Regarding the implications of Gödel's Proof:

- The Logistic School's central theme that "mathematics is meaningless" is still, in many ways prevalent today.
  - Ask the participants: "What do you think the implications were?"
  - Math, as we knew it, has changed. It demands that we think of math differently.
  - Hilbert (and Russell) wanted to give a firm foundation for all mathematics by eliminating intuition from mathematics. Gödel showed that math cannot proceed without intuition.
  - Goldstein's book, *Incompleteness*:
    - "It is extraordinary that a mathematical result can say anything about the nature of mathematical truth in general (metamathematics)."
    - "[Gödel believed] that mathematical reality must exceed all formal attempts to contain it."
    - The formalists and positivists were saying that math is a meaningless formal game, reduced to "manipulating meaningless symbols" according to the rules of the game. This game could be played by a machine. Gödel's theorems seem to imply that our minds are not just machines.
    - Gödel was a Platonist, and thereby believed that statements could be true or false even if they were unprovable.
- **<u>Continuum Hypothesis</u>** (CH). Cantor conjectured that there is no cardinal number between that of the integers (or counting numbers) and that of the real numbers (or irrational numbers). This is the Continuum Hypothesis.
    - It has been proven that CH cannot be proven either true or false. How this is interpreted is very different between the Platonists and Formalists. (See "What is Mathematics, Really?", p139):
    - Ask the students what the formalist and what the Platonist would say about this?
      - To the Platonist, this means that our system for expressing the CH isn't sufficient. The CH must, of course, either be true or false. We just can't understand things well enough to tell which is the case.
      - The Formalist doesn't agree largely because math isn't about whether something is really true or not. Math is about rearranging meaningless symbols (undefined terms). The game of math is about following the rules to create formal proofs. It's pointless to debate whether CH is true or not. Once we knew that it couldn't be proved or disproved, the game was over.
  - The irony is that even though Gödel had accomplished what his dream was, he was misunderstood.
    - The logical positivists saw it as evidence of the meaninglessness of math.

      "Some thinkers despaired at this result. Others could never accept it. And still others misunderstood it as a torpedo to the hull of rationality itself. For Gödel, however, it was evidence of an eternal, objective truth, independent of human thought, that can only be imperfectly apprehended by the human mind."
      --from Rebecca Goldstein's book *Incompleteness, The Proof and Paradox of Kurt Gödel.*
  - "We know that God exists because mathematics is consistent and we know that the devil exists because we cannot prove the consistency." -- Andre Weil
  - As humanity was cutoff more and more from the spiritual world, math education became less philosophical & more mechanical.
  - The Logical Positivist's (and Logistic School's) central theme that "mathematics is meaningless" is still, in many ways prevalent today, and has had a big impact on math education.
  - <u>Each epoch in history</u> can be thought of as having a dominant way of thinking or being.
  - In the world of math and science, it was the Greeks that began a new paradigm. This paradigm culminated in science with Newton, and in math with the likes of *Principia Mathematica*.
  - The new paradigm is now beginning. Some people have made the shift; most have not.

Solutions to Puzzles

- **Sum and Difference of Two Squares.**
  - *The difference of two square numbers. "Every prime number, except for 2, can be expressed as the difference of two square numbers in one and only one way."*
  - *The sum of two square numbers.*
    - *"If a number is prime and has a remainder of 1 after dividing it by 4, then it can be expressed as the sum of two square numbers in one and only one way."*
    - *"If a number is prime and has a remainder of 3 after dividing it by 4, then it is not possible to express it as a sum of two square numbers."*
    - *"If a number is not prime, then there are a variety of possibilities − it may be that the number can be expressed as the sum of two square numbers in one way, in multiple ways, or not at all."*

- **Perfect Numbers.** The first three perfect numbers are 6, 28, and 496. Find the next two perfect numbers. Find a formula that generates perfect numbers. See solution at the end of this document.
  - There are 21 even abundant numbers under 100. The first odd abundant number is 945.
  - The first 7 perfect numbers are:
    6; 28; 496; 8128; 33,550,336; 8,589,869,056; 137,438,691,328.
  - Euclid discovered this formula for calculating even perfect numbers: $P = (2^{N-1})\cdot(2^N - 1)$
    - Now the question becomes whether $(2^N - 1)$ is prime or not. It turns out to be prime only for these N values: 2, 3, 5, 7, 13, 17, 19, 31, 61
  - In the 1600's, Jean Prestet found the eighth perfect number: 2,305,843,008,139,952,128.
  - There are no known odd perfect numbers, and it is one of the great mysteries of mathematics whether or not an odd perfect number could possibly exist.
  - See *A Middle School Math Curriculum* for more detail.

- **4273!**

First some background… If a number has the prime factorization $2^9\cdot3^8\cdot5^6\cdot13^4$, then we only need to look at the exponents of the 2 and the 5, in order to conclude that the number ends in 6 zeroes. Likewise, if a number's prime factorization is $2^7\cdot5^{13}\cdot7^2\cdot11\cdot23^3$, then we know that that number must end in 7 zeroes.

Now, to address the question at hand… Let N = 4273! We know that the number of 2's in the prime factorization of N must be greater than the number of 5's (i.e., the exponent of the 2 must be greater than the exponent of the 5). Therefore, we simply need to determine the number of 5's in the prime factorization of 4273! We can systematically do this by asking ourselves the following questions:

How many numbers between 1 and 4273 are…

Divisible by 5?	Answer: 854.
Divisible by 25 (which is 5^2)?	Answer: 170.
Divisible by 125 (which is 5^3)?	Answer: 34.
Divisible by 625 (which is 5^4)?	Answer: 6.
Divisible by 3125 (which is 5^5)?	Answer: 1.

With a little bit of thought, we can now conclude that the number of zeroes in N must be equal to the sum of the above answers, which is <u>1065</u>.