

## 12<sup>th</sup> Grade Assignment – Week #25

### Individual Work

- Read Chapter Three of *Logicomix* before Tuesday's group meeting.
- Read Hersch's essay on Kant before Thursday's group meeting. To help you with some of the terms found in the reading, here are some notes (which I also covered in the lecture):
  - Idealism. Originating with Plato, Idealism is the philosophy that ideas form the basis of the essence of reality.
  - Rationalism. Rationalism is the view that knowledge is primarily gained through reasoning. The original rationalists were Descartes, Spinoza and Leibniz.
  - Empiricism. Empiricism asserts that knowledge comes primarily from sensory experience. Therefore, experiments and empirical data are critical for gaining proper knowledge.
  - Epistemology: The branch of philosophy concerned with the nature and scope of knowledge. It questions what knowledge is and how it is acquired. It addresses the question: *How do I come to know something is true?*
  - Metaphysics: The fundamental nature of being, existence, and the world. It addresses the questions: *What is there? What is it like?*
  - Ontology is the philosophical study of the nature of being, existence, or reality. Traditionally part of metaphysics, ontology deals with questions concerning what entities exist or can be said to exist, and how such entities can be grouped. (Not to be confused with ontogeny.)
  - A posteriori knowledge. Knowledge that is gained through experience or empirical evidence (e.g., "Cats have four legs").
  - A priori knowledge. Knowledge arrived at independent of experience. (e.g., "All bachelors are unmarried"). You don't have to examine the way things are in the physical world. You don't have to do any science. For Kant there were two kinds of a priori knowledge:
    - Analytic a priori (Descartes's "necessary deduction"). Conclusions of this sort are reached through logical deduction, such as mathematical proofs.
    - Synthetic a priori (Descartes's "evident intuition"). We can know some things are true purely through our intuition. Kant had 3 types of intuition, all of which are universal (completely objective). (See the below question.)

### Group Assignments:

for Tuesday.

- Discuss the *Logicomix* reading (Chapter #3)
  1. Give one word to describe Frege.
  2. What was Cantor obsessed with?
  3. How did BR find out that Gauss was upset at him?
  4. According to Poincaré, what does Hilbert think math is?
  5. In Hilbert's speech, he speaks of the "new mathematics".  
Fill in the blanks:
    - (a) He says that axioms (postulates) are the starting point of the logical process, and that: "we must abandon \_\_\_\_."
    - (b) With a mathematical proof... "it can be executed by \_\_\_\_."
- How do you think this "Great Story" gets resolved in 1931? Do you think PM will become the new foundation for math?
- *Puzzle: Sum of Two Squares – Part I.*
  - Choose any whole number less than 200. Find as many ways as possible to express your number as the sum of two square numbers. (E.g., If we choose 170, we can write it as  $1^2 + 13^2$  or as  $7^2 + 11^2$ .) Spend about 5 or 10 minutes with everyone trying a few numbers, and then report back to the group their findings.
  - Next, look at the table of *Sum of Two Squares* found at the end of this document. The question now is: How can we predict whether a given number can be expressed as the sum of two squares, or not?

for Thursday.

- **Hersch's Essay on Kant.**

Explain or discuss each of the following statements.

(Page numbers are found at the top of the page of the reading.)

1. (p130) What are Kant's three types of intuition (synthetic a priori)?
2. (p133) Which of the three great foundationalist gurus (Russell, Hilbert, Brouwer) were Kantians?
3. (p133) How did Kant justify the existence of God?
4. (p130) What is meant by: "Is metaphysics at all possible?"
5. (p132) "Physics doesn't prefer Euclid to non-Euclid." Explain.
6. (p135) What is the Euclid myth? Was it a problem for Kant? How is it a problem for his philosophy today?
7. (p135) In *Alice in Wonderland*, the Cheshire cat slowly disappears, but his grin remains. This ties into this statement: "The trouble with today's Platonism is that it gives up God, but wants to keep mathematics a thought in the mind of God." Explain what this means.
8. (p135) What is the dominant philosophy today?
9. (p136) How is mathematics an "embarrassing counter-example"?  
How does Mathematics play a special role in "the battle between rationalism and empiricism"?
10. (p136) "For many years the accepted assumptions in science have been materialism in ontology, empiricism in epistemology." Explain what this means.
11. Was Kant an empiricist or a rationalist?

## Numbers as the Sum of Two Squares (up to 442)

(0 is the first square number; Numbers that are missing cannot be expressed as the sum of two squares.)

<b>0</b> = $0^2 + 0^2$	<b>121</b> = $11^2 + 0^2$	<b>269</b> = $10^2 + 13^2$	<b>421</b> = $14^2 + 15^2$
<b>1</b> = $0^2 + 1^2$	<b>122</b> = $1^2 + 11^2$	<b>272</b> = $4^2 + 16^2$	<b>424</b> = $10^2 + 18^2$
<b>2</b> = $1^2 + 1^2$	<b>125</b> = $2^2 + 11^2$ ; $5^2 + 10^2$	<b>274</b> = $7^2 + 15^2$	<b>425</b> = $5^2 + 20^2$ ; $8^2 + 19^2$ ;
<b>4</b> = $2^2 + 0^2$	<b>128</b> = $8^2 + 8^2$	<b>277</b> = $9^2 + 14^2$	$13^2 + 16^2$
<b>5</b> = $1^2 + 2^2$	<b>130</b> = $3^2 + 11^2$ ; $7^2 + 9^2$	<b>281</b> = $5^2 + 16^2$	<b>433</b> = $12^2 + 17^2$
<b>8</b> = $2^2 + 2^2$	<b>136</b> = $6^2 + 10^2$	<b>288</b> = $12^2 + 12^2$	<b>436</b> = $6^2 + 20^2$
<b>9</b> = $3^2 + 0^2$	<b>137</b> = $4^2 + 11^2$	<b>289</b> = $17^2 + 0^2$ , $8^2 + 15^2$	<b>441</b> = $21^2 + 0^2$
<b>10</b> = $1^2 + 3^2$	<b>144</b> = $12^2 + 0^2$	<b>290</b> = $1^2 + 17^2$ ; $11^2 + 13^2$	<b>442</b> = $1^2 + 21^2$ ; $9^2 + 19^2$
<b>13</b> = $2^2 + 3^2$	<b>145</b> = $1^2 + 12^2$ ; $8^2 + 9^2$	<b>292</b> = $6^2 + 16^2$	
<b>16</b> = $4^2 + 0^2$	<b>146</b> = $5^2 + 11^2$	<b>293</b> = $2^2 + 17^2$	
<b>17</b> = $1^2 + 4^2$	<b>148</b> = $2^2 + 12^2$	<b>296</b> = $10^2 + 14^2$	
<b>18</b> = $3^2 + 3^2$	<b>149</b> = $7^2 + 10^2$	<b>298</b> = $3^2 + 17^2$	
<b>20</b> = $2^2 + 4^2$	<b>153</b> = $3^2 + 12^2$	<b>305</b> = $4^2 + 17^2$ ; $7^2 + 16^2$	
<b>25</b> = $5^2 + 0^2$ , $3^2 + 4^2$	<b>157</b> = $6^2 + 11^2$	<b>306</b> = $9^2 + 15^2$	
<b>26</b> = $1^2 + 5^2$	<b>160</b> = $4^2 + 12^2$	<b>313</b> = $12^2 + 13^2$	
<b>29</b> = $2^2 + 5^2$	<b>162</b> = $9^2 + 9^2$	<b>314</b> = $5^2 + 17^2$	
<b>32</b> = $4^2 + 4^2$	<b>164</b> = $8^2 + 10^2$	<b>317</b> = $11^2 + 14^2$	
<b>34</b> = $3^2 + 5^2$	<b>169</b> = $13^2 + 0^2$ , $5^2 + 12^2$	<b>320</b> = $8^2 + 16^2$	
<b>36</b> = $6^2 + 0^2$	<b>170</b> = $1^2 + 13^2$ ; $7^2 + 11^2$	<b>324</b> = $18^2 + 0^2$	
<b>37</b> = $1^2 + 6^2$	<b>173</b> = $2^2 + 13^2$	<b>325</b> = $1^2 + 18^2$ ; $6^2 + 17^2$ ;	
<b>40</b> = $2^2 + 6^2$	<b>178</b> = $3^2 + 13^2$	$10^2 + 15^2$	
<b>41</b> = $4^2 + 5^2$	<b>180</b> = $6^2 + 12^2$	<b>328</b> = $2^2 + 18^2$	
<b>45</b> = $3^2 + 6^2$	<b>181</b> = $9^2 + 10^2$	<b>333</b> = $3^2 + 18^2$	
<b>49</b> = $7^2 + 0^2$	<b>185</b> = $4^2 + 13^2$ ; $8^2 + 11^2$	<b>337</b> = $9^2 + 16^2$	
<b>50</b> = $1^2 + 7^2$ ; $5^2 + 5^2$	<b>193</b> = $7^2 + 12^2$	<b>338</b> = $7^2 + 17^2$ ; $13^2 + 13^2$	
<b>52</b> = $4^2 + 6^2$	<b>194</b> = $5^2 + 13^2$	<b>340</b> = $4^2 + 18^2$ ; $12^2 + 14^2$	
<b>53</b> = $2^2 + 7^2$	<b>196</b> = $14^2 + 0^2$	<b>346</b> = $11^2 + 15^2$	
<b>58</b> = $3^2 + 7^2$	<b>197</b> = $1^2 + 14^2$	<b>349</b> = $5^2 + 18^2$	
<b>61</b> = $5^2 + 6^2$	<b>200</b> = $2^2 + 14^2$ ; $10^2 + 10^2$	<b>353</b> = $8^2 + 17^2$	
<b>64</b> = $8^2 + 0^2$	<b>202</b> = $9^2 + 11^2$	<b>356</b> = $10^2 + 16^2$	
<b>65</b> = $1^2 + 8^2$ ; $4^2 + 7^2$	<b>205</b> = $3^2 + 14^2$ ; $6^2 + 13^2$	<b>360</b> = $6^2 + 18^2$	
<b>68</b> = $2^2 + 8^2$	<b>208</b> = $8^2 + 12^2$	<b>361</b> = $19^2 + 0^2$	
<b>72</b> = $6^2 + 6^2$	<b>212</b> = $4^2 + 14^2$	<b>362</b> = $1^2 + 19^2$	
<b>73</b> = $3^2 + 8^2$	<b>218</b> = $7^2 + 13^2$	<b>365</b> = $2^2 + 19^2$ ; $13^2 + 14^2$	
<b>74</b> = $5^2 + 7^2$	<b>221</b> = $5^2 + 14^2$ ; $10^2 + 11^2$	<b>369</b> = $12^2 + 15^2$	
<b>80</b> = $4^2 + 8^2$	<b>225</b> = $15^2 + 0^2$ , $9^2 + 12^2$	<b>370</b> = $3^2 + 19^2$ ; $9^2 + 17^2$	
<b>81</b> = $9^2 + 0^2$	<b>226</b> = $1^2 + 15^2$	<b>373</b> = $7^2 + 18^2$	
<b>82</b> = $1^2 + 9^2$	<b>229</b> = $2^2 + 15^2$	<b>377</b> = $4^2 + 19^2$ ; $11^2 + 16^2$	
<b>85</b> = $2^2 + 9^2$ ; $6^2 + 7^2$	<b>232</b> = $6^2 + 14^2$	<b>386</b> = $5^2 + 19^2$	
<b>89</b> = $5^2 + 8^2$	<b>233</b> = $8^2 + 13^2$	<b>388</b> = $8^2 + 18^2$	
<b>90</b> = $3^2 + 9^2$	<b>234</b> = $3^2 + 15^2$	<b>389</b> = $10^2 + 17^2$	
<b>97</b> = $4^2 + 9^2$	<b>241</b> = $4^2 + 15^2$	<b>392</b> = $14^2 + 14^2$	
<b>98</b> = $7^2 + 7^2$	<b>242</b> = $11^2 + 11^2$	<b>394</b> = $13^2 + 15^2$	
<b>100</b> = $10^2 + 0^2$ , $6^2 + 8^2$	<b>244</b> = $10^2 + 12^2$	<b>397</b> = $6^2 + 19^2$	
<b>101</b> = $1^2 + 10^2$	<b>245</b> = $7^2 + 14^2$	<b>400</b> = $20^2 + 0^2$ , $12^2 + 16^2$	
<b>104</b> = $2^2 + 10^2$	<b>250</b> = $5^2 + 15^2$ ; $9^2 + 13^2$	<b>401</b> = $1^2 + 20^2$	
<b>106</b> = $5^2 + 9^2$	<b>256</b> = $16^2 + 0^2$	<b>404</b> = $2^2 + 20^2$	
<b>109</b> = $3^2 + 10^2$	<b>257</b> = $1^2 + 16^2$	<b>405</b> = $9^2 + 18^2$	
<b>113</b> = $7^2 + 8^2$	<b>260</b> = $2^2 + 16^2$ ; $8^2 + 14^2$	<b>409</b> = $3^2 + 20^2$	
<b>116</b> = $4^2 + 10^2$	<b>261</b> = $6^2 + 15^2$	<b>410</b> = $7^2 + 19^2$ ; $11^2 + 17^2$	
<b>117</b> = $6^2 + 9^2$	<b>265</b> = $3^2 + 16^2$ ; $11^2 + 12^2$	<b>416</b> = $4^2 + 20^2$	

*The first number that can be expressed in 4 ways is...*

**1105** =  $4^2 + 33^2$ ;  $9^2 + 32^2$ ;  
 $12^2 + 31^2$ ;  $23^2 + 24^2$