Logical positivism

Logical positivism (later and more accurately called **logical empiricism**) is a school of philosophy that combines empiricism, the idea that observational evidence is indispensable for knowledge of the world, with a version of rationalism, the idea that our knowledge includes a component that is not derived from observation.

It grew from the discussions of the so-called "First Vienna Circle" that gathered at the Café Central before World War I. After the war Hans Hahn, a member of that early group, helped bring Moritz Schlick to Vienna. Schlick's Vienna Circle, along with Hans Reichenbach's Berlin Circle, propagated the new doctrines more widely in the 1920s and early 1930s. It was Otto Neurath's propaganda that made the movement self-conscious and more widely known. A 1929 pamphlet written by Neurath, Hahn, and Rudolf Carnap summarized the doctrines of the Vienna Circle at that time. These included especially: the opposition to all metaphysics, especially ontology and synthetic a priori propositions; the rejection of metaphysics not as wrong but as having no meaning; a criterion of meaning based on Ludwig Wittgenstein's early work; the idea that all knowledge should be codifiable in a single standard language of science; and above all the project of "rational reconstruction", in which ordinary-language concepts were gradually to be replaced by more precise equivalents in that standard language. In the early 1930s, the movement dispersed, mainly because of political upheaval and the untimely deaths of Hahn and Schlick. During this period of upheaval, Carnap proposed a replacement for the earlier doctrines in his "Logical Syntax of Language". This change of direction and the somewhat differing views of Reichenbach and others led to a consensus that the English name for the shared doctrinal platform, in its American exile from the late 1930s, should be "logical empiricism".

Origins

The chief influences on the early logical positivists were the positivist Ernst Mach and the young Ludwig Wittgenstein.

Mach's influence is most apparent in the logical positivists' persistent concern with metaphysics, the unity of science, and the interpretation of the theoretical terms of science, as well as the doctrines of reductionism and phenomenalism, later abandoned by many positivists.

Wittgenstein's *Tractatus* was a text of great importance for the positivists. The use of the tools of modern logic for linguistic reform, the conception of philosophy as a "critique of language," and the possibility of drawing a theoretically principled distinction between intelligible and nonsensical discourse were all appealing to the logical positivists. Many positivists adopted a correspondence theory of truth similar to that of the Tractatus, although some, like Otto Neurath, preferred a form of coherentism. Wittgenstein's influence is further evident in certain formulations of the verification principle. Compare, for example, *Proposition 4.024* of the *Tractatus*, where Wittgenstein asserts that we understand a proposition when we know what happens if it is true, with Schlick's assertion that "To state the circumstances under which a proposition is true is the same as stating its meaning".[1] The tractarian doctrine that the truths of logic are tautologies was widely held among the logical positivists. Wittgenstein also influenced the logical positivists' interpretation of probability. According to Neurath, not all the logical positivists liked the *Tractatus*; it was full of metaphysics.[2]

Contemporary developments in logic and the foundations of mathematics, especially Bertrand Russell and Alfred North Whitehead's monumental *Principia Mathematica*, impressed the more mathematically minded logical positivists such as Hans Hahn and Rudolf Carnap. "Language-planning" and syntactical techniques derived from these developments were used to defend logicism in the philosophy of mathematics and various reductionist theses. Russell's theory of types was employed to explosive effect in Carnap's early anti-metaphysical polemics.[3]

Immanuel Kant was something of a punching bag in many of the logical positivists' early debates, but his influence shows through. His doctrine of synthetic *a priori* truths was *the* view to overthrow, and his notion of the thing in itself commanded its fair share of attention. More positively, Kantian views about the nature of physical objects pervade the "protocol sentence" debate[4], and the positivists all shared somewhat Kantian views about the relationship between philosophy and science.[5]

Basic tenets

Although the logical positivists held a wide range of beliefs on many matters, they were all interested in science and skeptical of theology and metaphysics. Early on, most logical positivists believed that all knowledge is based on logical inference from simple "protocol sentences" grounded in observable facts. Many logical positivists supported forms of materialism, philosophical naturalism, and empiricism.

Perhaps the view for which the logical positivists are best known is the verifiability criterion of meaning, or verificationism. In one of its earlier and stronger formulations, this is the doctrine that a proposition is "cognitively meaningful" only if there is a finite procedure for conclusively determining whether it is true or false.[6] An intended consequence of this view, for most logical positivists, is that metaphysical, theological, and ethical statements fall short of this criterion, and so are not cognitively meaningful.[7] They distinguished

cognitive from other varieties of meaningfulness (e.g. emotive, expressive, figurative), and most authors concede that the non-cognitive statements of the history of philosophy possess some other kind of meaningfulness. The positive characterization of cognitive meaningfulness varies from author to author. It has been described as the property of having a truth value, corresponding to a possible state of affairs, naming a proposition, or being intelligible or understandable in the sense in which scientific statements are intelligible or understandable.[8]

Another characteristic feature of logical positivism is the commitment to "Unified Science"; that is, the development of a common language or, in Neurath's phrase, a "universal slang" in which all scientific propositions can be expressed.[9] The adequacy of proposals or fragments of proposals for such a language was often asserted on the basis of various "reductions" or "explications" of the terms of one special science to the terms of another, putatively more fundamental one. Sometimes these reductions took the form of set-theoretic manipulations of a handful of logically primitive concepts;[10] sometimes these reductions took the form of allegedly analytic or *a priori* deductive relationships.[11]. A number of publications over a period of thirty years would attempt to elucidate this concept.

Criticism and influences

Early critics of logical positivism said that its fundamental tenets could not themselves be formulated in a way that was clearly consistent. The verifiability criterion of meaning did not seem verifiable; but neither was it simply a logical tautology, since it had implications for the practice of science and the empirical truth of other statements. This presented severe problems for the logical consistency of the theory.[*citation needed*] Another problem was that, while positive existential claims ("there is at least one human being") and negative universals ("not all ravens are black") allow for clear methods of verification (find a human or a non-black raven), negative existential claims and positive universal claims do not allow for verification.

Universal claims could apparently never be verified: How can you tell that *all ravens are black*, unless you've hunted down every raven ever, including those in the past and future? This led to a great deal of work on induction, probability, and "confirmation", which combined verification and falsification.

Karl Popper, a well-known critic of logical positivism, published the book *Logik der Forschung* in 1934 (translated by himself as *The Logic of Scientific Discovery* published 1959). In it he presented an influential alternative to the verifiability criterion of meaning, defining scientific statements in terms of falsifiability. First, though, Popper's concern was not with distinguishing meaningful from meaningless statements, but distinguishing "scientific" from "metaphysical" statements. He did not hold that metaphysical statements must be meaningless; neither did he hold that a statement that in one century was "metaphysical" and unfalsifiable (like the ancient Greek philosophy about atoms), could not in another century become "falsifiable" and thus "scientific". About psychoanalysis he thought something similar: in his day it offered no method for falsification, and thus was not falsifiable and not scientific. However, he did not exclude it being *meaningful*, nor did he say psychoanalysts were necessarily "wrong" (it only couldn't be proven either way: that would have meant it was falsifiable), nor did he exclude that one day psychoanalysis could evolve into something falsifiable, and thus "scientific". He was, in general, more concerned with scientific practice than with the logical issues that troubled the positivists. Second, although Popper's philosophy of science enjoyed great popularity for some years, if his criterion is construed as an answer to the question the positivists were asking, it turns out to fail in exactly parallel ways. Negative existential claims ("there are no unicorns") and positive universals ("all ravens are black") can be falsified, but positive existential and negative universal claims cannot, although Popper thought himself these could be deemed as verifiable[12].

Logical positivists' response to the first criticism is that logical positivism is a philosophy of science, not an axiomatic system that can prove its own consistency (see Gödel's incompleteness theorem). Secondly, a theory of language and mathematical logic were created to answer what it really means to make statements like "all ravens are black".

A response to the second criticism was provided by A. J. Ayer in *Language, Truth and Logic*, in which he sets out the distinction between "strong" and "weak" verification. "A proposition is said to be verifiable, in the strong sense of the term, if, and only if, its truth could be conclusively established by experience." (Ayer 1946:50) It is this sense of verifiable that causes the problem of verification with negative existential claims and positive universal claims. However, the weak sense of verification states that a proposition is "verifiable... if it is possible for experience to render it probable" (ibid.). After establishing this distinction, Ayer goes on to claim that "no proposition, other than a tautology, can possibly be anything more than a probable hypothesis" (Ayer 1946:51), and therefore can only be subject to weak verification. This defense was controversial among logical positivists, some of whom stuck to strong verification, and claimed that general propositions were indeed nonsense.

Subsequent philosophy of science tends to make use of certain aspects of both of these approaches. W. V. O. Quine criticized the distinction between analytic and synthetic statements and the reduction of meaningful statements to immediate experience. Work by Thomas Kuhn has convinced many that it is not possible to provide truth conditions for science independent of its historical paradigm. But even this criticism was not

unknown to the logical positivists: Otto Neurath compared science to a boat which we must rebuild on the open sea.

Logical positivism was essential to the development of early analytic philosophy. It was disseminated throughout the European continent and, later, in American universities by the members of the Vienna Circle. A.J. Ayer is considered responsible for the spread of logical positivism to Britain. The term subsequently came to be almost interchangeable with "analytic philosophy" in the first half of the twentieth century. Logical positivism was immensely influential in the philosophy of language and represented the dominant philosophy of science between World War I and the Cold War. Many subsequent commentators on "logical positivism" have attributed to its proponents a greater unity of purpose and creed than they actually shared, overlooking the complex disagreements among the logical positivists themselves.

The **Vienna Circle**

The **Vienna Circle** (in German: der **Wiener Kreis**) was a group of philosophers who gathered around Moritz Schlick when he was called to the Vienna University in 1922, organized in a philosophical association named **Verein Ernst Mach** (**Ernst Mach Society**). Among its members were Moritz Schlick, chairman of the Ernst Mach Society, Gustav Bergmann, Rudolf Carnap, Herbert Feigl, Philipp Frank, Kurt Gödel, Hans Hahn, Victor Kraft, Karl Menger, Marcel Natkin, Otto Neurath, Olga Hahn-Neurath, Theodor Radakovic, Rose Rand and Friedrich Waismann. With the exception of Gödel, members of the Vienna Circle had a common attitude towards philosophy, characterized by two main beliefs: first, experience is the only source of knowledge; second, logical analysis performed with the help of symbolic logic is the preferred method for solving philosophical problems.

History of the Vienna Circle

The prehistory of the Vienna Circle began with meetings on the philosophy of science and epistemology from 1907 on, promoted by Philipp Frank, Hans Hahn and Otto Neurath.

Hans Hahn, the oldest of the three (1879-1934), was a mathematician. He received his degree in mathematics in 1902. Afterwards he studied under the direction of Ludwig Boltzmann in Vienna and David Hilbert, Felix Klein and Hermann Minkowski in Göttingen. In 1905 he received the Habilitation in mathematics. He taught at Innsbruck (1905-1906) and Vienna (from 1909).

Otto Neurath (1882-1945) studied sociology, economics and philosophy in Vienna and Berlin. From 1907 to 1914 he taught in Vienna at the Neuen Wiener Handelsakademie (Viennese Commercial Academy). Neurath married Olga, Hahn's sister, in 1911.

Philipp Frank, the youngest of the group (1884-1966), studied physics at Göttingen and Vienna with Ludwig Boltzmann, David Hilbert and Felix Klein. From 1912, he held the chair of theoretical physics in the German University in Prague.

Their meetings were held in Viennese coffeehouses from 1907 onward. Frank remembered:

> After 1910 there began in Vienna a movement which regarded Mach's positivist philosophy of science as having great importance for general intellectual life […] An attempt was made by a group of young men to retain the most essential points of Mach's positivism, especially his stand against the misuse of metaphysics in science. […] To this group belonged the mathematician H. Hahn, the political economist Otto Neurath, and the author of this book [i.e. Frank], at the time an instructor in theoretical physics in Vienna. […] We tried to supplement Mach's ideas by those of the French philosophy of science of Henri Poincaré and Pierre Duhem, and also to connect them with the investigations in logic of such authors as Couturat, Schröder, Hilbert, etc.
>
> – Uebel, Thomas, 2003, p.70).

Presumably the meetings stopped in 1912, when Frank went to Prague, where he held the chair of theoretical physics left vacant by Albert Einstein. Hahn left Vienna during World War I and returned in 1921. The following year Hahn, with the collaboration of Frank, arranged to bring into the group Moritz Schlick, who held the chair of philosophy of the inductive sciences at the University of Vienna. Schlick had already published his two main works *Raum und Zeit in die gegenwärtigen Physik* (*Space and Time in contemporary Physics*) in 1917 and *Allgemeine Erkenntnislehre* (*General Theory of Knowledge*) in 1918. A central work for the newly founded discussion group was the *Logisch-Philosophische Abhandlung* (Tractatus Logico-Philosophicus), published by Ludwig Wittgenstein in 1918.

Under the direction of Schlick, a new regular series of meetings began. In 1926 Schlick and Hahn arranged to bring in Rudolf Carnap at the University of Vienna. In 1928 the *Verein Ernst Mach* (Ernst Mach Society) was founded, with Schlick as chairman. In 1929 the Vienna Circle manifesto *Wissenschaftliche Weltauffassung. Der Wiener Kreis* (*The Scientific Conception of the World. The Vienna Circle*) was published. The pamphlet is

dedicated to Schlick, and its preface was signed by Hahn, Neurath and Carnap. In the appendix is a list of the members of the Vienna Circle.

The Vienna Circle was dispersed when the Nazi party came to power in Germany; many of its members emigrated to USA, where they taught in several universities. Schlick remained in Austria, but in 1936 he was killed by a Nazi sympathizer student in the University of Vienna.

The Vienna Circle manifesto

It states the scientific world-conception of the Vienna Circle, which is characterized "essentially by *two features*. *First* it is *empiricist and positivist*: there is knowledge only from experience […] *Second*, the scientific world-conception is marked by the application of a certain method, namely *logical analysis*." (*The Scientific Conception of the World. The Vienna Circle* in Sarkar, Sahotra, 1996, p. 331 – hereinafter *VC*).

Logical analysis is the method of clarification of philosophical problems; it makes an extensive use of symbolic logic and distinguishes the Vienna Circle empiricism from earlier versions. The task of philosophy lies in the clarification - through the method of logical analysis - of problems and assertions.

Logical analysis shows that there are two different kinds of statements; one kind includes statements reducible to simpler statements about the empirically given; the other kind includes statements which cannot be reduced to statements about experience and thus they are devoid of meaning. Metaphysical statements belong to this second kind and therefore they are meaningless. Hence many philosophical problems are rejected as pseudo-problems which arise from logical mistakes, while others are re-interpreted as empirical statements and thus becomes the subject of scientific inquiries.

One source of the logical mistakes that are at the origins of metaphysics is the ambiguity of natural language. "Ordinary language for instance uses the same part of speech, the substantive, for things ('apple') as well as for qualities ('hardness'), relations ('friendship'), and processes ('sleep'); therefore it misleads one into a thing-like conception of functional concepts" (*VC* p. 329). Another source of mistakes is "the notion that *thinking* can either lead to knowledge out of its own resources without using any empirical material, or at least arrive at new contents by an inference from given states of affair" (*VC* p. 330). The latter notion is typical in Kantian philosophy, according to which there are synthetic statements a priori that expand knowledge without using the experience. Synthetic knowledge a priori is rejected by the Vienna Circle. Mathematics, which at a first sight seems an example of necessarily valid synthetic knowledge derived from pure reason alone, has instead a tautological character, that is its statements are analytical statements, thus very different from Kantian synthetic statements. The only two kinds of statements accepted by the Vienna Circle are synthetic statements a posteriori (i.e. scientific statements) and analytic statements a priori (i.e. logical and mathematical statements).

However, the persistence of metaphysics is connected not only with logical mistakes but also with "social and economical struggles" (*VC* p. 339). Metaphysics and theology are allied to traditional social forms, while the group of people who "faces modern times, rejects these views and takes its stand on the ground of empirical sciences" (*VC* p. 339). Thus the struggle between metaphysics and scientific world-conception is not only a struggle between different kinds of philosophies, but it is also – and perhaps primarily – a struggle between different political, social and economical attitudes. Of course, as the manifesto itself acknowledged, "not every adherent of the scientific world-conception will be a fighter" (VC p. 339). Many historians of the Vienna Circle see in the latter sentence an implicit reference to a contrast between the so called 'left wing' of the Vienna Circle, mainly represented by Neurath and Carnap, and Moritz Schlick. The aim of the left wing was to facilitate the penetration of the scientific world-conception in "the forms of personal and public life, in education, upbringing, architecture, and the shaping of economic and social life" (*VC* p. 339-340). In contrast, Schlick was primarily interested in the theoretical study of science and philosophy. Perhaps the sentence "Some, glad of solitude, will lead a withdrawn existence on the icy slopes of logic" (*VC* p. 339) is an ironic reference to Schlick.

The manifesto lists Walter Dubislav, Josef Frank, Kurt Grelling, Hasso Härlen, Eino Kaila, Heinrich Loewy, F. P. Ramsey, Hans Reichenbach, Kurt Reidemeister, and Edgar Zilsel as "Those sympathetic to the Vienna Circle" and Albert Einstein, Bertrand Russell and Ludwig Wittgenstein as "Leading representatives of the scientific world-conception".

Unified science

The final goal pursued by the Vienna Circle was unified science, that is the construction of a "constitutive system" in which every legitimate statement is reduced to the concepts of lower level which refer directly to the given experience. "The endeavour is to link and harmonise the achievements of individual investigators in their various fields of science" (*VC* p. 328). From this aim follows the search for clarity, neatness, intersubjectivity, and for a neutral symbolic language that eliminates the problems arising from the ambiguity of natural language. The Vienna Circle published a collection, called *Einheitswissenschaft* (*Unified science*), edit by Rudolf Carnap, Philipp Frank, Hans Hahn, Otto Neurath, Joergen Joergensen (after Hahn's death) and Charles W. Morris (from 1938), whose aim was to present an unified vision of science. After the publication in Europe of seven monographs from 1933 to 1939, the collection was dismissed, because of the problems arising from the World

War II. In 1938 a new series of publications started in USA. It was the International Encyclopedia of Unified Science, an ambitious project never completed devoted to unified science. Only the first section *Foundations of the Unity of Sciences* was published; it contains two volumes for a total of twenty monographs published from 1938 to 1969. As remembered by Rudolf Carnap and Charles Morris in the *Preface* to the 1969 edition of the International Encyclopedia of Unified Science:

> The *Encyclopedia* was in origin the idea of Otto Neurath. It was meant as a manifestation of the unity of science movement […] Original plans for the Encyclopedia were ambitious. In addition to the two introductory volumes, there was to be a section on the methodology of the sciences, one on the existing state of the unification of sciences, and possibly a section on the application of the sciences. It was planned that the work in its entirety would comprise about twenty-six volumes (260 monographs)
>
> – *Foundations of the Unity of Sciences*, vol. 1, The University of Chicago Press, 1969, p. vii.

The well known Thomas Kuhn's work, *The Structure of Scientific Revolutions*, was published in this Encyclopedia in 1962, as the number two in the second volume.

The elimination of metaphysics

The attitude of Vienna Circle towards metaphysics is well expressed by Carnap in the article 'Überwindung der Metaphysik durch Logische Analyse der Sprache' in *Erkenntnis*, vol. 2, 1932 (English translation 'The Elimination of Metaphysics Through Logical Analysis of Language' in Sarkar, Sahotra, ed., *Logical empiricism at its peak: Schlick, Carnap, and Neurath*, New York : Garland Pub., 1996, pp. 10-31). A language – says Carnap – consists of a vocabulary, i.e. a set of meaningful words, and a syntax, i.e. a set of rules governing the formation of sentences from the words of the vocabulary. Pseudo-statements, i.e. sequences of words that at first sight resemble statements but in reality have no meaning, are formed in two ways: either meaningless words occur in them, or they are formed in an invalid syntactical way. According to Carnap, pseudo-statements of both kinds occur in metaphysics.

A word W has a meaning if two conditions are satisfied. First, the mode of the occurrence of W in its elementary sentence form (i.e. the simplest sentence form in which W is capable of occurring) must be fixed. Secondly, if W occurs in an elementary sentence S, it is necessary to give an answer to the following questions (that are – according to Carnap – equivalent formulation of the same question):

- (1.) What sentences is S deducible from, and what sentences are deducible from S?
- (2.) Under what conditions is S supposed to be true, and under what conditions false?
- (3.) How S is to verified?
- (4.) What is the meaning of S?

(Carnap, 'The Elimination of Metaphysics Through Logical Analysis of Language' in Sarkar, Sahotra, cit., p12)

An example offered by Carnap concerns the word 'arthropod'. The sentence form "the thing *x* is an arthropod" is an elementary sentence form that is derivable from "*x* is an animal", "*x* has a segmented body" and "*x* has jointed legs". Conversely, these sentences are derivable from "the thing *x* is an arthropod". Thus the meaning of the words 'arthropod' is determined.

According to Carnap, many words of metaphysics do not fulfil these requirements and thus they are meaningless. As an example, Carnap considers the word 'principle'. This word has a definite meaning, if the sentence "*x* is the principle of *y*" is supposed to be equivalent to the sentence "*y* exists by virtue of *x*" or "*y* arises out of *x*". The latter sentence is perfectly clear: *y* arises out of *x* when *x* is invariably followed by *y*, and the invariable association between *x* and *y* is empirically verifiable. But – says Carnap – metaphysicians are not satisfied with this interpretation of the meaning of 'principle'. They assert that no empirical relation between *x* and *y* can completely explain the meaning of "*x* is the principle of *y*", because there is something that cannot be grasped by means of the experience, something for which no empirical criterion can be specified. It is the lacking of any empirical criterion – says Carnap - that deprives of meaning the word 'principle' when it occurs in metaphysics. Therefore, metaphysical pseudo-statements such as "water is the principle of the world" or "the spirit is the principle of the world" are void of meaning because a meaningless word occurs in them.

However, there are pseudo-statements in which occur only meaningful words; these pseudo-statements are formed in a counter-syntactical way. An example is the word sequence "Caesar is a prime number"; every word has a definite meaning, but the sequence has no meaning. The problem is that "prime number" is a predicate of numbers, not a predicate of human beings. In the example the nonsense is evident; however, in natural language the rules of grammar do not prohibit the formation of analogous meaningless word sequences that are not so easily detectable. In the grammar of natural languages, every sequence of the kind "*x* is *y*", where *x* is a noun and *y* is a predicate, is acceptable. In fact, in the grammar there is no distinction between predicate which can be affirmed of human beings and predicate which can be affirmed of numbers. So "Caesar is a general" and "Caesar is a prime number" are both well-formed, in contrast for example with "Caesar is and", which is ill-formed. In a logically constructed language – says Carnap – a distinction between the various kinds of predicate

is specified, and pseudo-statements as "Caesar is a prime number" are ill-formed. Now, and this is the main point of Carnap's argument, metaphysical statements in which do not occur meaningless words, are indeed meaningless because they are formed in a way which is admissible in natural languages, but not in logically constructed languages. Carnap attempts to indicate the most frequent sources of errors from which metaphysical pseudo-statements can arise. One source of mistakes is the ambiguity of the verb 'to be', which is sometimes used as a copula ("I am hungry") and sometimes to designate existence ("I am"). The latter statement incorrectly suggests a predicative form, and thus it suggests that existence is a predicate. Only modern logic, with the introduction of an explicit sign to designate existence (the sign $\exists$), which occurs only in statements such as $\exists x P(x)$, never as a predicate, has showed that existence is not a predicate, and thus has revealed the logical error from which pseudo-statements such as "cogito, ergo sum" has aroused.

Another source of mistakes is type confusions, in which a predicate of a kind is used as a predicate of another kind. For example the pseudo-statements "we know the Nothing" is analogous to "we know the rain", but while the latter is well-formed, the former is ill-formed, at least in a logically constructed language, because 'Nothing' is incorrectly used as a noun. In a formal language, 'Nothing' only means $\neg \exists x$, such as "there is nothing which is outside", i.e. $\neg \exists x O(x)$, and thus 'Nothing' never occurs as a noun or as a predicate.

What is the role of metaphysics? According to Carnap, although metaphysics has not theoretical content, it has content indeed: metaphysical pseudo-statements express the attitude of a person towards life. Metaphysics is an art like lyrical poetry. The metaphysician, instead of using the medium of art, works with the medium of the theoretical; he confuses art with science, attitude towards life with knowledge, and thus produces an unsatisfactory and inadequate work. "Metaphysicians are musicians without musical ability" (Carnap, 'The Elimination of Metaphysics', in Sarkar, Sahotra, cit.,p. 30).

Intuitionism

In the philosophy of mathematics, **intuitionism**, or **neointuitionism** (opposed to preintuitionism), is an approach to mathematics as the constructive mental activity of humans. That is, mathematics does not consist of analytic activities wherein deep properties of existence are revealed and applied. Instead, logic and mathematics are the application of internally consistent methods to realize more complex mental constructs.

The fundamental distinguishing characteristic of intuitionism is its interpretation of what it means for a mathematical statement to be true. As the name suggests, in Brouwer's original intuitionism, the truth of a statement is taken to be equivalent to the mathematician being able to intuit the statement. The vagueness of the intuitionistic notion of truth often leads to misinterpretations about its meaning. Kleene formally defined intuitionistic truth from a realist position, however Brouwer would likely reject this formalization as meaningless, given his rejection of the realist/Platonist position. Intuitionistic truth therefore remains somewhat ill defined. Regardless of how it is interpreted, intuitionism does not equate the truth of a mathematical statement with its provability. However, because the intuitionistic notion of truth is more restrictive than that of classical mathematics, the intuitionist must reject some assumptions of classical logic to ensure that everything he/she proves is in fact intuitionistically true. This gives rise to intuitionistic logic.

To claim an object with certain properties exists, is, to an intuitionist, to claim to be able to construct a certain object with those properties. Any mathematical object is considered to be a product of a construction of a mind, and therefore, the existence of an object is equivalent to the possibility of its construction. This contrasts with the classical approach, which states that the existence of an entity can be proved by refuting its non-existence. For the intuitionist, this is not valid; the refutation of the non-existence does not mean that it is possible to find a *constructive* proof of existence. As such, intuitionism is a variety of mathematical constructivism; but it is not the only kind.

As well, to say *A* or *B*, to an intuitionist, is to claim that either *A* or *B* can be *proved*. In particular, the law of excluded middle, *A* or not *A*, is disallowed since one can construct, via Gödel's incompleteness theorems, a mathematical statement that can be neither proven nor disproved.

The interpretation of negation is also different. In classical logic, the negation of a statement asserts that the statement is *false*; to an intuitionist, it means the statement is *refutable* (i.e., that there is a proof that there is no proof of it). The asymmetry between a positive and negative statement becomes apparent. If a statement *P* is provable, then it is certainly impossible to prove that there is no proof of *P*; however, just because there is no proof that there is no proof of *P*, we cannot conclude from this absence that there *is* a proof of *P*. Thus *P* is a stronger statement than *not-not-P*.

Intuitionistic logic substitutes justification for truth in its logical calculus. The logical calculus preserves justification, rather than truth, across transformations yielding derived propositions. It has given philosophical support to several schools of philosophy, most notably the Anti-realism of Michael Dummett.

Intuitionism also rejects the abstraction of actual infinity; i.e., it does not consider as given objects infinite entities such as the set of all natural numbers or an arbitrary sequence of rational numbers. This requires the reconstruction of the foundations of set theory and calculus as constructivist set theory and constructivist analysis respectively.