8th Grade Stereometry and Loci Lesson Plans

February 2008

Comments:

- Stereometry is the study of 3-D solids, which includes the Platonic and Archimedean solids.
- Loci is the study of 2-D curves, which includes the conic sections.
- Most of the contents of these topics are explained in detail my book *MS Math Curriculum*.
- Ideally, there should be two math main lessons in 8th grade. One would be Number Bases and Loci, and the other would be Stereometry and Mensuration. Idea here is that that number bases and mensuration are more "head oriented" and they are units in my workbook, while Loci and stereometry are more artistic and imaginative.
- If there is only one math main lesson, then it could be Stereometry and Mensuration (and Loci would be dropped), or it could be Stereometry and Loci, and then something from the 8th grade math curriculum might need to be dropped due to lack of time.
- These lesson plans have been written from the perspective that stereometry and loci may very well be done in separate main lessons.
- The plan is based upon a three-week main lesson, again where each topic occupies roughly half of the main lesson. Therefore each day below is actually half a main lesson (although generally, I have spent more time on stereometry than on loci). I have written it as an 12-day plan, thinking that a couple days may be missed during the three weeks, or that on a couple of the days the main lesson's other topic might take the entire main lesson time for the day.

Stereometry Day #1

- Course expectations. Include what I will be grading on. Each day, homework should be on your desk at the start of main lesson.
- Why study this? The students need to be convinced that this ML is important.
  - Plato's study of mathematics included**:** Arithmetic, Geometry, Astronomy, and Stereometry.
  - Few people have the opportunity to study stereometry and loci – you are quite lucky!
  - Nobody will ever expect you to know this material – hardly anyone knows it. So why are we teaching it?
  - Employers today are looking for people who can problem solve and think creatively. (Roger Loeb's story.) The kind of imaginative thinking in this main lesson, will help to develop your spatial imagination, and help to develop your ability to think creatively.
  - The work you will be doing is an exercise in exactness and care.
  - Tell a story that helps to spark their interest.
- Clay: Practice making only a cube.
- Meditation: on the Cube → Octahedron transformation.
- Mention equipment that is needed: Compass, ruler, box, etc.
- HW: Practice the cube→octa meditation

Stereometry Day #2

- Give instructions on how to do paper models; hand out "Tips" sheet.
- Questions: What is the least # of edges for a polygon? What is least # of faces for a polyhedron?
- Terms: polygon, polyhedron, hexahedron, tetrahedron, prism, bipyramid, vertex (vertices), face.
- Clay: Cube to Octahedron
- Challenge question: How many nets are there for the cube?
- HW: Cube in paper

Stereometry Day #3

- Before class have students draw all of the cube nets on the board.
- Terms: Equilateral, equiangular, regular, anti-prism, trapezohedron.
- Clay: Redo cube→Octa
- Meditation: Cube→dodeca by adding roofs
- HW: Practice the cube→dodeca meditation

Stereometry Day #4

- Show all of the paper models in the Cube→Octa transformation.
- The Platonic Solids are the regular solids. So of all of the solids that we've seen so far, which ones are the Platonics (i.e. regular)?
- Review how to make equilateral triangle in order to make tetrahedron.
- Clay: Cube→Dodeca by adding roofs
- HW: Tetra in Paper (5" edges); What are all of the tetra nets?

Stereometry Day #5

- Before class have students draw all of the tetra nets on the board.
- Show pennies on a desk being pushed together to form hexagonal pattern. Explain that since circles fall naturally into a hexagonal pattern, my question was what the 3-D equivalent – i.e. what pattern do spheres fall into?
- Have the students (as a class) come up with what the characteristics are of the Platonic solids:
    - Faces are regular.
    - Faces are identical.
    - Vertices are identical.
    - Dihedral angles are equal.
        - (students may not come up with the last one, and may come up with extra ones.)
- Clay catch-up day, or perhaps no clay on this day.
- Do group area efficiency problem from group worksheet #3 of Mensuration.
- HW: Octahedron in Paper (3½" edges); What are all of the Octahedron nets (challenge)?

Stereometry Day #6

- Before class have students draw all of the octahedron nets on the board.
- Review:
    - The types of solids that we have learned about.
    - The characteristics of the Platonic solids; intro to dihedral angle.
    - The 3 questions that have emerged from the puzzle:
        - What do you get when packed tennis balls expand to full up the space?
        - What shape do water balloons take?
        - What is the most efficient way that space can be enclosed (and close-pack)?
- The idea of duality; the tetra and octa in a cube model.
- There are 43380 distinct nets for the dodecahedron (the same number as for the icosahedron).
- Meditation: Dodeca→Icosa
- Clay catch-up day, or perhaps no clay on this day.
- HW: Dodecahedron in Paper (due in two days); practice dodeca→icosa transformation

Stereometry Day #7

- Show the 5 paper models of the Dodeca→Icosa transformation.
- The Archimedean solids.
- Clay: Dual of a tetrahedron by pushing in points.
- Meditation: Cube→Rhombic-D transformation by growing pyramids.
- HW: Practice the cube→rhombic-D meditation

Stereometry Day #8

- Proof that there are only five Platonic solids.
- Show the paper rhombic-D; show paper models of pushing in points of a tetrahedron.
- What are the properties of the Archimedean solids.
- Clay: Cube→Rhombic-D transformation by growing pyramids.
- Meditation: Stretching a cube to form a rhombicuboctahedron.
- HW: Icosahedron in Paper 2½" edges (due in two days); practice the cube stretching meditation.

Stereometry Day #9

- Show paper rhombicuboctahedron, and the snub cube.
- In Groups: Do the transformation worksheet.
- Properties of the Archimedean duals.
- Plato's Academy. Above the door: "*Let no one unversed in geometry enter here"*.
  - Above the door of the Academy was written "*Let no one unversed in geometry enter here*".
  - In his book *Timaeus*, Plato said that when God created the universe, he first created the element of *fire*, for which he used the tetrahedron. Then he created the element *air*, for which he used the octahedron. He then created *water* using the icosahedron, and then used the hexahedron (cube) to create the *earth* element. Lastly, he used the dodecahedron to create the *life* element, or *quintessence*.

Stereometry Day #10

- Display paper models of all of the Archimedean solids and their duals.
- HW: Final project in Paper (due on last day of main lesson)

Stereometry Day #11

- Kepler's Universe.
  - In 1596 he published the book, *Mysterium Cosmographicum* in which he stated his hypothesis on the movement of the planets.
    - The order was: sphere (Mercury), octahedron, sphere (Venus), icosahedron, sphere (Earth), dodecahedron, sphere (Mars), tetrahedron, sphere (Jupiter), cube, sphere (Saturn).
  - In 1609, Kepler published his remarkable (and correct) laws of planetary motion, which included the fact that the planets travel along elliptical paths around the sun.
- Orthagonal views. Show how a rhombic dodecahedron can appear to be a cube.
- Discover Euler's Formula in groups

Stereometry Day #12

- The tennis ball demonstration, and close-packing – one solid from each group close packs.
    - The β-tetrakaidecahedron has faces 57% pentagonal, 29% hexagonal, 14% quad., which closely matches the occurrence in nature (fat cells, soap bubbles, etc.), which is 67% pentagonal, 22% hexagonal, 10½% quad, 0.4% heptagonal.
- Give the inner-tube problem.

Loci Day #1

- The idea of a treasure problem.
- Loci is the study of 2-D curves.
- By very specific about requirements for doing drawings.
    - Color and shading-in on each page is used to highlight the important part of the drawing.
    - Use sharp pencil and compass (sand paper).
    - Graded mostly on accuracy.
- Make sure that everyone has proper equipment.
- HW: Plates no. 1,2 &3 (A circle, 2 parallel lines, 2 concentric circles)

Loci Day #2

- In Groups: Discover what the curves are for plates no.4, 5 & 6 (Perpendicular bisector, 2 angle bisectors, a parabola).
- HW: Finish drawings for plates no. 4&5; think about plate no.6 (parabola)

Loci Day #3

- Have students give ideas about how to do parabola drawing, with just a compass and straightedge.
- HW: Question to think about: How can we find a point that is exactly a given distance away from both the fence (directrix) and the tree (focus)?

Loci Day #4

- Have students share ideas with class from yesterday's ending question.
- Instruct how to do parabola drawing; given students different distances between point and line.
- HW: Do plate no. 6; think about how the parabola changes as the focus (point) moves away from the directrix (fence). (Give different distances from tree to fence: 2, 4, 6, or 8cm.)

Loci Day #5

- Have one student for each distance (focus to directrix) line up in front of the class to show what happens as the focus moves away from, or closer to, the directrix (for the parabola drawing).
- In Groups: figure out how to do plate no.7 (ellipse); assign different groups to different distances (focus to directrix).
- HW: Do plate no.7; think about how the ellipse changes as the focus moves away from the directrix. (Fence radius = 8cm. Distances from tree to fence can be 1, 2, 4, 6cm.)

Loci Day #6

- Have one student for each distance (focus to directrix) line up in front of the class to show what happens as the focus moves away from, or closer to, the directrix (for the ellipse drawing).
- In Groups: figure out how to do plate no.8 (hyperbola); assign different groups to different distances (focus to directix).
- HW: Do plate no.8. (Fence radius = 5cm. Distances from tree to fence can be 1, 2, 4, 6cm.)
    - Think about how the hyperbola changes as the focus moves away from the directrix.

Loci Day #7

- Have one student for each distance (focus to directrix) line up in front of the class to show what happens as the focus moves away from, or closer to, the directrix (for the hyperbola drawing).
- Conic sections from cones of clay, including the transformation process as the cutting plane rotates, thereby showing the transformation of ellipse→parabola→hyperbola. The parabola as the magical instant between an ellipse and a hyperbola.
- HW: Meditate on conic section transformation of ellipse→parabola→hyperbola.

Loci Day #8

- Review yesterday: Conic sections from cones of clay, especially the transformation of ellipse→parabola→hyperbola. The parabola as the magical instant between an ellipse and a hyperbola.

Alternative Definitions

- *An ellipse is the locus of points such that* ***the sum of the distances*** *to the two focal points is constant*.
- *A hyperbola is the locus of points such that* ***the difference of the distances*** *to the two focal points is constant*.

- HW: Drawing that shows the *Movement of the Focus to the outside of the Directrix Circle.*

Loci Day #9

- Cassini curves.
  - For 8½ by 11inch paper, use C value (the constant product) of 64 (for each group).
  - The following table shows pairs of values that multiply to 64. Although these values are the same for all f values, some points won't work for certain f values; and some f values will require more values.
    Distance to focal point #1: 8 9 10 11 12 14 16 18 20
    Distance to focal point #2: 8 7.11 6.4 5.82 5.33 4.57 4 3.56 3.2
  - Give each group different f values (distance b/t the focal points in cm) of 8, 11.3, 15, 16, 17, and 20.

c (constant product)	f (distance between focal points, cm)	Shape of Curve	$X_{out} = \frac{\sqrt{f^2+4c} - f}{2}$ (Focal point to outside of curve)	$X_{in} = \frac{f - \sqrt{f^2-4c}}{2}$ (Focal point to inside of curve)
64	8 cm	Oval	4.94 cm	n/a
64	11.3 ($f = \sqrt{2C}$)	Flat Oval	4.14	n/a
64	15	Indented Oval	3.47	n/a
64	16 ($f = 2\sqrt{C}$)	Lemniscate	3.31	8 cm
64	17	Two Eggs	3.17	5.63cm
64	20	Two Eggs	2.81	4 cm

- Double cone of light demonstration.
- HW: Drawing: Turning the directrix circle inside-out.

Loci Day #10

- Finish Cassini curve drawing.
- Have one student for each f value line up in front of the class to show what happens as the two foci move apart (for the Cassini curve drawing).
- HW: Freehand drawing of the transformation of the Cassini curve.

Loci Day #11

- Go outside and move the Cassini curves.
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