How fast do the planets move on their orbits?

We can calculate the relationship of the speed of one planet compared with the others, using Kepler's third law.

We start with the third law: $\frac{P_1^2}{P_2^2} = \frac{R_1^3}{R_2^3}$

Assuming a circular orbit, the circumference of the orbit is $2 \cdot \pi \cdot r$. Therefore, the speeds of the two planets can be written as $V_1 = \frac{2\pi R_1}{P_1} \rightarrow P_1 = \frac{2\pi R_1}{V_1}$

and $V_2 = \frac{2\pi R_2}{P_2} \rightarrow P_2 = \frac{2\pi R_2}{V_2}$

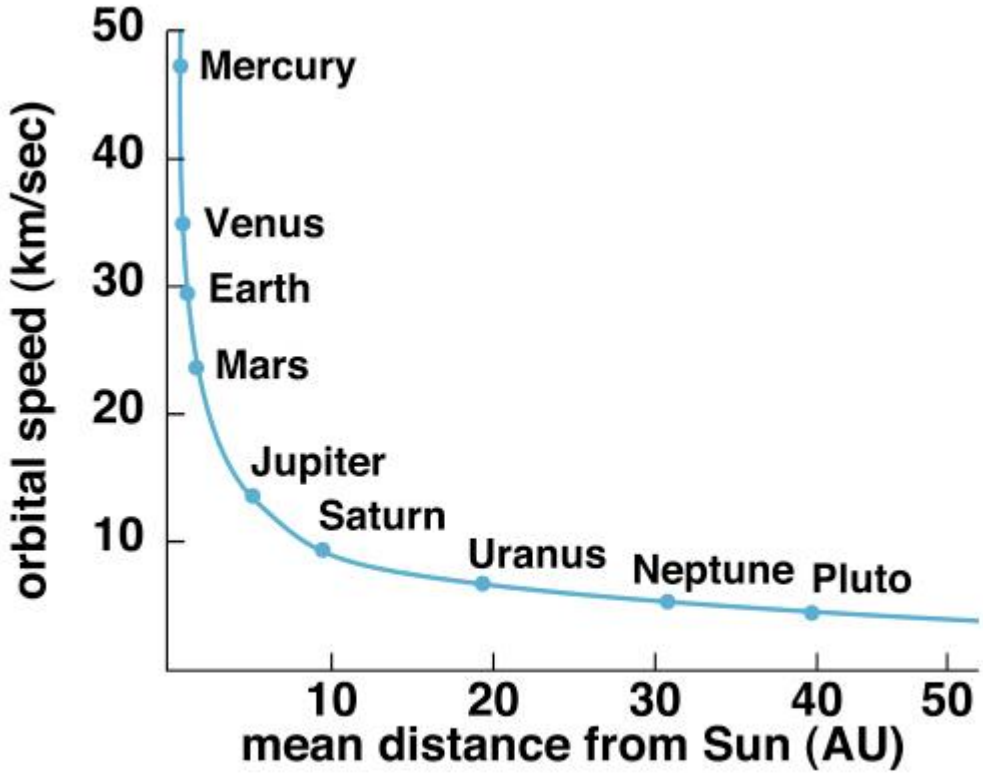
Subbing these P values into the third law eventually yields: $\frac{V_2^2}{V_1^2} = \frac{R_1}{R_2} \rightarrow \frac{V_2}{V_1} = \frac{\sqrt{R_1}}{\sqrt{R_2}}$

or $R_2 \cdot V_2^2 = R_1 \cdot V_1^2 \rightarrow V_2\sqrt{R_2} = V_1\sqrt{R_1}$

Therefore we can express this relationship between a planet's speed and its average distance from the sun in one of two ways:

- The speed is inversely proportional to the square root of the distance.
- The distance is inversely proportional to the square of the speed.

That is, the speed goes inversely with the square root of the distance from the sun.
Note that the earth is moving at about 30 km/sec, or about 67,000 miles per hour! (from Bennett et al., The Essential Cosmic Perspective)



(b)

(From an email I wrote to Beth Moger Jan 26, 2013.)

I just tried to derive the law that you spoke about: that the speed of a planet is inversely proportional to the square of its mean distance from the sun.

Instead I got the reverse: that the speed of a planet is inversely proportional to the square root of its mean distance from the sun. The inverse square root law!

I figured that I made a mistake, so I looked it up, and apparently I am correct. See this website: http://ircamera.as.arizona.edu/NatSci102/NatSci102/text/planetspeed.htm.

I like my proof better, but that doesn't matter.

So yes, this does seem odd. I liked your logic – that this should follow the inverse square law. E.g., if a planet were 9 times further from the sun, then it would have 1/81 the gravitation, and should therefore have 1/81 the speed. However, the truth seems instead to be that if a planet were 9 times further from the sun, then it would have 1/3 the speed. A big difference!

I think this can partially be explained by this fact: the centripetal force needed to keep an object moving in uniform circular motion is $F_c = m \cdot v^2 / r$. I again show through manipulation of equations that the "inverse square root law" is valid, but it still seems surprising to me.

What do you think?