$$P_1 = P_0(1+r) - M$$

substituting

$$P_2 = P_1(1+r) - M$$

$$= \left[P_0(1+r) - M\right](1+r) - M$$

$$P_2 = P_0(1+r)^2 - M(1+r) - M$$

$$P_3 = P_2(1+r) - M$$

$$= \left[P_0(1+r)^2 - M(1+r) - M\right](1+r) - M$$

$$P_3 = P_0(1+r)^3 - M(1+r)^2 - M(1+r) - M$$

$$P_4 = P_3(1+r) - M$$

$$= \left[P_0(1+r)^3 - M(1+r)^2 - M(1+r) - M\right](1+r) - M$$

$$P_4 = P_0(1+r)^4 - M(1+r)^3 - M(1+r)^2 - M(1+r) - M$$

See the pattern?

$$P_n = P_0(1+r)^n - M(1+r)^{n-1} - M(1+r)^{n-2} - M(1+r)^{n-3} \ldots - M(1+r)^2 - M(1+r) - M$$

$$P_n = P_0(1+r)^n - M\left[(1+r)^{n-1} + (1+r)^{n-2} \ldots (1+r) + 1\right]$$

Now remember $x^n + x^{n-1} + x^{n-2} \ldots + x + 1$ equals $\dfrac{x^{n+1} - 1}{x - 1}$

$$P_n = P_0(1+r)^n - M\left[\frac{(1+r)^n - 1}{(1+r) - 1}\right]$$

$$\boxed{P_n = P_0(1+r)^n - M\left[\frac{(1+r)^n - 1}{r}\right]}$$

Amount owed after "n" months

OR easier

$$\boxed{P_n = (1+r)^n\left(P_0 - \frac{M}{r}\right) + \frac{M}{r}}$$

furthermore to pay-off everything in exactly N months $P_N = 0$

Therefore

$$0 = P_0(1+r)^N - \frac{M(1+r)^N - 1}{r}$$

$$P_0(1+r)^N = M\left[\frac{(1+r)^N - 1}{r}\right]$$

solving for "M"

$$\frac{rP_0(1+r)^N}{(1+r)^N - 1} = M$$

or better

$$\boxed{M = rP_0\left[1 + \frac{1}{(1+r)^N - 1}\right]}$$

Monthly payment given r, P_0, N

How Mortgages are Done

The standard is to have the same payment every month.

$M \rightarrow$ the monthly payment

$P_0 \rightarrow$ the amount of the initial loan

$r \rightarrow$ the monthly interest rate (yearly rate ÷ 12)

$P_n \rightarrow$ the amount still owed after "n" months

Note the difference between "n" and "N"

$N \rightarrow$ the total # of months of the mortgage (12Y)

Key Idea: The amount of money owed on any month is the amount owed on the previous month, plus the additional interested accumulated during that month, minus the monthly payment paid for that month.

In algebra, this is expressed by the equation

$$P_n = \underbrace{P_{n-1}}_{\text{Money owed the previous month}} + \underbrace{r \cdot P_{n-1}}_{\text{Interest on that amount}} - M$$

Hopefully, the monthly payment is greater than the monthly interest, so that as the months move along, ~~your debt~~ the amount you owe decreases.
In other words P_n is less than P_{n-1}.

Using the formula, we can say also (by factoring):

$$P_n = P_{n-1}(1+r) - M$$

as well as:

$$P_5 = P_4(1+r) - M$$

$$P_{17} = P_{16}(1+r) - M$$

$$P_1 = P_0(1+r) - M$$

(over)