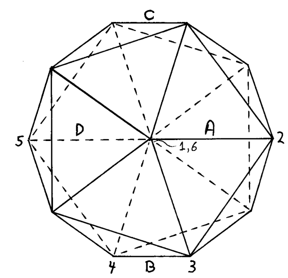
Instructions for Drawing an Icosahedron

<u>Notes for the teacher</u>:

- It is helpful to make a model of an icosahedron from Zometools in order to best visualize what the various views look like.
- The process for drawing an icosahedron is very similar to the process for drawing a dodecahedron.
- Once the students have learned how to draw the dodecahedron, it may be best to have them figure out a process for drawing the icosahedron. They might figure out a different process than what is shown here.

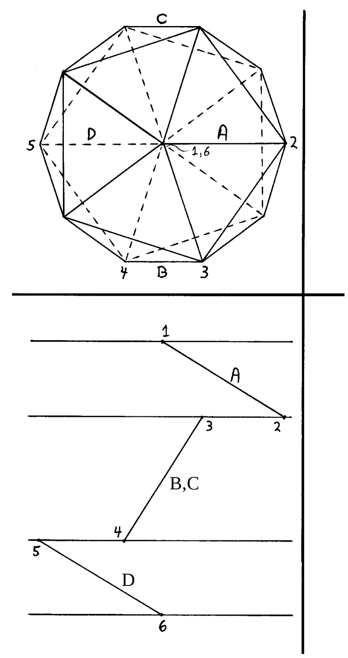
<u>The Top View</u>:

- Start with ten evenly spaced points on a circle, such that two of the points are placed on the circle's horizontal diameter. In black ink, draw the decagon that connects these ten points. Now the top view can be completed as shown in the drawing on the right.



<u>The Front View</u>: (Read through these instructions before trying them!)

- In the front view, one point is at the top, five points are located on a horizontal line a bit lower than the top, five more points are then located on another horizontal line somewhat further down, and the last point is the bottom point. *In order to draw the front view, we must first determine exactly how far apart the spacing between these four horizontal lines must be*. This spacing is not arbitrary!
- Of the icosahedron's 30 edges, four of them will be drawn to actual size in the front view. These edges are the ones that are parallel to the front view, and are drawn horizontally across the page; they are labeled as A, B, C, D in the top-view drawing, shown here. The end points of edge A are labeled #1 and #2.
- Through each point in the top view, draw vertical lines (lightly in pencil) that run down through the front view. In most cases, two points share the same vertical line. Label the vertical line through point #2 as line ℓ, and the vertical line through point #1 as ***m***. (These lines are not shown in the drawing here.)
- Point #1 in the front view of the drawing must be located somewhere on line ***m***. Choose point #1 to be conveniently about one inch from the top of the front view. (The rest of the points of the icosahedron are now fixed.)
- Now, we need to determine the location of point #2 in the front view. The location of point #2 in the front view is determined by two things: the length of edge A must be actual size, and point #2 must be located on line ℓ. To do this, we set our compass to the length of any edge that is of actual size in the top view drawing (i.e., the edge connecting points #2 and #3), place the compass needle on point #1 in the front view drawing, and then mark the place where point #2 must be located on line ℓ.



- Lightly draw a horizontal line through point #2, and locate the positions of the other four points of the icosahedron that must also appear on this same horizontal line. (These four points, along with point #2, are all at the same elevation above the table on which the icosahedron sits.)
- In the same way that we used edge A to determine the spacing between the top two horizontal lines, we can now determine the spacing between the middle two horizontal lines by using edge B (or C), and we can determine the spacing between the bottom two horizontal lines by using edge D. Again, the key to this stems from the fact that these edges appear in actual size in the front view. (See above drawing.)
- Once the placement of the four horizontal lines has been determined, the rest of the front view, as well as the side view, can easily be completed according to the normal procedures. Notice that there should be no dotted lines in the front view. See the completed drawing on page 39.

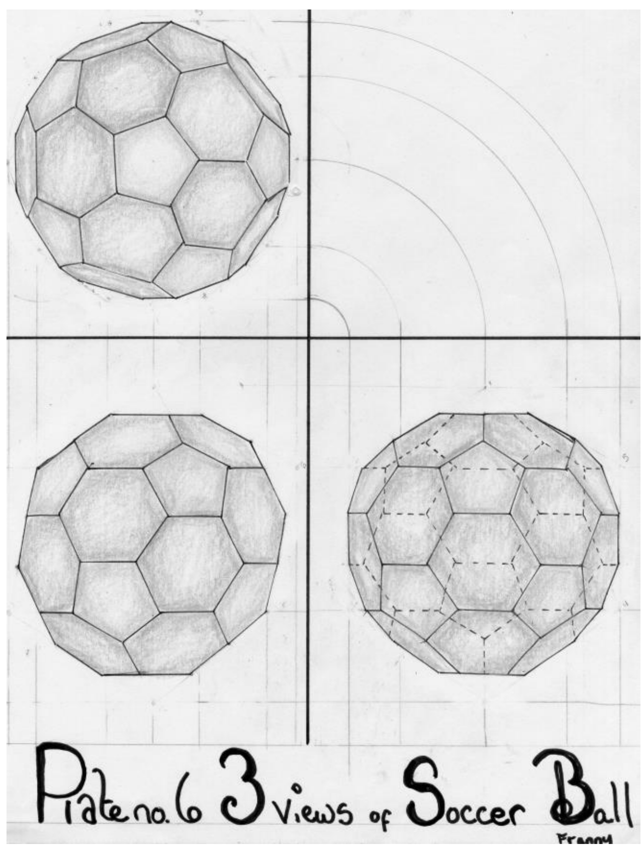
Instructions for Drawing a
Truncated Icosahedron (a.k.a. Soccer Ball)

Notes for the teacher:

- The truncated icosahedron is quite challenging, and should only be attempted by advanced students.
- It is helpful to make a model of a truncated icosahedron from Zometools (all blue sticks) in order to best visualize what the various views look like.
- The truncated icosahedron has 32 faces, 60 points, and 90 edges. However, once the drawing of the icosahedron has been mastered, the truncated icosahedron is quite workable.
- Conceptually, the truncated icosahedron is the product of "chopping off" the 12 points of the icosahedron, whereby each point is replaced with a regular pentagon and each triangular face of the icosahedron transforms into a regular hexagon.
- Once the students have learned how to draw the dodecahedron and icosahedron, it may be best to have the students figure out a process for drawing the truncated icosahedron. They might figure out a different process than what is shown here.

Outline of the process for the drawing:

- Lightly in lead pencil, draw the top view of an icosahedron according to the instructions given for the icosahedron. (Some of the lines will be erased later.)
- With the front view of the icosahedron, the points were located on four different levels shown by horizontal lines. (The top and bottom lines only had one point, so drawing these lines weren't actually necessary. But we will leave them here for reasons that will soon be evident.) Lightly in lead pencil, draw these four lines (with the proper spacing) across the front view of the truncated icosahedron.
- With the front view and the four lines just drawn, draw two more horizontal *evenly-spaced* lines (do you understand why?) between the top two lines (lightly in lead pencil), two more horizontal *evenly-spaced* lines between the middle two lines, and two more horizontal *evenly-spaced* lines between the bottom two lines. Erase the original top and bottom lines. You should now have eight horizontal lines across the front view.
- *Drawing the Top View*. Each of the 30 edges of the icosahedron in the top view (including dotted edges that are in the background) needs to have two evenly spaced points (at distances of ⅓ and ⅔ the length of the edge) marked on it. Carefully and accurately locate these 60 new points with very small dots in black ink. Erase the lead pencil lines in the top view, and then connect the 60 inked points to complete the top view. Start by drawing the small pentagon in the center, and then the five hexagons connected to that pentagon. This top view should have dotted background lines (which aren't shown in the drawing here).
- *Drawing the Front View*. We have already drawn 8 horizontal lines in the front view. Of the 60 points in the front view, the middle four horizontal lines should have ten points each, and the top two and bottom two lines should have five points each. Locate these 60 points and then complete the front view. Notice that there should be no dotted lines in the front view.
- The *Side View* can now be completed.

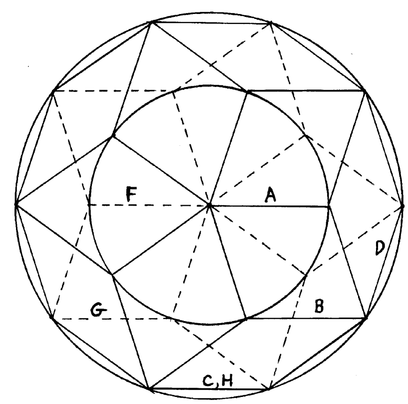


Instructions for Drawing a
Rhombic Triacontahedron

Notes for the teacher:

- The rhombic triacontahedron is quite challenging, and should only be attempted by advanced students.
- It is helpful to make a model of a rhombic triacontahedron from Zometools (all red sticks) in order to best visualize what the various views look like.
- The rhombic triacontahedron has 30 rhombic faces, 32 points and 60 edges.
- It emerges as the result of either growing (shallow) pentagonal pyramids off the faces of a dodecahedron, or by growing (shallow) triangular pyramids off the faces of an icosahedron.
- If the short diagonals are drawn onto every rhombic face of the rhombic triacontahedron, and then the rhombic triacontahedron disappears (but the diagonals remain), we are left with a dodecahedron.
  If the long diagonals are drawn onto every rhombic face of the rhombic triacontahedron, and then the rhombic triacontahedron disappears (but the diagonals remain), we are left with an icosahedron.
- Once the students have learned how to draw the dodecahedron and icosahedron, it may be best to have the students figure out a process for drawing the rhombic triacontahedron. They might figure out a different process than what is shown here.

The Top View:



- As with the drawing for the dodecahedron, start with ten evenly spaced points on a circle, such that two of the points are placed on the circle's horizontal diameter. In black ink, draw the decagon that connects these ten points. Draw an inner circle (lightly in pencil) that has a radius equal to the length of the side of the decagon. Locate and mark the ten points on the inner circle that fall on the diagonals of the outer decagon. (The two circles should be erased in the end.)
- Now, in black ink, draw the five rhombuses that come to a point at the top center, and then the five rhombuses that fit into the between spaces around the perimeter of the circle.
- With dotted lines in black ink, draw in all of the edges that are in the background.

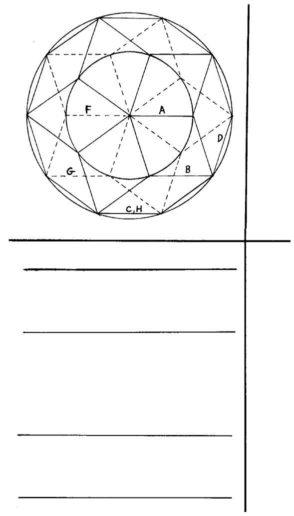
Some key observations:

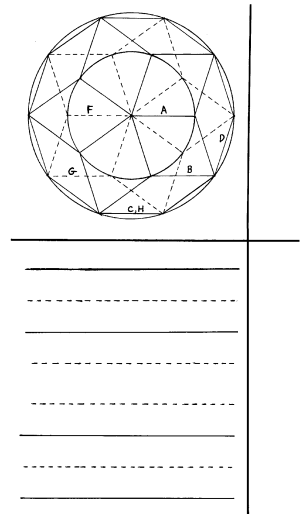
- We have now drawn a total of 40 edges (15 of which are dotted), and 20 faces (ten of which are in the background). Where are the other 20 edges and ten faces? To fully understand this, it is best to look at a model of a rhombic triacontahedron. It is then clear that ten of the rhombic faces are "hiding" (i.e., lying on planes that are perpendicular to the top view) under the edges of the outer decagon in the top view. For example, under the edge labeled C, there is a hidden rhombic face. Edge H sits under and is parallel to edge C; they are both part of this same rhombus. This rhombus is the center face in the front view (as shown in the last drawing on the next page). The ten faces that are hidden in the top view (under the outer decagon) account for the 20 hidden edges – ten of the hidden edges sit under another edge (in the same way that edge H sits under edge C), and *ten of the hidden edges have a vertical orientation, meaning that they are both perpendicular to the top view and parallel to the front and side views*. (This last fact is important.)
- There are no edges parallel to the top view or the side view. Therefore, none of the edges in the top view are actual size. We will find a way to work around this.
- This particular view is unique because every edge that is drawn in the top view appears as the same length. (Do you understand why?) Because of this fact, we can say that the drawn length of edges D and A must be the same. *This allows us to understand why, when we drew the <u>top view of the dodecahedron</u>, the inner circle has a radius equal to the length of the edge of the outer decagon.*

(Continued on next page →)

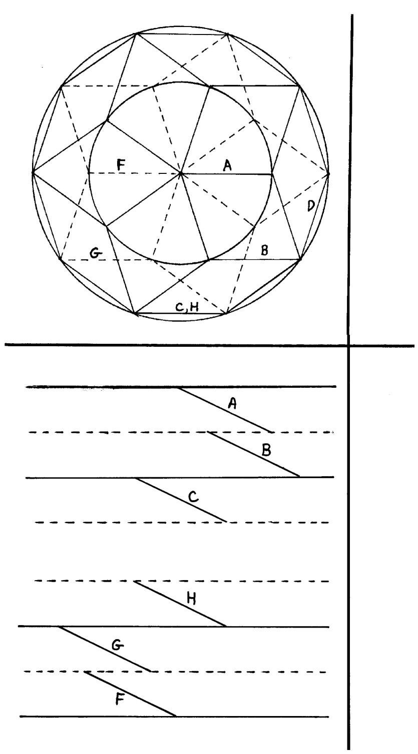
(Instructions for Drawing a Rhombic Triacontahedron, continued)

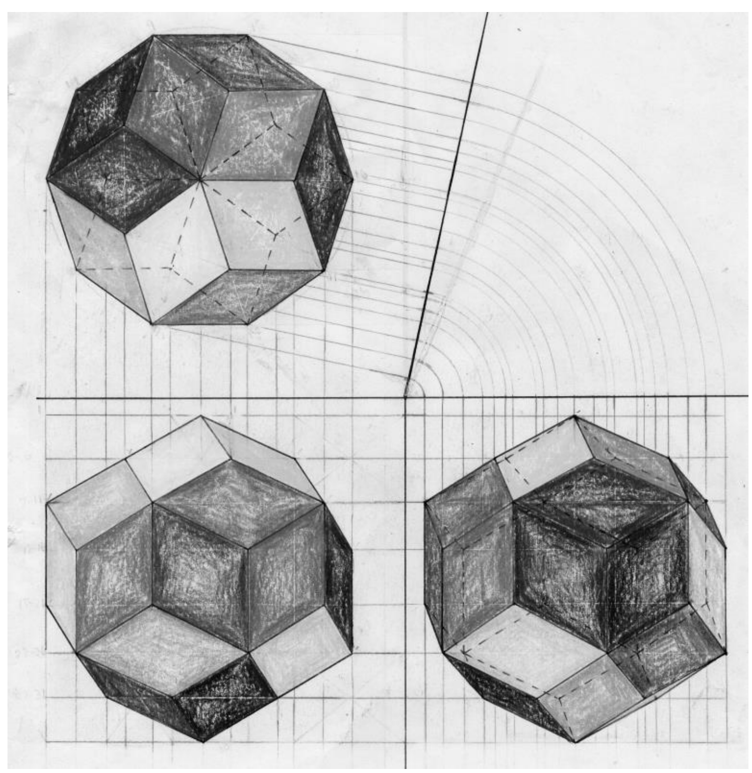
<u>The Front and Side Views</u>:





- The rhombic triacontahedron's 32 points lie on eight horizontal lines in the front (and side) view. The top four of these lines are evenly spaced, and the bottom four of these lines are evenly spaced.
- We will start by drawing the first four lines, which come from the icosahedron. (Recall that we get an icosahedron by drawing the long diagonals on the rhombic faces.) Follow the instructions for the icosahedron's front view in order to draw the first four horizontal lines. (See the first drawing shown here.)
- The rhombic triacontahedron sits balanced on its point. Therefore, one point falls on the top horizontal line of the front view, and five points fall on the second horizontal line (shown in the above-left drawing). These five points fall on the top view's outer circle. There are five more points in the front view which fall on a horizontal line that is exactly halfway between the two above-mentioned lines. These five points fall on the top view's inner circle. Once we draw this "in-between line", we add three more lines with this same spacing. All four of these new lines in the front view are shown as dotted lines in the above-right drawing. Note that the two dotted lines in the middle are spaced slightly further apart than the other lines.
- In the below-left drawing, we have begun to draw in some of the edges in the front view. These edges (labeled A, B, C, H, G, F) are six of the ten edges which are parallel to the front view *and* actually appear in the top view. Therefore, they must appear actual size in the front view. There are also ten edges with a vertical orientation. These edges are also parallel to the front view (and therefore actual size) and they are perpendicular to the top view, which caused them to not appear in the top view.
- The rest of the front view, as well as the side view, can be completed according to the normal procedures. Notice that there should be no dotted lines in the front view.
- In the completed drawing (shown below, right), the third view is a rotated view (rotated by about 13°), which is explained on the next page.

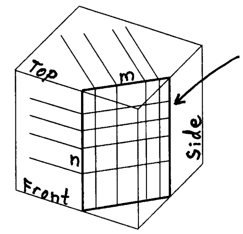




Instructions for Drawing a Rotated View

<u>Creating the image plane for a rotated view</u>:

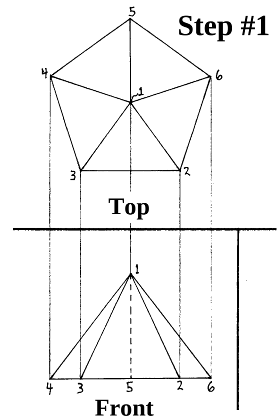
- The desired rotated view is achieved by creating an image plane which is a vertical section of the viewing box. The rotation we are dealing with here is between the front view and the side view. We can imagine that this rotated image plane serves as our painter's canvas, upon which we draw our view of the solid. (See drawing on the right.)



The **rotated** new image plane is perpendicular to the top view, and rotated 42° from the side view toward the front view.

- A rotated view (also called a primary auxiliary view) must be perpendicular to one of the three principle views – top view, front view, or side view. Here, we will create a rotated view that is perpendicular to the top view.

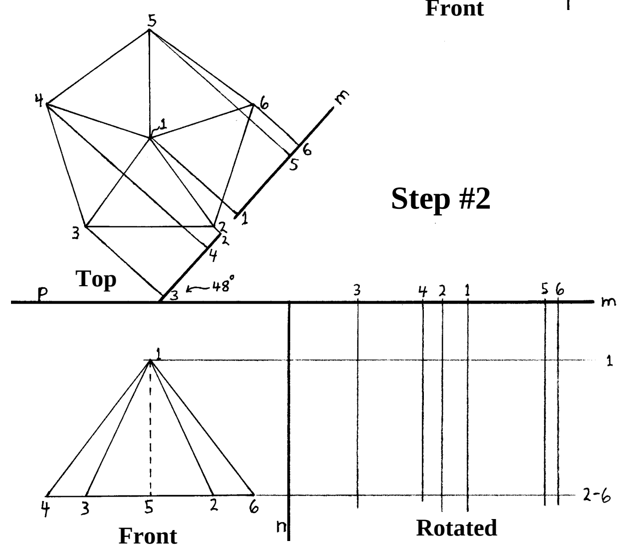
Step #1



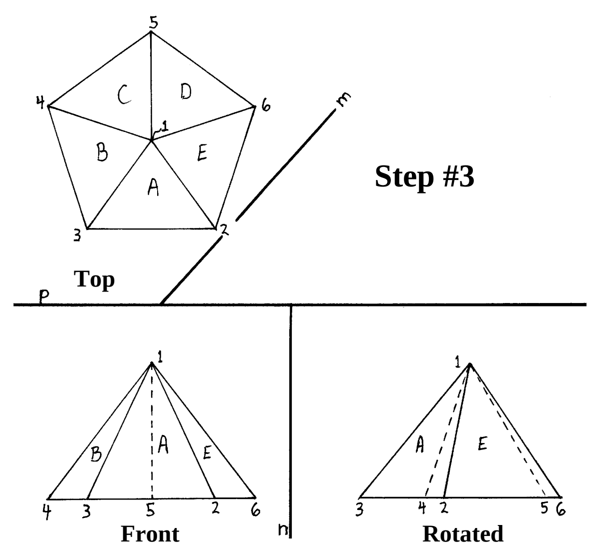
<u>Drawing a rotated view</u>:

- As we mentioned before, once we know the top view and the front view, we have enough information to construct any desired view. In the same manner that we employed to construct the side (or profile) view, we simply follow the below steps (in this case, to draw a 42° rotated view of a pentagonal pyramid with an arbitrary height):
  - **Step #1.** Draw the top view and the front view.
  - **Step #2.** From each point in the front view, draw (horizontal) construction lines perpendicularly to the fold line (lines n in the drawing). Extend these lines past and perpendicular to the fold line in the rotated view. The fold line, m, is drawn at a 48° angle in the top view. Therefore the construction lines from the points in the top view do not go horizontally across the page (as they did in for the normal side view construction); the construction lines run at a 42° angle. In the below drawing, note the following:
    - The points on line m (which is the folding line between the top and rotated views) are copied with a compass from the top view onto the rotated view.
    - In the rotated view, line m looks as if it is the same line as line p (the folding line between the top and front views).
    - In the top view, line m can be drawn starting anywhere on line P, as long as the 48° angle is maintained, and all of the points of the solid can be successfully projected (by the perpendicular construction lines) onto line m.
  - **Step #3.** We can now locate and label all of the points in the rotated view, and finally draw all of the edges of the solid, carefully considering which edges are in the background, and should therefore be dotted. With the final drawing, we have erased all construction lines (which the students shouldn't do) and have labeled the faces that are in the foreground (which the students should instead show with color).

Step #2



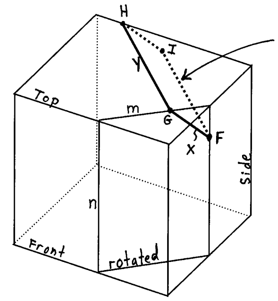
Step #3



- Additional drawings of rotated views are shown on the following pages.

Instructions for a Rotated and Lifted View

(Four Views of a Solid)



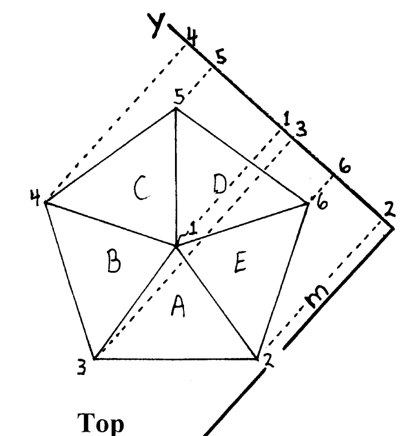
The **rotated and lifted** image plane is perpendicular to the rotated view, and then lifted 53° from vertical.

Creating the image plane for a rotated and lifted view:

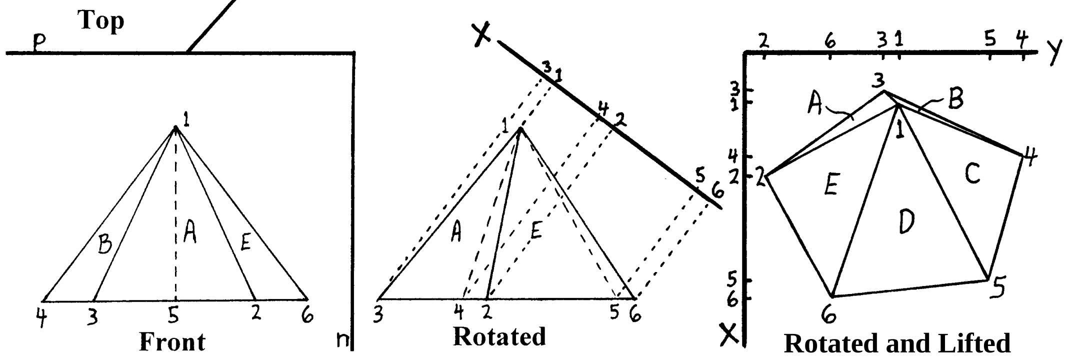
- An arbitrary rotated and lifted view (also called a secondary auxiliary view) must have an image plane that is perpendicular to a rotated view. We will therefore build upon the rotated view from the previous page.
- Recall that the previous page's rotated view came from rotating 42° from the side view toward the front view. The new image plane takes that rotated view and turns it counter-clockwise by 90°. This results in the new view being a rotation of 42° from the back view toward the side view. Lastly, we rotate (lift) upwards by 53°. The resulting image plane is shown as rectangle FGHI in the above drawing. Note that point I is not on the viewing box.

Drawing a rotated and lifted view:

- The below explanation shows how to construct a view of a pentagonal pyramid (as shown on the previous page) which has been rotated clockwise 42° from the back view toward the side view, and then lifted 53° from vertical. All of the steps are reflected in the below drawing.
  - **Step #1.** Draw the top view, the side view, and the rotated view as shown on the previous page. Note that the final image plane for the rotated and lifted view will be additionally rotated counter-clockwise by 90°, and then lifted by 53°. (See above drawing.)
  - **Step #2.** Starting with the drawing of the three views from the previous page, we need to first add the two folding lines (lines x and y in the above drawing) that result from adding the new image plane. Because the new image plane must be perpendicular to the rotated view, folding line y, in the top view, must be perpendicular to folding line m (which was used in the previous drawing). Folding line x is the intersection of the new image plane with the plane of the rotated view. Since the new image plane is 53° from vertical, it is drawn above the rotated view at an angle of 37° from horizontal. (See below drawing.)



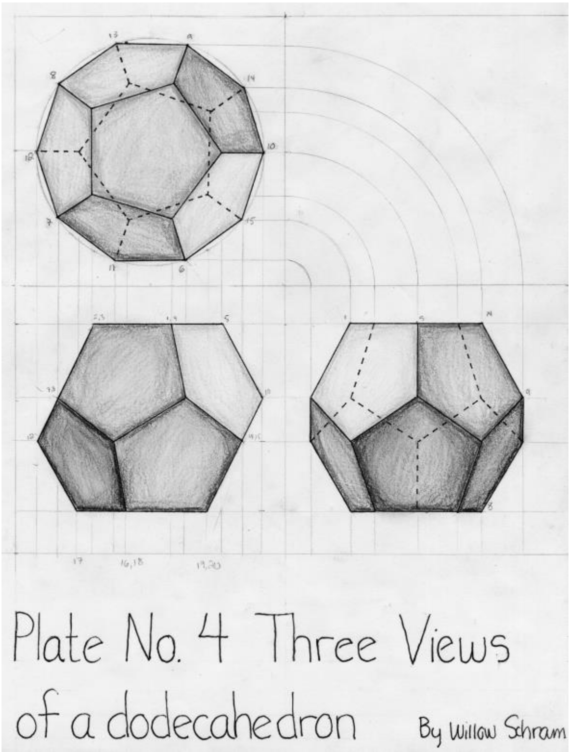
  - **Step #3.** In the top view, draw construction lines from points 1-6 at right angles to line y. In the rotated view, draw construction lines from points 1-6 at right angles to line x.
  - **Step #4.** Copy line x (and the points on it) to a vertical orientation (shown below, right) and copy line y (and the points on it) at a right angle to line x.
  - **Step #5.** As usual, construction lines (which aren't shown in the drawing) can be drawn from lines x and y in order to locate all of the points in the desired rotated and lifted view.
  - **Step #6.** The edges of the pyramid are drawn and the faces are shaded in with colors that are consistent with the other views.

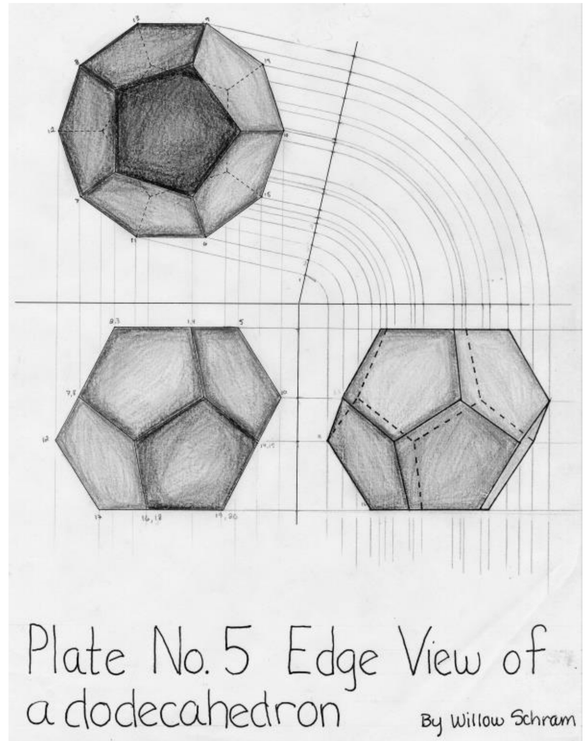


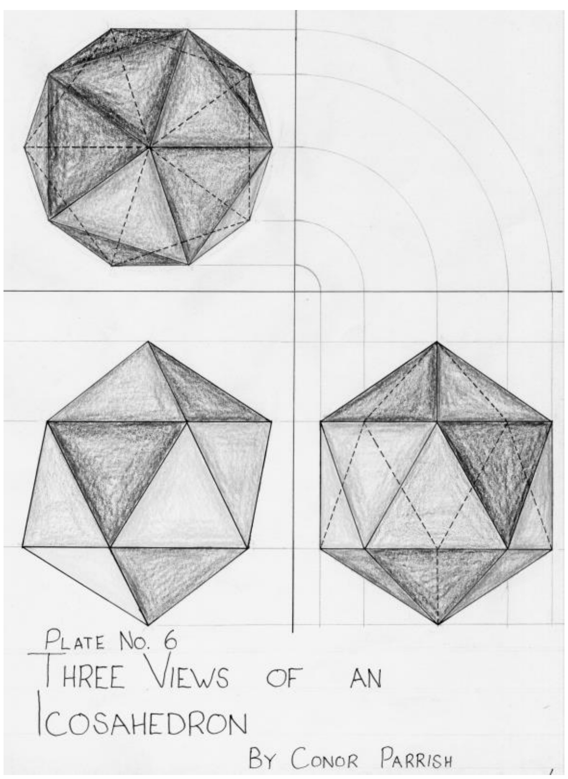
- Additional drawings of rotated and lifted views are shown on the following pages.

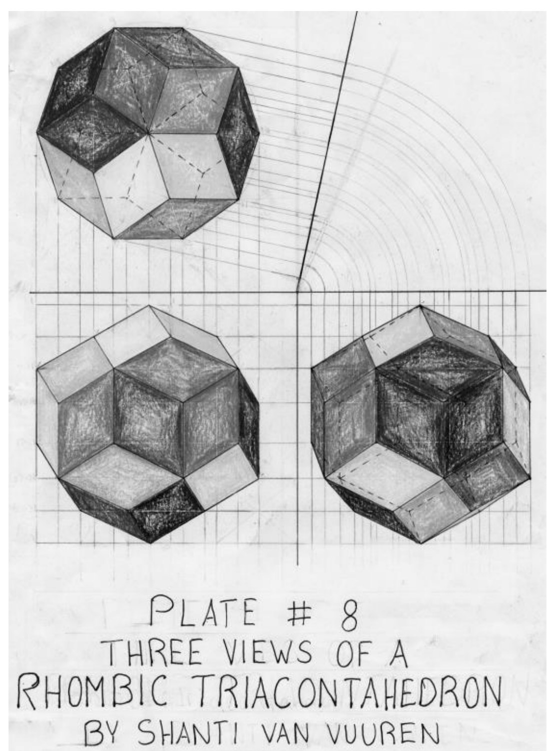
Three Views of Various Solids

(Note that the drawings on the right are “rotated” views.)

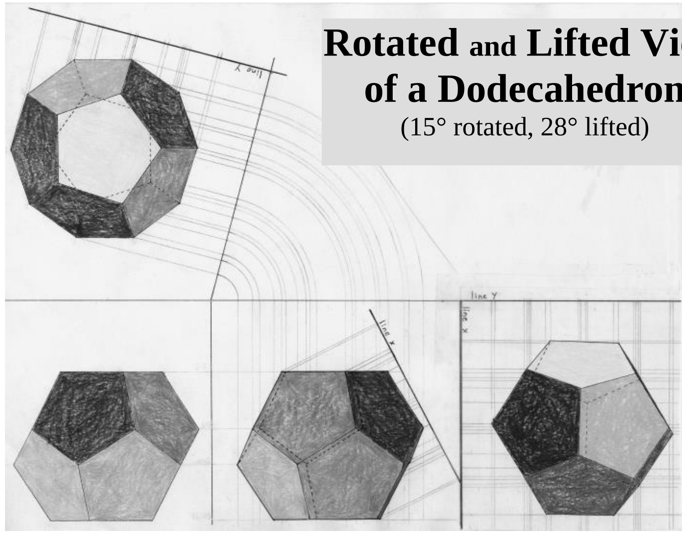


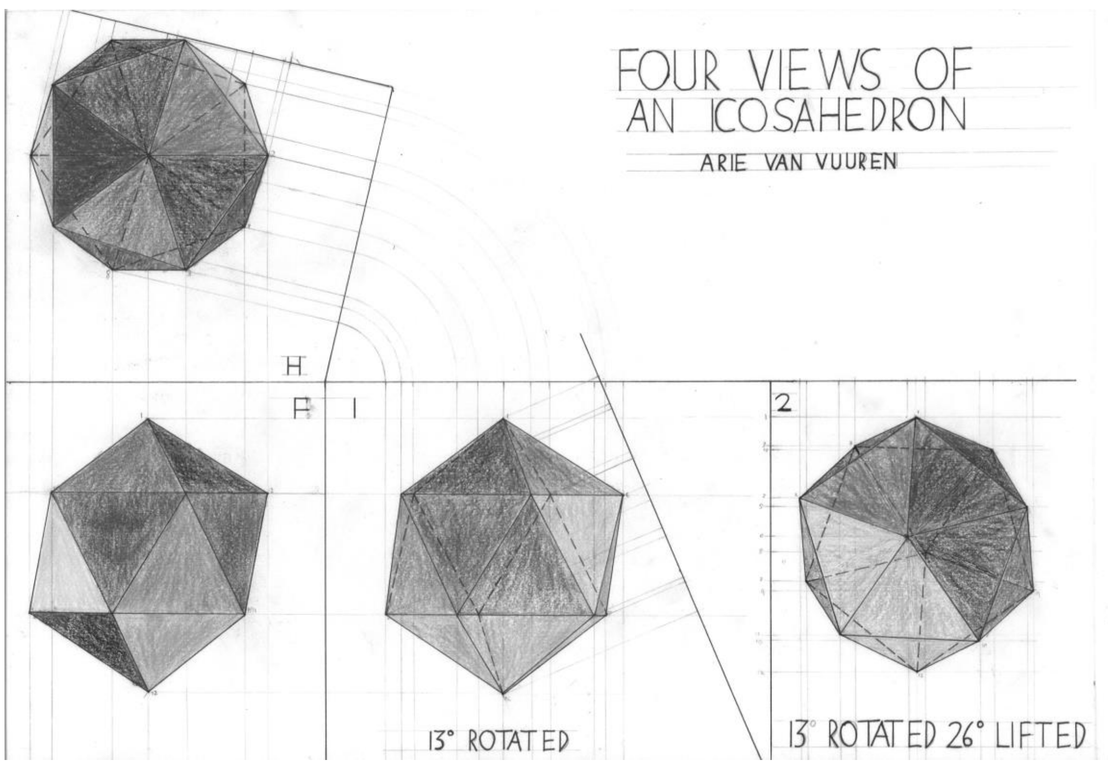


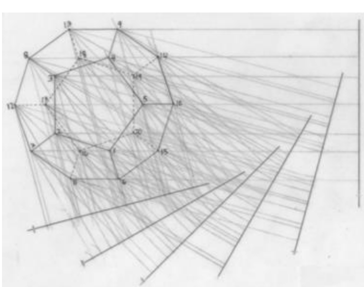




Rotated and Lifted View of a Dodecahedron
(15° rotated, 28° lifted)







Multiple Views of a Rotating Dodecahedron