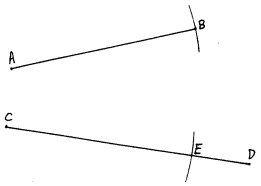
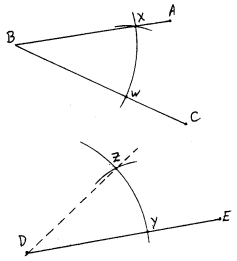
Basic Euclidean Constructions

Copying a Line Segment



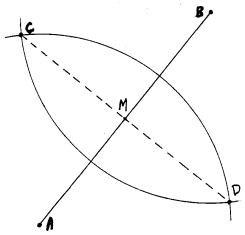
Instructions: *The intention is to copy line segment AB onto line CD*. Make sure that CD is longer than AB. Set the compass's width equal to AB. Put the needle of the compass on C, and mark an arc that passes through line CD at point E. CE is now equal in length to AB.

Copying an Angle



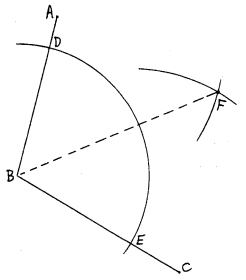
Instructions: *The intention is to copy angle ABC onto line DE.* Set the compass at a width that is a bit less than the shortest of the line segments AB, BC, and DE. Using this width of the compass, draw an arc with the needle at B that passes through both AB (at X) and BC (at W), and then draw an arc with the needle at D that passes through DE at Y. Place the needle at W and adjust the compass so that it reaches to X, and then draw a short arc through X. Keeping this width of the compass, draw an arc, with the needle at Y, that crosses through the previously drawn arc at point Z. Angle ZDY is now equal to angle ABC.

Bisecting a Line Segment (and construction of the perpendicular bisector).



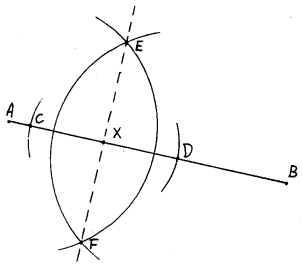
Instructions: *The intention is to bisect the line segment AB, and to draw a perpendicular bisector* through *it.* Set the compass width so that it is a bit more than half the length of AB. Using this compass width, draw two arcs, one with the needle at A and the other with the needle at B, so that they cross one another in two places – at points C and D. CD is the perpendicular bisector of AB, and M is the midpoint of AB. **Note:** This same technique is used to bisect an arc.

Bisecting an Angle



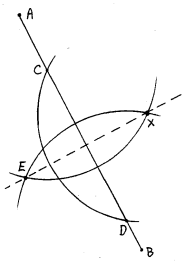
Instructions: *The intention is to bisect angle ABC.* Set the compass width a bit less than both AB and BC. Draw an arc, with the needle at B, that passes through AB at D, and passes through BC at E. Now draw two arcs, both with the same compass width, with the needle at D and then at E, so that the two arcs cross inside angle ABC at point F. The line BF is the desired bisector of the angle ABC. Notice that this will still work if the two arcs are made to intersect "outside" (in this case, to the left of) the angle.

Constructing a Perpendicular Line through a Point on that Line:



Instructions: *The intention is to construct a line perpendicular to AB that passes through X, which is a point on AB.* First, draw two arcs, each one using the same compass width and with the needle at X – one arc passing through AX at C and the other passing through XB at D. Now lengthen the compass somewhat and draw two long arcs – one with the needle at C and the other with the needle at D, such that they cross each other at points E and F. Line EF is the desired line; it passes through X and is perpendicular to AB.

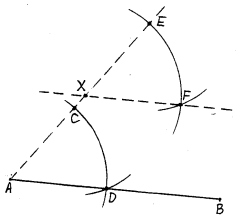
Constructing a Perpendicular Line through a Point *Not* on that Line:



Instructions: *The intention is to construct a line perpendicular to AB that passes through X, which is NOT on AB.* First, set the compass width a bit longer than the distance that X is from line AB and then draw an arc, with the needle at X, that passes through AB in two points, C and D. Now draw two long arcs, using the same compass width, one with the needle at C and the other with the needle at D. They should cross each other at X and at another point E, which is on the other side of AB from X. Line EX is the desired line – it passes through X and is perpendicular to AB.

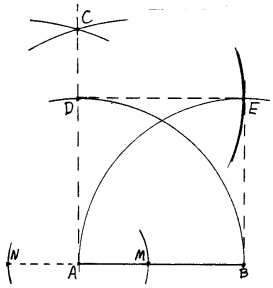
Constructing a Parallel Line

Instructions: *The intention is to construct a line that is parallel to AB and that passes through X, which is not on AB.* Draw line AX significantly beyond X. Set the compass width so that it is a bit shorter than AX, and draw an arc, with the needle at A, that passes through both AB at D and AX at C. Using that same compass width, draw an arc, with the needle at X, that passes through both the extended line AX (at E) and the line (not yet drawn) that passes through X and is parallel to AB. Now, adjusting the compass width, draw a short arc, with the needle at C, that passes through D, and then using the same compass width, draw an arc, with the needle at E that crosses, at point F, the arc that was drawn earlier that passed through E. The line XF is the desired line – it passes through X and is parallel to AB.



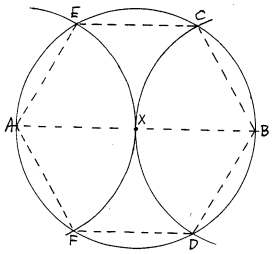
Constructing a Square, Given One Side.

Instructions: *The intention is to construct a square that has each side equal in length to AB.* Extend AB past A to N, and then mark point M on AB such that the length of NA is equal to the length of AM. Adjust the compass so that it is somewhat wider than AB and draw two arcs – one with the needle at N, and the other with the needle at M, so that they intersect vertically above A, at point C. Line AC is now perpendicular to AB. Set the compass width equal to AB and draw an arc, with the needle at A, so that it crosses line AC at point D. Using the same compass width, draw two more arcs**:** one that is horizontally to the right of D, with the needle at D, and a second arc that is above B, with the needle at B. These two arcs cross at point E. Finish the square by connecting the four points ABED.



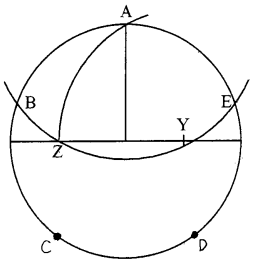
Constructing a Hexagon, Inside a Given Circle.

Instructions: *The intention is to construct a regular hexagon inside the given circle.* Draw diameter AB passing through the center of the circle, X. Then set the width of the compass equal to the radius of the circle, and draw one arc with the needle at B, which crosses the circle at points C and D, and another arc, with the needle at A, which crosses the circle at points E and F. The desired hexagon is AFDBCE.



Constructing a Pentagon, Inside a Given Circle.

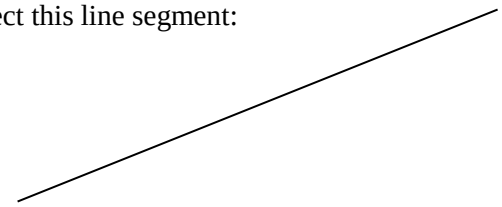
Instructions: *The intention is to construct a regular pentagon inside the given circle.* Draw a circle, and a diameter of that circle. Find the midpoint, Y, of the radius (Y is $^1/_4$ way along the diameter). Draw the perpendicular bisector of the diameter, which intersects the circle at point A. Placing the needle of the compass at Y, draw an arc through A to point Z on the diameter. The distance from A to Z is precisely the length of the sides of the desired pentagon. Place the needle of the compass at A, and draw an arc through Z that intersects the circle at points B and E. Points A, B and E are three of the points of the pentagon. Now, keeping the compass at the same width, place the needle at B and draw an arc that crosses the circle at C. Similarly, place the needle at E and draw an arc that crosses the circle at D. The sides of the pentagon are now drawn by connecting, in order, the points A, B, C, D, and E. With your compass check to see that the five sides have equal length.



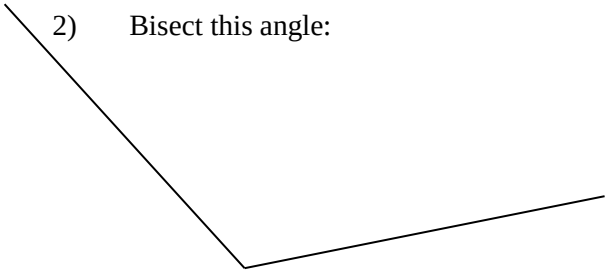
Euclidean Constructions: **Sheet #1**

Instructions: Use only a compass and a straight edge for each construction.

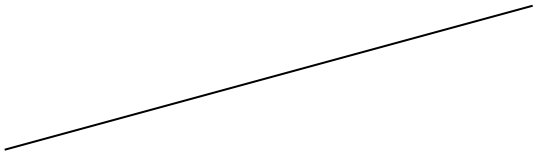
1) Bisect this line segment:



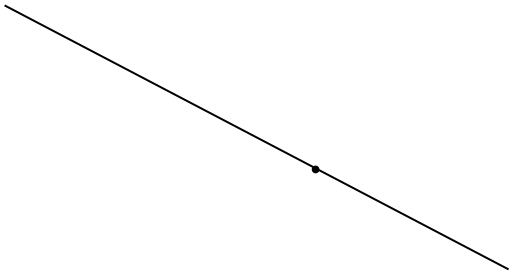
2) Bisect this angle:



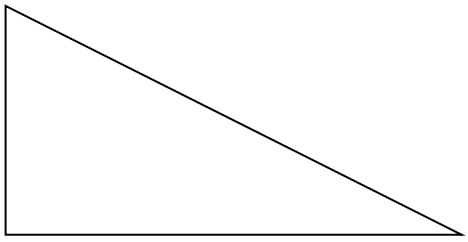
3) Draw a line through the point that is perpendicular to the line:



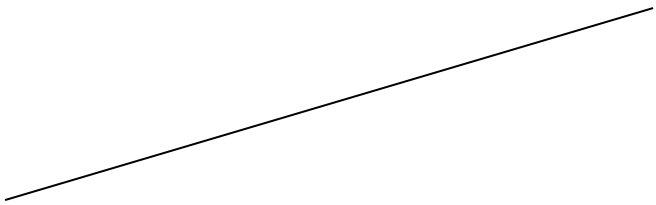
4) Draw a line through the point that is perpendicular to the line:



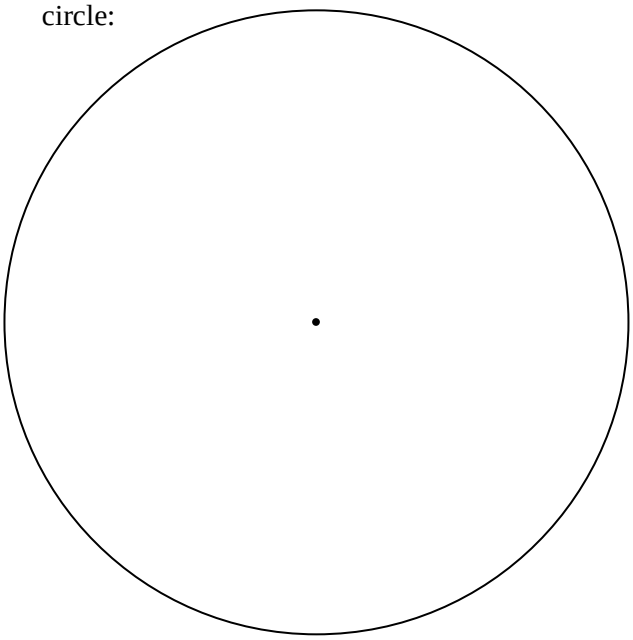
5) Construct a square onto the hypotenuse of this right triangle:



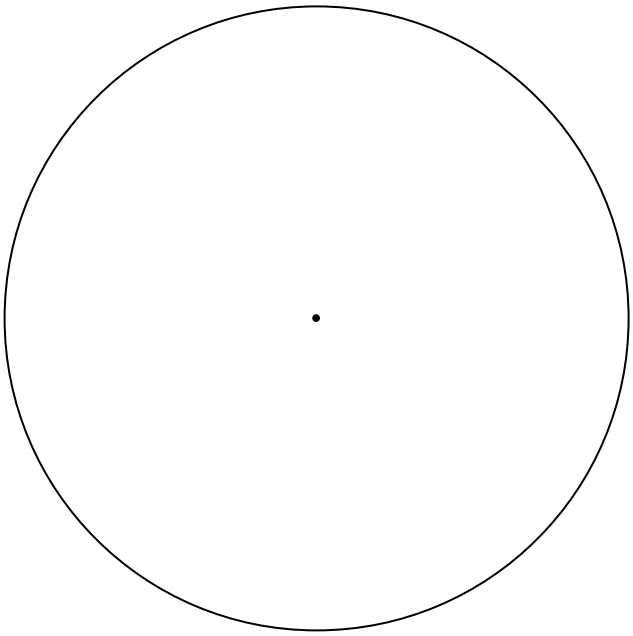
6) Draw a line through the point that is parallel to the line:



7) Construct a regular hexagon inside this circle:



8) Construct a pentagon inside this circle:



Instructions: Describe, in general terms, how you could do each construction. If you get stuck on one, then move to the next one and come back to it later.

9) The division of an angle into four equal parts.

10) The division of an angle into three equal parts. (The trisection of an angle.)

11) The construction of a 12-gon.

12) The construction of a 10-gon.

13) The construction of a 15-gon.

14) The construction of a 7-gon.

15) The construction of a 17-gon.

16) Finding the center of a given circle.

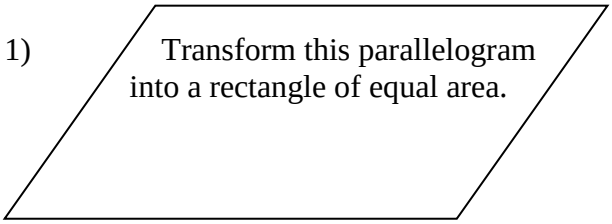
17) The division of a line into 7 equal parts.

18) The construction of a line tangent to a given circle (where the center of the circle is given) through a given point outside the circle.

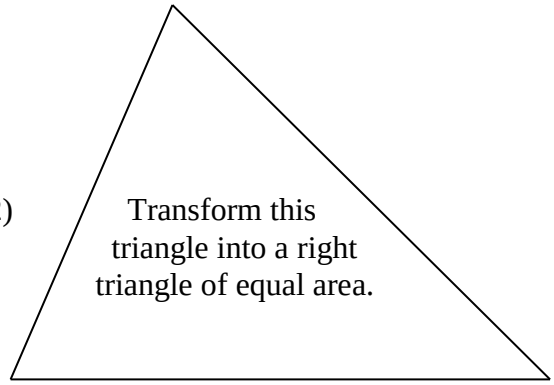
Euclidean Constructions: **Sheet #2**

Instructions: Use only a compass and a straight edge for each transformation.

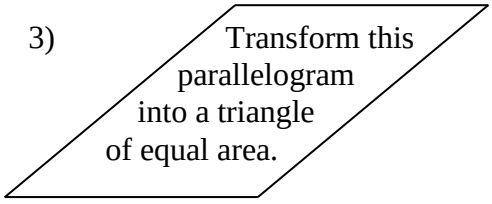
1) Transform this parallelogram into a rectangle of equal area.



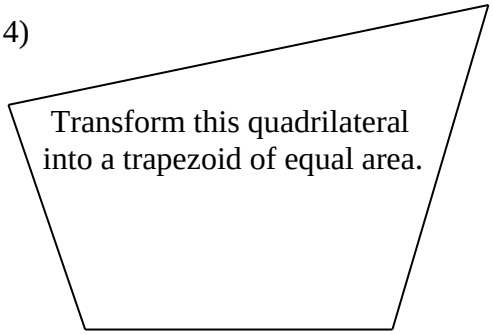
2) Transform this triangle into a right triangle of equal area.



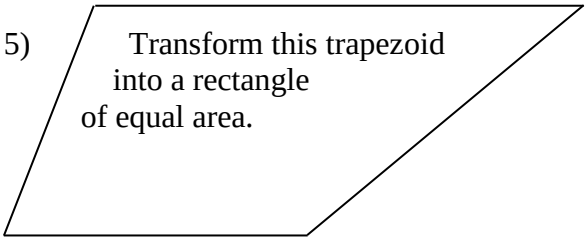
3) Transform this parallelogram into a triangle of equal area.



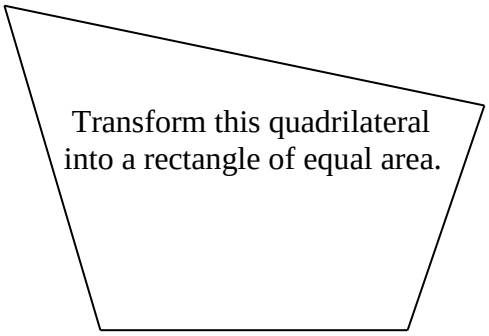
4) Transform this quadrilateral into a trapezoid of equal area.



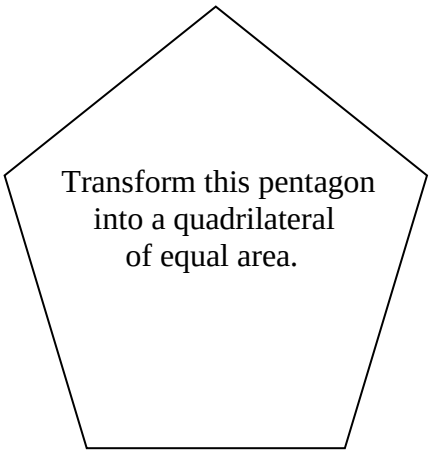
5) Transform this trapezoid into a rectangle of equal area.



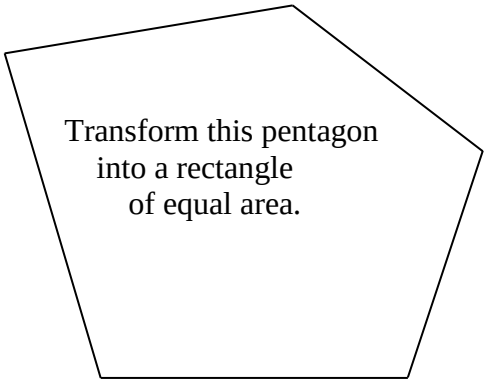
6) Transform this quadrilateral into a rectangle of equal area.



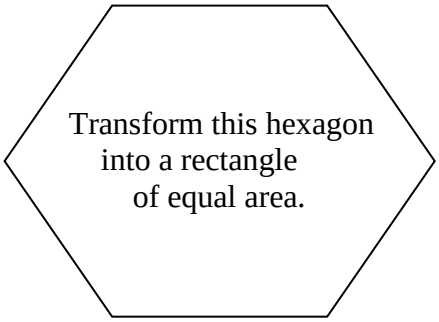
7) Transform this pentagon into a quadrilateral of equal area.



8) Transform this pentagon into a rectangle of equal area.



9) Transform this hexagon into a rectangle of equal area.



10) *Challenge!* Transform this rectangle into a square of equal area.