

Trigonometric Identities and Laws

Opposite Angle Identities

$$\begin{aligned}\sin(-\theta) &= -\sin\theta \\ \cos(-\theta) &= \cos(\theta) \\ \tan(-\theta) &= -\tan\theta\end{aligned}$$

Supplementary Angle Identities

$$\begin{aligned}\sin(\pi-\theta) &= \sin\theta \\ \cos(\pi-\theta) &= -\cos\theta \\ \tan(\pi-\theta) &= -\tan\theta\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= 1 \\ 1 + \tan^2\theta &= \sec^2\theta \\ \cot^2\theta + 1 &= \csc^2\theta\end{aligned}$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{or} \quad a:b = \sin A : \sin B$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

Law of Tangents

$$\frac{a+b}{a-b} = \frac{\tan[\frac{1}{2}(A+B)]}{\tan[\frac{1}{2}(A-B)]}$$

Sum/Difference Identities

$$\begin{aligned}\sin(u \pm v) &= \sin u \cdot \cos v \pm \cos u \cdot \sin v \\ \cos(u \pm v) &= \cos u \cdot \cos v \mp \sin u \cdot \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}\end{aligned}$$

Double-Angle Identities

$$\begin{aligned}\sin 2u &= 2 \sin u \cdot \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

Half-Angle Identities

$$\begin{aligned}\sin \frac{1}{2}u &= \pm \sqrt{\frac{1}{2}(1 - \cos u)} \\ &\text{(Sign depends on quadrant of angle } \frac{1}{2}u) \\ \cos \frac{1}{2}u &= \pm \sqrt{\frac{1}{2}(1 + \cos u)} \\ &\text{(Sign depends on quadrant of angle } \frac{1}{2}u) \\ \tan \frac{1}{2}u &= \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}\end{aligned}$$

Co-Function Identities

$$\begin{aligned}\sin(\frac{\pi}{2} - x) &= \cos x & \cos(\frac{\pi}{2} - x) &= \sin x \\ \tan(\frac{\pi}{2} - x) &= \cot x & \cot(\frac{\pi}{2} - x) &= \tan x \\ \sec(\frac{\pi}{2} - x) &= \csc x & \csc(\frac{\pi}{2} - x) &= \sec x\end{aligned}$$

Power Reducing Identities

$$\begin{aligned}\sin^2 u &= \frac{1}{2}(1 - \cos 2u) \\ \cos^2 u &= \frac{1}{2}(1 + \cos 2u) \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u}\end{aligned}$$

Sum-to-Product Identities

$$\begin{aligned}\sin u + \sin v &= 2 \left(\sin \frac{u+v}{2} \cdot \cos \frac{u-v}{2} \right) \\ \sin u - \sin v &= 2 \left(\cos \frac{u+v}{2} \cdot \sin \frac{u-v}{2} \right) \\ \cos u + \cos v &= 2 \left(\cos \frac{u+v}{2} \cdot \cos \frac{u-v}{2} \right) \\ \cos u - \cos v &= -2 \left(\sin \frac{u+v}{2} \cdot \sin \frac{u-v}{2} \right)\end{aligned}$$

Product-to-Sum Identities

$$\begin{aligned}\sin u \cdot \sin v &= \frac{1}{2}[\cos(u-v) - \cos(u+v)] \\ \cos u \cdot \cos v &= \frac{1}{2}[\cos(u-v) + \cos(u+v)] \\ \sin u \cdot \cos v &= \frac{1}{2}[\sin(u+v) + \sin(u-v)]\end{aligned}$$