Tutorial Session Notes
Grade 9
Quarter #2 (Week 9-16)

<u>About these notes</u>:

- These notes are primarily for those who are acting as the tutor — either a parent or a class teacher.
- During the second year of JYMA, Mr. Messner (our JYMA tutor) kept scribbled records of some of the proceedings of his Friday tutorial sessions. These notes are a reconstruction of those scribbles.
- These lessons were often a spontaneous collaboration with the students present and incorporate their questions. In the process of clarifying details, examples occasionally stepped beyond the skills presented in lectures. This is not an ideal script, but only an offering of possible tutorial activities.
- In order to support those who are acting as the tutor for their child or a class, I am sharing these notes with those who are acting as the tutor.
- Of course, these tutorial sessions are also an opportunity for the students to ask their tutor questions.
- If you are acting as the tutor, it may be helpful to read the section of the JYMA handbook titled "The Role of the Tutor".

Week #9

- Exponents and Polynomials Problem Set #10 Q36 …

  Simplify: $\dfrac{5x^{-1}y^3z^{-2}}{3y^{-2}} \rightarrow \dfrac{5y^5}{3xz^2}$

- Exponents and Polynomials Problem Set #11 Q8 …

  Simplify: $\left(\dfrac{2x^{-3}}{3}\right)^{-3} \rightarrow \dfrac{27}{8}x^9$ -or- $\dfrac{27x^9}{8}$

- Create some radical simplification questions to practice.

- Binomial multiplication practice …

  $(x^2+x)(x+5) \rightarrow x^3+6x^2+5x$

  $(x^3+x)(x^2+5) \rightarrow x^5+6x^3+5x$

  $(x^2+5)(x+5) \rightarrow x^3+5x^2+5x+25$

  $(x^3+5)(x^3-5) \rightarrow x^6-25$

  $(x^3+5)(x^3+5) \rightarrow x^6+10x^3+25$

  Ask students how many terms are possible when multiplying binomials. Ask them how we can predict how many terms will be in the final result. Note that binomials of form (A+B) and (A−B) are called *conjugates*, and as we can see, they are quite special.

- Do a couple quick scientific notation exercises, at least one in each direction.
- Create and practice more involved polynomial work as time allows.

Week #10

- Factor the following, discussing technique in detail …

$x^2 + 10x + 24 \quad \rightarrow \quad (x+6)(x+4)$

$x^2 + 10x - 24 \quad \rightarrow \quad (x+12)(x-2)$

$x^2 - 10x + 24 \quad \rightarrow \quad (x+2)(x-12)$

$x^2 - 10x - 24 \quad \rightarrow \quad (x-6)(x-4)$

- Factor something of the form "$x^2 + Bx + C$" and "$x^8 + Bx^4 + C$", replacing B and C with constants. Show how, despite the higher powers, the latter can be thought of as a quadratic also. For example, if we let $z = x^4$, then the latter can be re-written as "$z^2 + Bz + C$", an obvious quadratic.
- Factor something of the form "$x^{10}y^2 + Bx^5y + C$" expanding on the above understanding.
- Factor something of the form "$x^2 + Bxy + Cy^2$". Explain how this can be thought of as a quadratic in terms of x going from left to right, and a quadratic in terms of y going from right to left. And then how that understanding can be used in the factoring process.
- Factor something of the form "$Fx^3y + FBx^2y + FCxy$" where F is a common factor of constants FB and FC. Emphasize that regardless of the factoring problem and the final approach, we should always do the first thing first, that is, look for and if possible factor out the GCF of all terms.
- Practice some more involved factoring problems that only have this first step. That is, all that needs to happen for them to be fully factored is to take out the GCF.

Week #11

- Factor the following, discussing technique in detail …

$x^2 + 5x + 6 \quad \rightarrow \quad (x+2)(x+3)$

$x^2 + 5x - 6 \quad \rightarrow \quad (x+6)(x-1)$

$x^2 - 5x + 6 \quad \rightarrow \quad (x+1)(x-6)$

$x^2 - 5x - 6 \quad \rightarrow \quad (x-2)(x-3)$

- Factor: $4x^3 + 32x^2 - 132x$ → $4x(x+11)(x-3)$

- Factor: $x^2 - 3xy - 28y^2$ → $(x+4y)(x-7y)$

- Difference of Squares … Factor "$x^2 - 25$", explaining the process and what to look for carefully. In particular, a Difference of Squares polynomial has two terms of opposite signs, has perfect squares for all constants and coefficients (big numbers) and exponents that are even (little numbers).

- Difference of Squares … Factor a more involved problem with higher powers and more variables.

- Difference of Squares … Factor a more involved problem that has a GCF to remove first.

- Difference of Squares … Factor "$x^8 - 1$". Note that any time we produce a binomial factor we must ask ourselves if it is a Difference of Squares. This is analogous to Mr. York's rule of "factor again" until you cannot. Note also that it is helpful organizationally to always put the "+" conjugate before the "–" conjugate as the latter has the possibility of expanding (being factored) further.

- Conduct a little bit of negative exponent review and practice if the group is willing.

- In Lecture 2 of Week 10 (previous week) we saw the following problem. Re-do it, then show how it is possible to factor the four-term result by grouping. *In his 9th-grade lectures, Mr. York does <u>not</u> teach Factoring by Grouping; this should be considered <u>optional</u> enrichment for the curious only.*

  $$(x^3+8)(x^2+10)$$
  $$x^5 + 10x^3 + 8x^2 + 80$$
  $$[x^5 + 10x^3] + [8x^2 + 80]$$
  $$x^3(x^2+10) + 8(x^2+10)$$
  $$(x^3+8)(x^2+10)$$

Week #12

- Review how to solve basic equations, including the step of testing the result.
  Solve: $-2[4-(3x+2)] = 5 - 2(3x+6)$ → $x = -\frac{1}{4}$

- Factoring Problem Set #6 Q42 …
  Solve: $\dfrac{-\frac{1}{2}}{3x + 1\frac{1}{4}} = \dfrac{4}{\frac{3}{4}x - \frac{1}{2}}$ → $x = -\frac{38}{99}$

- Factoring Problem Set #6 Q30 …
  Simplify: $\dfrac{5x^{-4}y^{-3}}{15x^{-3}y^{5}}$ → $\dfrac{1}{3xy^{8}}$

- Factoring Problem Set #6 Q29 …
  Simplify: $\left(\dfrac{3y^{-3}}{2x^{3}}\right)^{-2}$ → $\frac{4}{9}x^{6}y^{6}$ -or- $\dfrac{4x^{6}y^{6}}{9}$

- Factoring Problem Set #6 Q40 …
  Solve for x in terms of y: $y = \frac{2}{3}x - 2$ → $x = \frac{3}{2}y + 3$

- Simplify: $\sqrt{81x^{16}y^{10}}$ → $9x^{8}y^{5}$

- Factor: $81x^{16}y^{10} - 4z^{4}$ → $(9x^{8}y^{5} + 2z^{2})(9x^{8}y^{5} - 2z^{2})$

- Factor something of the form "$x^{2E} + Bx^{E} - C$".

Week #13

- Factor: $2x^{9}y^{2} - 14x^{5}y^{2} - 36xy^{2}$ → $2xy^{2}(x^{4} + 2)(x^{2} + 3)(x^{2} - 3)$

- Factoring Problem Set #7 Q45 …
  Factor: $6x^{6} + 10x^{3}y^{2} - 4y^{4}$ → $2(x^{3} + 2y^{2})(3x^{3} - y^{2})$

- Factor: $8x^{2} + 17x + 9$ → $(x + 1)(8x + 9)$

- Solve: $(x - 8)(x + 4) = -35$ → $x = 1, 3$

- Solve: $2x^{2} + 7x = 4$ → $x = \frac{1}{2}, -4$

Week #14

- Solve: $6x^{3} - 9(3x^{2} - 1) = 3(5x + 3)$ → $x = -\frac{1}{2}, 0, 5$

- Solve: $\dfrac{2x}{x + 4} = \dfrac{3}{x - 1}$ → $x = -\frac{3}{2}, 4$

- Factoring Problem Set #10 Q20 …
  Factor: $20x^8 - 32x^4y^3 - 16y^6$ → $4(5x^4 + 2y^3)(x^4 - 2y^3)$
- Solve: $(x+4)(x+3) = (x+2)(x+5) + 2$ → $\mathbb{R}$
- Solve: $(x+4)(x+3) = (x+2)(x+5)$ → $\emptyset$
- Factor: $12x^5 - 75xy^8$ → $3x(2x^2 + 5y^4)(2x^2 - 5y^4)$

Week #15

- Factor: $13x + x^2 + 30$ → $(x+3)(x+10)$
- Factor: $x^5 + 13x^4 - 30x^3$ → $x^3(x+3)(x+10)$
- Factor: $2x^2 - 26x + 60$ → $2(x-3)(x-10)$
- Factor: $3x^3y^2 - 39x^2y^2 - 90xy^2$ → $3xy^2(x+2)(x-15)$
- Factor: $x^2 + 12x - 45$ → $(x+15)(x-3)$
- Factor: $x^4 + 12x^2 - 45$ → $(x^2+15)(x^2-3)$
- Factor: $x^{10} + 12x^5 - 45$ → $(x^5+15)(x^5-3)$
- Factor: $x^2 + 12xy - 45y^2$ → $(x+15y)(x-3y)$
- Factor: $10x^{13} - 40x^7y^2 - 450xy^4$ → $10x(x^6 + 5y^2)(x^3 + 3y)(x^3 - 3y)$
- Solve: $\dfrac{x}{x-1} = \dfrac{8}{x+2}$ → $x = 2, 4$
- Solve: $x(x+3) = 0$ → $x = 0, -3$
- Solve: $x(x+3) = 28$ → $x = 4, -7$
- Factoring Problem Set #12 Q23 …
  Solve: $2x^3(x-4)(3x+5) - (6x)(2x^2) = 2x^5 + 34x^4$ → $x = -1, 0, 13$

Week #16

- Factor / Solve some basic expressions / equations of the form "$x^2 + Bx - C\ (= 0)$".

- Note that most equations of this form (that is, for most values of B and C) cannot be factored, and yet they can be solved. (For example, "$x^2 + 13x - 42$".) This will be returned to in the future.

- Share with students the single page **Translation Guide** found at the end of this document.

- Consider the problem … *The difference of a number and half of the sum of seven and the square of three is two less than twice the quotient of ten and four. Find the number. ...* Note that having purely symbolic means of expressing such relationships is a relatively new development in the history of mathematics. All algebra problems were word problems in the past. Appreciate that, and help them decode the problem to arrive at the following:
  $N - \frac{1}{2}(7 + 3^2) = 2\left(\frac{10}{4}\right) - 2 \quad \rightarrow \quad N = 11$

- Solve for x and for y: $3x - 6y = 15 \quad \rightarrow \quad x = 2y + 5$ -and- $y = \frac{1}{2}x - \frac{5}{2}$

★ *Be aware that Mr. York does* <u>*not*</u> *teach how to solve systems by use of the elimination method (alternatively known as the addition method, or the linear combination method) in 9th-grade. Sticking with the substitution method limits the number of skills students have to keep track of for algebra-1, and also provides greater opportunity for arithmetic (especially fraction) practice.*

- Solve: $\begin{array}{l} x - 2y = 4 \\ 3x + 4y = 2 \end{array} \quad \rightarrow \quad (2, -1)$

- Solve: $\begin{array}{l} 3x - y = 11 \\ 2x + 5y = 13 \end{array} \quad \rightarrow \quad (4, 1)$

- Emphasize that, when solving a system by substitution, the choice of which variable to solve for initially can make a dramatic difference in the difficulty of the problem. If time allows, solve one of the above systems using a less optimal initial choice.

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From English to Algebra (Translating Verbal Expressions into Variable Expressions)

ADDITION	added to	6 added to y	$y + 6$	←
	more than	8 more than x	$x + 8$	←
	the sum of	the sum of x and z	$x + z$	$\square + \square$
	increased by	n increased by 9	$n + 9$	
	the total of	the total of 5 and y	$5 + y$	$\square + \square$
	plus	b plus 17	$b + 17$	
SUBTRACTION	minus	x minus 2	$x - 2$	
	less than	7 less than m	$m - 7$	←
	less	7 less m	$7 - m$	
	subtracted from	5 subtracted from d	$d - 5$	
	decreased by	n decreased by 3	$n - 3$	
	the difference between	the difference between y and 4	$y - 4$	$\square - \square$
MULTIPLICATION	times	10 times g	$10g$	
	of	one half of x	$\frac{1}{2}x$	
	the product of	the product of y and z	yz	$\square \times \square$
	multiplied by	11 multiplied by y	$11y$	
	twice	twice n	$2n$	
	thrice	thrice n	$3n$	
DIVISION	divided by	x divided by 12	$\frac{x}{12}$	
	the quotient of	the quotient of y and z	$\frac{y}{z}$	$\frac{\square}{\square}$
	the ratio of	the ratio of n to 9	$\frac{n}{9}$	$\frac{\square}{\square}$
EXPONENTS	the square of	the square of x	x^2	←
	the cube of	the cube of m	m^3	←
	the power of	z to the power of n	z^n	

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