Tutorial Session Notes
Grade 9
Quarter #4 (Week 25-32)

<u>About these notes</u>:
- These notes are primarily for those who are acting as the tutor — either a parent or a class teacher.
- During the second year of JYMA, Mr. Messner (our JYMA tutor) kept scribbled records of some of the proceedings of his Friday tutorial sessions. These notes are a reconstruction of those scribbles.
- These lessons were often a spontaneous collaboration with the students present and incorporate their questions. In the process of clarifying details, examples occasionally stepped beyond the skills presented in lectures. This is not an ideal script, but only an offering of possible tutorial activities.
- In order to support those who are acting as the tutor for their child or a class, I am sharing these notes with those who are acting as the tutor.
- Of course, these tutorial sessions are also an opportunity for the students to ask their tutor questions.
- If you are acting as the tutor, it may be helpful to read the section of the JYMA handbook titled "The Role of the Tutor".

Week #25

- Review briefly how to simplify square roots, including the rules…
  $\sqrt{\frac{A}{B}} \Leftrightarrow \frac{\sqrt{A}}{\sqrt{B}}$, $\sqrt{A}\sqrt{B} \Leftrightarrow \sqrt{AB}$ …and the need to rationalize denominators.
- Simplify: $\sqrt{\frac{4}{9}} \rightarrow \frac{2}{3}$, $\sqrt{\frac{7}{16}} \rightarrow \frac{\sqrt{7}}{4}$, $\sqrt{\frac{3}{5}} \rightarrow \frac{\sqrt{15}}{5}$
- Note that Lecture 2 had a solution of $x = -3 \pm \sqrt{12}$ which simplifies to $x = -3 \pm 2\sqrt{3}$.
- The Quadratic Formula Problem Set 3 Q11 …
  $\left(x - \frac{1}{3}\right)^2 = \frac{5}{9} \rightarrow \frac{1 \pm \sqrt{5}}{3}$
- Solve: $x^2 + 6x + 1 = 0 \rightarrow -3 \pm 2\sqrt{2}$
- Solve: $2x^2 + 5x - 3 = 0 \rightarrow \frac{1}{2}, -3$
  Accomplish this by factoring, and then again by completing the square. Guide students into understanding how to handle quadratic equations when the coefficient of x^2 is not 1 ($a \neq 1$).
- Simplify: $\sqrt{45} \rightarrow 3\sqrt{5}$

Week #26

- Solve: $6x^2 + 5x - 4 = 0$ → $\frac{1}{2}, -\frac{4}{3}$
- Solve: $x^2 + 4x - 8 = 0$ → $-2 \pm 2\sqrt{3}$
- Solve for **P**: $\frac{S\boldsymbol{P}R}{I} - N = G$ → $\boldsymbol{P} = \frac{IG + IN}{SR}$
- Solve for **E**: $S(N + \boldsymbol{E}) + \boldsymbol{E} = Z\boldsymbol{E}$ → $\boldsymbol{E} = -\frac{SN}{S + 1 - Z} = \frac{SN}{Z - S - 1}$
- Solve for **A**: $H\boldsymbol{A} = P(P - Y)$ → $\boldsymbol{A} = \frac{P^2 - PY}{H}$
- Solve: $\frac{x^2}{4} - \frac{x}{2} = 5$ → $1 \pm \sqrt{21}$
- Simplify: $(-2x^{-2})^{-3}$ → $-\frac{1}{8}x^6$

Week #27

- Show the geometric technique for solving for the root (x) if: *A square minus 8 roots is equal to 20*.
- Simplify: $\frac{6}{x^2} + \frac{3}{4x} - \frac{2}{5x}$ → $\frac{7x + 120}{20x^2}$
- Simplify: $\frac{3x - 2}{12x} - \frac{x - 3}{18x}$ → $\frac{7}{36}$
- Simplify: $\frac{x}{x - 4} + \frac{5}{x + 5} - \frac{11x - 8}{x^2 + x - 20}$ → $\frac{x + 3}{x + 5}$
- Solve for **E**: $\frac{N\boldsymbol{E}}{AR} - L = Y$ → $\boldsymbol{E} = \frac{YAR + LAR}{N}$
- Solve for **M**: $SU + \boldsymbol{M} = \boldsymbol{M}E + R$ → $\boldsymbol{M} = \frac{R - SU}{1 - E} = \frac{SU - R}{E - 1}$
- Solve: $4x + 2 = (x - 1)(x + 3)$ → $1 \pm \sqrt{6}$

Week #28

- Ask what -3^2 is equal to and then review why $-3^2 \neq (-3)^2$.

- Review why Mr. York says "the opposite of b" rather than "negative b" when reciting the midnight formula. Also remind students that whenever evaluating an expression, best practice is to replace each variable with a set of parentheses into which its value is placed.

- The Quadratic Formula Problem Set #11 Q11 …
  Solve by factoring and solve by completing the square: *A rectangle has a length of 10 inches and a height equal to the length of the side of a square. Find the side of the square such that the square has an area that is 56 square inches greater than the rectangle.*
  Students may appreciate assistance decoding this into: $x^2 = 10x + 56$

- Show diagrams for how Al-Khwarizmi would have gone about solving this equation (almost the same as the above but not quite) geometrically: $x^2 + 10x = 56$

- Solve by completing the square, and by factoring:
  $(2x - 3)(x + 5) = (x + 1)(x + 3) \quad \rightarrow \quad -6, 3$

- Solve by completing the square, and by using the quadratic formula:
  $5x^2 + 3x - 7 = 0 \quad \rightarrow \quad -\frac{3 \pm \sqrt{149}}{10}$

- Ask what would have happened if the c-value above had instead been positive 7. Were that the case, the value under the square root would be negative, and thus the equation would have no real solution ($x \notin \mathfrak{R}$). Explain that "b^2-4ac" is therefore referred to as *the discriminant*. When $b^2-4ac > 0$ there are two unique solutions. When $b^2-4ac = 0$ there is only a single solution (the binomial factors would be identical). And when $b^2-4ac < 0$ there are no real solutions.

★ *By careful use of language, a teacher can allow for the existence of imaginary numbers without ever leading students to consider the same. This misdirection is aided tremendously by the fact that the word "real" is not only a formal mathematical designation, but also an everyday word. Thus, if we encounter the square root of a negative we can be quite clear how (+)(+) = (+) and (−)(−) = (+), and therefore while one might imagine there could be such a thing, there is no real number whose square is negative. Regardless, it is best not to dwell too much on these things, but leave the very wonderful surprise of the meaningful existence of so-called imaginary numbers for 11th-grade.*

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Week #29

- Logarithms Problem Set #1 Q25 …
  Simplify without negative exponents: $(3x^{-3})^2$ → $\frac{9}{x^6}$

- Logarithms Problem Set #1 Q26 …
  Simplify without negative exponents: $(9x^{-4}y^3)^4$ → $\frac{6561y^{12}}{x^{16}}$

- Logarithms Problem Set #1 Q29 …
  Simplify without negative exponents: $(4x^{-6}y^2)^{-4}$ → $\frac{x^{24}}{256y^8}$

- Logarithms Problem Set #1 Q31 …
  Simplify without negative exponents: $\left(\frac{x^{-3}}{y^2}\right)^3$ → $\frac{1}{y^6x^9}$

- Logarithms Problem Set #1 Q32 …
  Simplify without negative exponents: $\frac{15x^{-4}y^{-3}}{6x^{-7}y^0}$ → $\frac{5x^3}{2y^3}$

- Logarithms Problem Set #1 Q33 …
  Simplify without negative exponents: $\frac{8x^{-4}y^7}{6x^3y^3}$ → $\frac{4y^4}{3x^7}$

- Logarithms Problem Set #1 Q34 …
  Simplify without negative exponents: $\left(\frac{8x^{-4}y^7}{6x^3y^3}\right)^{-1}$ → $\frac{3x^7}{4y^4}$

- Solve for **E**: $R(\boldsymbol{E} - V) + I = \boldsymbol{E} - W$ → $\boldsymbol{E} = \frac{RV - I - W}{R - 1}$

- Simplify: $\frac{x^2 + x - 6}{x^2 + 2x - 8} \cdot \frac{x^2 + 5x + 4}{x^2 + 2x - 3} - \frac{2}{x - 1}$ → 1

- Solve: $2x(x + 4) = -5(x + 3)$ → $-\frac{3}{2}, -5$
  Allow students to choose a technique for solving. Then note that this equation can be solved by factoring, completing the square, or using the quadratic formula.

Week #30

- Note that some calculators do not allow the base of logarithms to be set. This should not cause difficulty for any questions given in assignment work, but it did show up in the lectures. Explain that when not otherwise specified, the standard base is 10.

- Note that it is *sometimes* possible to take the logarithm of a negative number. For example…
  $(-2)^3 = -8 \quad \Leftrightarrow \quad \log_{-2}(-8) = 3$

- Review…
  $\sqrt{x} \cdot \sqrt{x} \rightarrow \sqrt{x^2} \rightarrow \boldsymbol{x}$
  $\sqrt{x} \cdot \sqrt{x} \rightarrow (\sqrt{x})^2 \rightarrow \boldsymbol{x}$
  $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \rightarrow x^{\frac{1}{2}+\frac{1}{2}} \rightarrow \boldsymbol{x}$
  $\left(x^{\frac{1}{2}}\right)^2 \rightarrow x^{\frac{1}{2}\cdot 2} \rightarrow \boldsymbol{x}$

- Explain how scientific notation works and give conversion to and from examples.

- Logarithms Problem Set #3 Q9 …
  Simplify without negative exponents: $\left(\frac{10x^{-2}y^{-5}}{6x^{-6}y^{3}}\right)^{-3} \rightarrow \frac{27y^{24}}{125x^{12}}$

- Simplifying radicals. Practice examples such as …
  $\sqrt{18} \rightarrow 3\sqrt{2}$, $\sqrt{150} \rightarrow 5\sqrt{6}$, $\sqrt{54} \rightarrow 3\sqrt{6}$, $7\sqrt{56} \rightarrow 14\sqrt{14}$,
  $\sqrt{x^2y^5} \rightarrow xy^2\sqrt{y}$, $-8x^3y^2\sqrt{540x^5y^8} \rightarrow -48x^5y^6\sqrt{15x}$

Week #31

- Review / discussion of independent events: P(n) = P(A) × P(B)

- Year-End Review Q5: Given x = −1, y = 3, z = −½ …
  Evaluate: $z^3 - 4xz^2 \rightarrow \frac{7}{8}$

- Logarithms Problem Set #5 Q35 …
  Solve: $8 + 3\log_5(2x - 7) = 17 \rightarrow 66$

- Logarithms Problem Set #5 Q34 …
  Solve: $\frac{1}{9} \cdot 6^{3x-5} - 6 = 18 \rightarrow 2\frac{2}{3}$ -or- $\frac{8}{3}$

- Solve: $3\,(5x+2)+2 \;=\; 10x+5\,[x-(3x-1)]$ → $-\frac{1}{5}$

- Solve: $x-\frac{3}{2} \;=\; \frac{2}{3}+\frac{1}{2}(x+3)$ → $\frac{22}{3}$
  Solve by working with fractions as presented, then show how to clear fractions before solving by multiplying through by 6.

- Last call for any logarithms or possibility & probability questions students may have, then move into core review. This could include equation systems, quadratics, basic equations, &c. by request.

Week #32

- Solve: $\begin{aligned} 3x+5y &= -11 \\ 2x-7y &= 3 \end{aligned}$ → $(-2,-1)$

- Solve: $\dfrac{x}{x+12} \;=\; \dfrac{1}{x+5}$ → $-6,\ 2$

- List topics covered on the year-end test, noting that although we have studied so very much more over the year (perhaps ask or remind them about these things), the test deliberately focuses on the central and most important topics of algebra-1.
  - Simplifying polynomial expressions (+, −, ×, ÷, exponentiation, negative exponents)
  - Evaluating expressions given values for each variable
  - Factoring
  - Solving equation systems for x and y
  - Solving single equations (basic, variables in denominators, quadratics: factoring & formula)

- Create a varied set of polynomial simplification exercises as examples. Allow students to decide which they are confident in simplifying, and which they wish to review.

- Factor: $12x^3y^4-27x^5z^{16}$ → $3x^3(2y^2+3xz^8)(2y^2-3xz^8)$

- How many lines do 6 non-collinear points determine? → 15 (${}_6C_2$ -or- 5^{th} triangular number)