Tutorial Session Notes
Grade 9
Quarter #3 (Week 17-24)

About these notes:

- These notes are primarily for those who are acting as the tutor — either a parent or a class teacher.
- During the second year of JYMA, Mr. Messner (our JYMA tutor) kept scribbled records of some of the proceedings of his Friday tutorial sessions. These notes are a reconstruction of those scribbles.
- These lessons were often a spontaneous collaboration with the students present and incorporate their questions. In the process of clarifying details, examples occasionally stepped beyond the skills presented in lectures. This is not an ideal script, but only an offering of possible tutorial activities.
- In order to support those who are acting as the tutor for their child or a class, I am sharing these notes with those who are acting as the tutor.
- Of course, these tutorial sessions are also an opportunity for the students to ask their tutor questions.
- If you are acting as the tutor, it may be helpful to read the section of the JYMA handbook titled "The Role of the Tutor".

Week #17

- Provide a reminder of why $(x + A)(x - B) = 0$ yields $x = -A, B$. Explain the inner steps that cause the signs to seemingly shift.
- *Twice the difference between a number and twenty-five is three times the number. Find the number.* $\rightarrow -50$
- *The sum of two numbers is twenty. Three times the smaller is equal to two times the larger. Find the two numbers.* $\rightarrow$ 8 and 12
- *Find three consecutive integers whose sum is negative twenty-four.* $\rightarrow -9, -8, -7$
- *Find three consecutive odd integers such that three times the middle integer is six more that the sum of the first and third integers.* $\rightarrow \emptyset$: There is no solution to this word problem, although there is a solution to the equation which describes it. If even integers were allowed, then 4, 6, 8 would work.
- Solve the following equation for x and for y. (Create equations beginning with "x =" and "y =".)
  $4x - 9y = 12 \rightarrow x = \frac{9}{4}y + 3 \rightarrow y = \frac{4}{9}x - \frac{4}{3}$
- Word Problems Problem Set #4 Q1 …
  Find the Numbers: *The sum of two numbers is 17 and the sum of their squares is 185.* $\rightarrow$ 4 and 13

Week #18

Note: When time is limited, it is more valuable to take students through the setup of word problems (arriving at equation(s)) than the actual algebra of solving equations.

- List the types of word problems we have encountered so far:
  - Number Riddles (involving one or two numbers, and sometimes their squares)
  - Age Puzzles (often involving past or future as well as present age)
  - Amount & Value Problems (often involving coins, tickets, stamps, and the like)
  - Weighted Averages (often involving grades and test scores)

- *Four years ago, Amy was three times Ben's current age. In 8 years, Ben will be half as old as Amy is now. What are their present ages?* $\rightarrow$ Amy is 40, Ben is 12

- *Two coin banks contain only nickels (5¢) and quarters (25¢). The total value of the coins in the first bank is $2.90. In the second bank, there are two more quarters than in the first bank and twice as many nickels. The total value of the coins in the second bank is $3.80. Find the number of nickels and the number of quarters in the first bank.* → 8 nickels, and 10 quarters

- Word Problems Problem Set #5 Q12 … *Joe received a 95% on the final exam which was worth 60% of the class grade. If he received a 67% on the midterm exam, what was his final grade in the class if the midterm exam was worth 40% of the class grade?* → 83.8%

- *The total value of dimes (10¢) and quarters (25¢) in a coin bank is $3.30. If the quarters were dimes and the dimes were quarters, the total value of the coins would be $3.00. Find the number of dimes and the number of quarters in the bank.* → 8 dimes, and 10 quarters

Week #19

- Word Problems Problem Set #6 Q4 …

  $8x + 3y = 12$

  $8x + 12y = -39 \rightarrow (\frac{29}{8}, -\frac{17}{3})$-or-$(3\frac{5}{8}, -5\frac{2}{3})$
- *A total of 26 bills are in a cash box. Some of the bills are one-euro bills, and the rest are five-euro bills. The total amount of money in the box is €50 (50 euros). How many of each type of bill are in the box?* → 20 one-euro bills, and 6 five-euro bills
- *A collection of stamps consists of 2¢ stamps, 8¢ stamps, and 14¢ stamps. The number of 2¢ stamps is five more than twice the number of 8¢ stamps. The number of 14¢ stamps is three times the number of 8¢ stamps. The total value of the collection is $2.26. How many of each type of stamp are there?* → 13 of 2¢ stamps, 4 of 8¢ stamps, and 12 of 14¢ stamps
- *The sum of the squares of two consecutive positive integers is forty-one. Find the integers.* → 4, 5
- *A skater starts from one end of a 15-mile course at 8:00am. One hour later, a cyclist starts from the other end of the course and rides toward the skater. If the rate of the skater is 6 mph and the rate of the cyclist is 9 mph, at what time will the two meet?* → 9:36am
- More equation systems. If time is limited, focus on the steps up to and including substitution. Once students have a new equation in one-variable, they should be on comfortable ground.

  • $6x - 5y = -7; y = 3x - 4 \rightarrow (3,5)$
  • $5x + 2y = 8; 2x - 4y = 8 \rightarrow (2, -1)$
  • $4x - 9y = 14; 6x - 5y = 4 \rightarrow (-1, -2)$
  • $2x + y = -13; 3y = 2x - 15 \rightarrow (-3, -7)$

Week #20

Note: There is something especially challenging about word problems involving uniform motion. Take time to show how to decode and diagram these kinds of problems. Throughout the unit, Mr. York's answer keys often provide considerable detail on how to solve specific problems.

- Word Problems Problem Set #7 Q10 … *On Saturday, Ben jogged for 2½ hours. On Sunday, he jogged for two hours, but went 2 km further, and jogged at a rate that was 3 km/h faster than he did on Saturday. How far did he jog on Saturday?* → 20 km
- Word Problems Problem Set #7 Q14 … *Margaret biked up a hill at 4 mph and came back down at 18 mph. What was her average speed?* → 6.54 mph -or- $6\frac{6}{11}$
- Word Problems Problem Set #8 Q10 … *Thomas and Keith start out 12 miles apart. At what time do they pass each other if they both start biking toward each other at 2:20pm, and Thomas bikes at 15 mph and Keith bikes at 21 mph?* → 2:40pm

Week #21

- Conduct a thorough review of percents. Practice example problems such as …
  (a) What is 40% of 21? → → → → → → → → [8.4 -or- 8 ⅖]
  (b) 75% of 48 is what number? → → → → → → [36]
  (c) 14 is what precent of 18? → → → → → → → [77⅞% -or- 77.7%]
  (d) 72 is 66⅔% of what number? → → → → → → [108]
  (e) What is 56 decreased by 37½%? → → → → → [35]
  (f) Going from 40 to 30 is what percent decrease? → [25% decrease]
  (g) Going from 84 to 98 is what percent increase? → [16⅔% increase]
  (h) Cost to purchase $90 item with 5% sales tax? → → [$94.50]

- Remind students (perhaps prior to the above) about the value of memorizing basic single-digit denominator fraction to decimal to percent conversions. If necessary, review how they can be calculated by hand.

- Complete the following table (leave bracketed values blank) …

FRACTION	DECIMAL	PERCENT
[$\frac{2}{25}$]	**0.08**	**[8%]**
[$\frac{13}{20}$]	**[0.65]**	**65%**
$\frac{4}{11}$	**[0.36]**	**[36.36%]**

- *A bus traveling at a rate of 60mph overtakes a car traveling at a rate of 45mph. If the car had a 1-hour head start, how far from the starting point does the bus overtake the car?* → 180 miles

- Midyear Review Problem Set #1 Q40 … *What is 7% of 5000?* → 350

- Midyear Review Problem Set #1 Q45 … *What is 180% of 450?* → 810

- Midyear Review Problem Set #1 Q49 … *What is 7000 increased by 3%?* → 7210

- If you have time, do a very simple problem involving negative exponents.

Week #22

- With student input, select questions from Midyear Review Problem Sets #3-4. Questions working with percents and/or PDA (proportional and dimensional analysis) will likely be the focus.

- Midyear Review Problem Set #4 Q57 … is a rather extensive problem about cheese. (Mr. York loves cheese.) It takes some time to work through. Doing so was a request from our group during the year these notes were transcribed, but it may not be the best choice for your student(s).

Week #23

- Midyear Review Problem Set #5 Q44 … *If a model of the Earth were made exactly to scale with a diameter of one meter, how far above the surface of the model would Mount Everest stick out? (Mount Everest has a height of about 8800m and the Earth has a radius of about 6400km.)* → 0.0006875m = 0.6875mm • (Students may be interested to know that although Mt. Everest is the highest above sea level, its peak is not the furthest point from the earth's center. That distinction goes to Mt. Chimborazo in the Andes of Ecuador, though it is "only" 6310m above sea level.)
- Review and practice unit conversions.

- Midyear Review Problem Set #5 Q49 …
  $3x^3(x+3)^2 = 6x^3(3x+17) \rightarrow 0, \pm 5$

- Midyear Review Problem Set #5 Q5 …
  $\frac{4x^{-3}y^{-2}}{7y^5z^{-4}} \rightarrow \frac{4z^4}{7x^3y^7}$

- Midyear Review Problem Set #5 Q6 …
  $(\frac{5x^{-2}}{4y^3})^{-3} \rightarrow \frac{64x^6y^9}{125}$

- Midyear Review Problem Set #6 Q34 …
  $\frac{3x+6}{12} = \frac{x+2}{4} \rightarrow \mathbb{R}$(All real numbers are valid solutions.)

- Simplify: $(3x^{-3}y^4)(-2x^{-1}y^{-2})^{-3} \rightarrow -\frac{3y^{10}}{8}$-or- $-\frac{3}{8}y^{10}$

- Solve: $(x+3)(2x+3) = 5 \rightarrow -\frac{1}{2}, -4$

Week #24

- The Quadratic Formula Problem Set #1 Q12 …
  Complete the Square: $x^2 + 5x + \_\_\_ \rightarrow \frac{25}{4}$

- The Quadratic Formula Problem Set #1 Q19 …
  Solve: $|½x - 5| + 3 = 11 \rightarrow -6, 26$

- The Quadratic Formula Problem Set #1 Q21 …
  The sum of two numbers is 13, and the difference of their squares in 39. Find the numbers.
  $x + y = 13, y^2 - x^2 = 39 \rightarrow (5,8)$

- Review negative exponents (create basic practice problems) and exponent work more generally.

- Simplify: $(3x^{-2}y^{-3}z^4)^{-2} \cdot (2x^4y^6z^{-3})^3 \rightarrow \frac{8x^{16}y^{24}}{9z^{17}}$