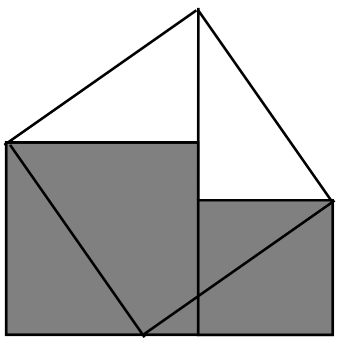


Six Proofs of the Pythagorean Theorem (Continued.)

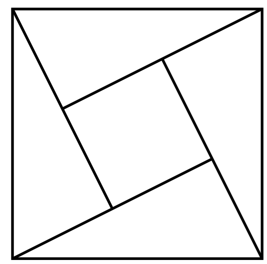
3. *Rearranging the Triangles.* This is, perhaps, similar to what Pythagoras did.



4. *From Tabit ibn Qorra* (ca. 870AD)
This proof becomes most obvious if you color in the small and medium-sized squares, and then visualize moving the triangles that are outside the largest square on top of the triangles that are inside the largest square. This shows that the areas of the two smaller squares combine to equal the area of the largest square.



5. *From Bhaskara* (ca. 1150AD) This proof requires a bit of algebra. The inner square's side has length (b-a), so its area is $(b-a)^2$. Set up the equation and simplify it.



$c^2 = (b-a)^2 + 4(\frac{1}{2}ab)$
$c^2 = b^2 - 2ab + a^2 + 2ab$
$\therefore \mathbf{c^2 = b^2 + a^2}$

6. *Proportion Proof.* This proof is found in many modern high school geometry books, and it is the only one of the six that doesn't involve squares or area.

$a/y = c/a$ therefore $a^2 = cy$
$b/x = c/b$ therefore $b^2 = cx$
$a^2 + b^2 = cy + cx$
$a^2 + b^2 = c(y + x)$
$\therefore \mathbf{a^2 + b^2 = c^2}$