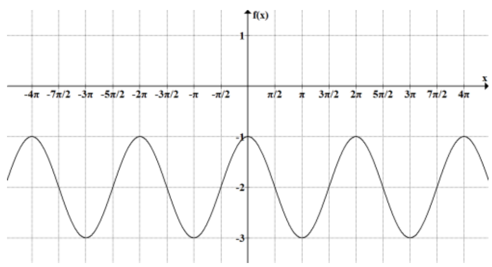
Trigonometry – Part IV ANSWERS

Problem Set #1

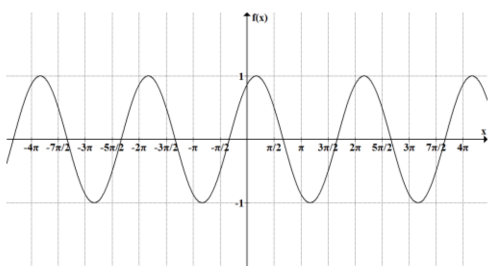
1)	2)	3)	4)	5)
a) 30°	a) $\frac{\pi}{4}$	a) ½	a) $\theta \approx 33.6°$	a) $\frac{\pi}{3}, \frac{5\pi}{3}$
b) 135°	b) $\frac{20\pi}{3}$	b) $-\frac{\sqrt{2}}{2}$	b) $x \approx 16.7$	b) $\frac{\pi}{3}, \frac{2\pi}{3}$
c) 330°	c) $\frac{7\pi}{6}$	c) $\frac{\sqrt{3}}{3}$	c) $x \approx 36.9$	c) $\frac{3\pi}{4}, \frac{7\pi}{4}$
d) ≈57.3°	d) $\frac{3\pi}{2}$	d) 2	d) $x \approx 19.3$	d) $\frac{\pi}{4}, \frac{7\pi}{4}$
		e) $-\sqrt{2}$	e) $\theta \approx 121°$	
		f) $\sqrt{3}$	f) $\theta \approx 52.1°$	

6) a-c) See graphs on *Cartesian Geometry Part III*, Problem Set #6, problem #6

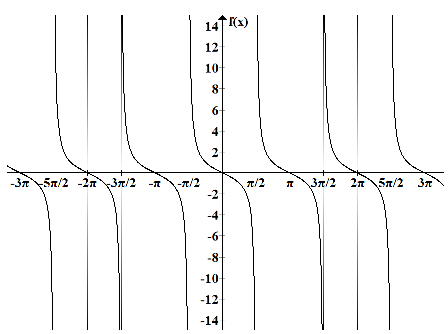
6) d)



6) e)



6) f)



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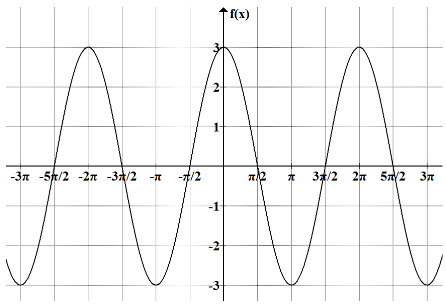
Problem Set #2

1) Answers may vary.

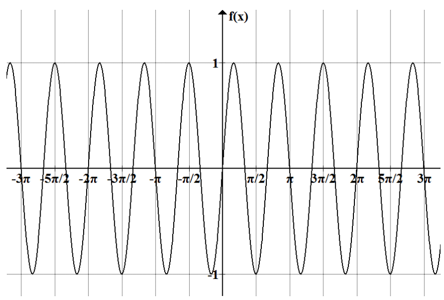
2) a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 0 d) $-\sqrt{2}$ e) 2 f) -1

3) a) $\frac{2\pi}{3}, \frac{4\pi}{3}$ b) $\frac{5\pi}{4}, \frac{7\pi}{4}$ c) $\frac{\pi}{3}, \frac{4\pi}{3}$ d) $\frac{5\pi}{6}, \frac{7\pi}{6}$

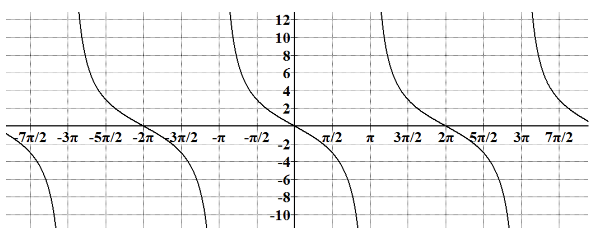
4) a)



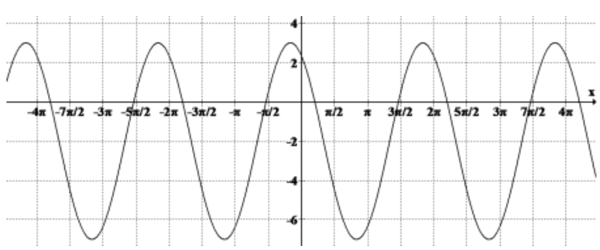
4) b)



4) c)



4) d)



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5)
- a) $x \approx 13.3$
- b) $\theta \approx 32.01°$
- c) Not a possible triangle.
- d) $x \approx 13.15$
- e) $\theta \approx 66.29°$
- f) $\theta \approx 65.74°$ <u>or</u> $\theta \approx 114.26°$
- g) $x \approx 4.133$ <u>or</u> $x \approx 9.435$
- h) $\theta \approx 32.16°$

6)
- a) $(\sec\theta + 1)(\sec\theta - 1)$
- b) $(\tan\theta + 4)(\tan\theta - 1)$
- c) $4\cos^2 x\,(\sin x - 2\cos x)$

7)
- a) $-\sin x$
- b) $\cos x$
- c) $\csc x$

Problem Set #3

1)
- a) $x \approx 7.25$
- b) $\theta \approx 48.2°$
- c) $\theta \approx 51.54°$
- d) $x \approx 124.85$
- e) Not a possible triangle.
- f) $\theta \approx 22.41°$

2)
- a) $\tan(\theta)$
- b) $-\cos(x)$
- c) $\sin(x)$
- d) $-\sin(x)$
- e) $-\tan(x)$
- f) $\csc(\theta)$
- g) $\sec(x)$
- h) $\cos^2(x)$

3) a) $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

$\sin^2 x - (1 - \sin^2 x) = 2\sin^2 x - 1$

$\sin^2 x - 1 + \sin^2 x = 2\sin^2 x - 1$

$2\sin^2 x - 1 = 2\sin^2 x - 1$

3) b) $\frac{2\cot x}{1+\cot^2 x} = 2\sin x\cos x$

$\frac{2\cot x}{\csc^2 x} = 2\sin x\cos x$

$\frac{2\cot x}{1} \cdot \frac{1}{\csc^2 x} = 2\sin x\cos x$

$\frac{2\cos x}{\sin x} \cdot \frac{\sin^2 x}{1} = 2\sin x\cos x$

$2\cos x\sin x = 2\sin x\cos x$

3) c) $\cot^2 x = (\csc x - 1)(\csc x + 1)$

$\cot^2 x = \csc^2 x - 1$

$\cot^2 x = \cot^2 x$

3) d) $\frac{\csc x}{\cot^2 x} = (\tan x)(\sec x)$

$\frac{\csc x}{1} \cdot \frac{1}{\cot^2 x} = (\tan x)(\sec x)$

$\frac{1}{\sin x} \cdot \frac{\tan^2 x}{1} = (\tan x)(\sec x)$

$\frac{1}{\sin x} \cdot \tan x \cdot \tan x =$

$(\tan x)(\sec x)$

$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \tan x =$

$(\tan x)(\sec x)$

$\frac{1}{\cos x} \cdot \tan x = (\tan x)(\sec x)$

$\sec x \cdot \tan x = (\tan x)(\sec x)$

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3) e) $\tan^2 A - \sin^2 A = \sin^2 A \cdot \tan^2 A$

$$\tan^2 A - \sin^2 A = \sin^2 A \tan^2 A$$

$$\frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \sin^2 A \tan^2 A$$

$$\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 A \cos^2 A}{\cos^2 A} = \sin^2 A \tan^2 A$$

$$\frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A} = \sin^2 A \tan^2 A$$

$$\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A} = \sin^2 A \tan^2 A$$

$$\frac{\sin^2 A \sin^2 A}{\cos^2 A} = \sin^2 A \tan^2 A$$

$$\sin^2 A \frac{\sin^2 A}{\cos^2 A} = \sin^2 A \tan^2 A$$

$$\sin^2 A \tan^2 A = \sin^2 A \tan^2 A$$

3) f) $\sec\theta - \cos\theta = \sin\theta \tan\theta$

$$\frac{1}{\cos\theta} - \frac{\cos\theta}{1} = \sin\theta \tan\theta$$

$$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} = \sin\theta \tan\theta$$

$$\frac{1 - \cos^2\theta}{\cos\theta} = \sin\theta \tan\theta$$

$$\frac{\sin^2\theta}{\cos\theta} = \sin\theta \tan\theta$$

$$\frac{\sin\theta}{1} \cdot \frac{\sin\theta}{\cos\theta} = \sin\theta \tan\theta$$

$$\sin\theta \tan\theta = \sin\theta \tan\theta$$

3) g) $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$$

3) h) $\frac{1+\csc x}{\sec x} = \cos x + \cot x$

$$\frac{1}{\sec x} + \frac{\csc x}{\sec x} = \cos x + \cot x$$

$$\cos x + \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} = \cos x + \cot x$$

$$\cos x + \frac{\cos x}{\sin x} = \cos x + \cot x$$

$$\cos x + \cot x = \cos x + \cot x$$

4) Using these identities:

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\sin 2u = 2 \sin u \cdot \cos u$

$\cos 2u = 1 - 2\sin^2 u$

$\cos^2 x = 1 - \sin^2 u$

We then get:

$\sin(3x) = \sin(2x+x)$

$= \sin(2x)\cos x + \cos(2x)\sin x$

$= [2\sin x \cos x]\cos x + [1 - 2\sin^2 x]\sin x$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x[1 - \sin^2 x] + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$\mathbf{= 3\sin x - 4\sin^3 x}$

5) $\frac{1}{1+\sin\theta} \rightarrow \frac{1}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$

$\rightarrow \frac{1-\sin\theta}{1-\sin^2\theta} \rightarrow \frac{1-\sin\theta}{\cos^2\theta}$

$\rightarrow \frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos^2\theta}$

$\rightarrow \sec^2\theta - \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$\rightarrow \sec^2\theta - \sec\theta\tan\theta$

6)

a) $\frac{\pi}{3}, \frac{5\pi}{3}$

b) $\frac{3\pi}{4}, \frac{7\pi}{4}$

Trigonometry – Part IV ANSWERS

Problem Set #4

1)

a) $-\sin(x)$
b) $-\sin(x)$
c) $\sin^2(x)$
d) $\cot(x)$
e) Using the identity:

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$$

$$\tan(x + \frac{\pi}{4}) = \frac{\tan x + 1}{1 - \tan x}$$

f) $\sec^2(x)$
g) $\sec^4(x)$
h) $-\sin^3(x)$

2) a) $\frac{\tan x \cot x}{\cos x} = \sec x$

$$\frac{1}{\cos x} = \sec x$$

2) b) $(1 - \tan\theta)^2 = \frac{1 - \sin 2\theta}{\cos^2\theta}$

$$1 - \frac{2\sin\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} - \frac{2\sin\theta\cos\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$$

$$\frac{1 - 2\sin\theta\cos\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$$

$\frac{1 - \sin 2\theta}{\cos^2\theta} = \frac{1 - \sin 2\theta}{\cos^2\theta}$ because

$\sin 2\theta = 2\sin\theta\cos\theta$

2) c) $\frac{1 + \tan\theta}{1 + \cot\theta} = \tan\theta$

$$\frac{1 + \frac{\sin\theta}{\cos\theta}}{1 + \frac{\cos\theta}{\sin\theta}} = \tan\theta$$

$$\frac{\frac{\cos\theta + \sin\theta}{\cos\theta}}{\frac{\sin\theta + \cos\theta}{\sin\theta}} = \tan\theta$$

$$\frac{\cos\theta + \sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta + \cos\theta} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

2) d) $\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x\,(1 + \cos x)}{1 - \cos^2 x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x\,(1 + \cos x)}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{1 + \cos x}{\sin x}$$

2) e) $(1 - \sin^2 x)\csc^2 x = \cot^2 x$

$$\cos^2 x \cdot \frac{1}{\sin^2 x} = \cot^2 x$$

2) f) $\frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = 2\csc x$

$$\frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} + \frac{\sin x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} = 2\csc x$$

$$\frac{\sin x\,(1 + \cos x) + \sin x\,(1 - \cos x)}{1 - \cos^2 x} = 2\csc x$$

$$\frac{2\sin x + \cos x\sin x - \cos x\sin x}{\sin^2 x} = 2\csc x$$

$$\frac{2\sin x}{\sin^2 x} = 2\csc x$$

$$\frac{2}{\sin x} = 2\csc x$$

3) a) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

b) $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

c) No solution.

d) $x = \frac{\pi}{3}, \frac{5\pi}{3}$

e) $2\cot x\cos^2 x = \cot x$

(Don't divide by $\cot x$)

$2\cot x\cos^2 x - \cot x = 0$

$\cot x\,(2\cos^2 x - 1) = 0$

either $\cot x = 0$

solutions: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

or $2\cos^2 x - 1 = 0$

$\cos x = \pm\frac{\sqrt{2}}{2}$

solutions: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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3) f) $2\sin^2 x + 3\cos x = 0$

$2(1 - \cos^2 x) + 3\cos x = 0$

$2\cos^2 x - 3\cos x - 2 = 0$

$(2\cos x + 1)(\cos x - 2) = 0$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

4) Using these identities:

$\cos(u \pm v) = \cos u \cdot \cos v \mp \sin u \cdot \sin v$
$\sin 2u = 2 \sin u \cdot \cos u$
$\cos 2u = 2\cos^2 u - 1$
$\sin^2 u = 1 - \cos^2 u$

We then get:

$\cos(3x) = \cos(2x + x)$
$= \cos(2x) \cdot \cos x - \sin(2x) \cdot \sin x$
$= [2\cos^2 x - 1]\cos x - [2\sin x \cdot \cos x]\sin x$
$= 2\cos^3 x - \cos x - 2\sin^2 x \cdot \cos x$
$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$
$= \mathbf{4\cos^3 x - 3\cos x}$

5) Using the identity:

$\cos^2 u = \frac{1}{2}(1 + \cos 2u)$

We get:

$\cos^4(x) = \cos^2(x) \cdot \cos^2(x)$

$= \frac{1}{2}[1 + \cos(2x)] \frac{1}{2}[1 + \cos(2x)]$

$= \frac{1}{4}[1 + 2\cos(2x) + \cos^2(2x)]$

$= \frac{1}{4}[1 + 2\cos(2x) + \frac{1}{2}[1 + \cos(4x)]] = \mathbf{\frac{3}{8} + \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)}$

6) $\theta = \tan^{-1}\left(\frac{x}{55}\right)$

Trigonometry – Part IV ANSWERS

Problem Set #5

1)

a)	-1	e)	$\sin(x)$
b)	$\tan(x)$	f)	$-\tan(x)$
c)	$\cos^2(x)$	g)	$\sec^4(x)$
d)	$\cot(x)$		

2) a)

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$(1 + \tan^2\theta) - \tan^2\theta = 1$$
$$1 = 1$$

2) b)

$$(\cot x + \tan x)^2 - \cot^2 x = \tan^2 x + 2$$
$$\cot^2 x + 2\cot x \tan x + \tan^2 x - \cot^2 x = \tan^2 x + 2$$
$$2\cot x \tan x + \tan^2 x = \tan^2 x + 2$$
$$2 + \tan^2 x = \tan^2 x + 2$$

2) c) $\dfrac{\cos^2 x + \cot x}{\cos^2 x - \cot x} = \dfrac{\cos^2 x \tan x + 1}{\cos^2 x \tan x - 1}$

$$\frac{\cos^2 x + \frac{\cos x}{\sin x}}{\cos^2 x - \frac{\cos x}{\sin x}} = \frac{\cos^2 x \frac{\sin x}{\cos x} + 1}{\cos^2 x \frac{\sin x}{\cos x} - 1}$$

$$\frac{\frac{\cos^2 x \sin x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\cos^2 x \sin x}{\sin x} - \frac{\cos x}{\sin x}} = \frac{\cos x \sin x + 1}{\cos x \sin x - 1}$$

$$\frac{\frac{\cos x(\cos x \sin x + 1)}{\sin x}}{\frac{\cos x(\cos x \sin x - 1)}{\sin x}} = \frac{\cos x \sin x + 1}{\cos x \sin x - 1}$$

$$\frac{\cos x \sin x + 1}{\cos x \sin x - 1} = \frac{\cos x \sin x + 1}{\cos x \sin x - 1}$$

2) d) $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \csc\theta$

$$\frac{\sin\theta}{1+\cos\theta} + \cot\theta = \csc\theta$$

$$\frac{\sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin\theta - \sin\theta\cos\theta}{\sin^2\theta} + \frac{\sin\theta\cos\theta}{\sin^2\theta} = \csc\theta$$

$$\frac{\sin\theta - \sin\theta\cos\theta + \sin\theta\cos\theta}{\sin^2\theta} = \csc\theta$$

$$\frac{\sin\theta}{\sin^2\theta} = \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$\csc\theta = \csc\theta$$

2) e) $\cos^4 x - \sin^4 x = \cos(2x)$

$$\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$$
$$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$$
$$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$$

2) f) $\sec x - \tan x \cdot \sin x = \cos x$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \cos x$$

$$\frac{1 - \sin^2 x}{\cos x} = \cos x$$

$$\frac{\cos^2 x}{\cos x} = \cos x$$

$$\cos x = \cos x$$

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3)

a) $x = \pm\frac{2\pi}{3}$

b) $x = \frac{\pi}{3}, -\frac{2\pi}{3}$

c) $x = 0, \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pi$

d) $x = \pm\frac{\pi}{2}, \pi$

e) $x \approx \pm 1.37$

f) Because $-\pi < x \le \pi$ we know that $-3\pi < 3x \le 3\pi$. Quickly we get $\cos(3x) = -½$ There are two locations on the trig unit circle where the cosine is -½ : at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. But since $3x$ can go up to 3π, we can also add 2π to these answers. This leads to $3x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{8\pi}{3}$.
Therefore $x = \pm\frac{2\pi}{9}, \pm\frac{4\pi}{9}, \pm\frac{8\pi}{9}$

3) g) $\sec^2(x) - 2\tan(x) = 4$
$1 + \tan^2(x) - 2\tan(x) = 4$
$\tan^2(x) - 2\tan(x) - 3 = 0$
$(\tan(x) - 3)(\tan(x) + 1) = 0$
$\tan(x) = 3$ or $\tan(x) = -1$
$x \approx 1.25, -1.89, -\frac{\pi}{4}, \frac{3\pi}{4}$

4) $\frac{\pi}{6} + \pi n$ for any integer n.

5) Using these identities:

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

We then get:

$$\tan(3x) = \tan(2x + x) = \frac{\tan(2x) + \tan x}{1 - \tan(2x)\cdot\tan x} = \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x}\cdot\tan x}$$

$$= \frac{\frac{3\tan x - \tan^3 x}{1-\tan^2 x}}{\frac{1-3\tan^2 x}{1-\tan^2 x}} = \mathbf{\frac{3\tan x + \tan^3 x}{1 - 3\tan^2 x}}$$

Another option is using $\tan(3x) = \frac{\sin(3x)}{\cos(3x)}$ to find the same answer.

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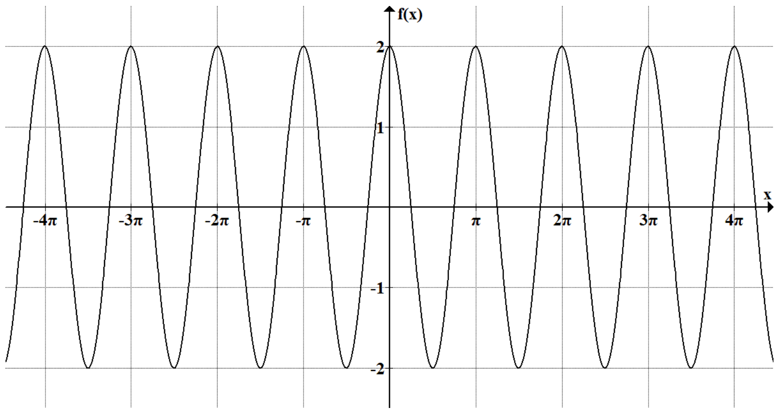
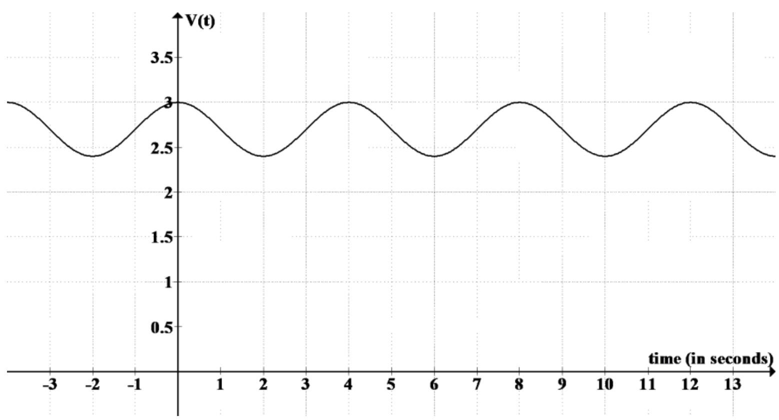
6) $f(x) = 4\cos^2 x - 2 = 2\cos(2x)$



7) $V(t) = 2.7 + 0.3\cos(\frac{\pi}{2}t)$

- 2.7 is the average of the maximum and minimum volume so is therefore the vertical shift of the graph.
- 0.3 is the amplitude (the difference between the max/min and the middle).
- 15 breathes per minute means 1 breathe every 4 seconds therefore the period is 4 thus we plug in $\frac{2\pi}{4} = \frac{\pi}{2}$.
- Henry has fully inhaled at time zero so therefore cosine has not shifted right or left at all.



Trigonometry – Part IV ANSWERS

Problem Set #6

1)

a) $x \approx 4.114$

b) $x \approx 6.55$ <u>or</u> $x \approx 5.43$

c) $x \approx 8.144$

d) $\alpha \approx 67.46°$

2)

a) $\tan(\theta)$ d) $\sin(\theta)$

b) $\sin^2(x)$ e) $-\sin(x)$

c) $\csc(y)$ f) $-\csc(x)$

3)a) $\sin x \cos x \tan x = 1 - \cos^2 x$

$\sin x \cos x \tan x = \sin^2 x$

$\sin x \cos x \frac{\sin x}{\cos x} = \sin^2 x$

$\sin^2 x = \sin^2 x$

3) b) $\frac{\cos 3x}{6} + \frac{\cos x}{2} = \frac{2\cos^3 x}{3}$

$\frac{\cos 3x}{6} + \frac{3\cos x}{6} = \frac{2\cos^3 x}{3}$

$\frac{\cos 3x + 3\cos x}{6} = \frac{2\cos^3 x}{3}$

$(\cos 3x = 4\cos^3 x - 3\cos x)$

$\frac{4\cos^3 x - 3\cos x + 3\cos x}{6} = \frac{2\cos^3 x}{3}$

$\frac{4\cos^3 x}{6} = \frac{2\cos^3 x}{3}$

$\frac{2\cos^3 x}{3} = \frac{2\cos^3 x}{3}$

3) c) $\frac{2\cot x}{1+\cot^2 x} = \sin(2x)$

$\frac{2\cot x}{\csc^2 x} = \sin(2x)$

$\frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = \sin(2x)$

$\frac{2\cos x}{\sin x} \cdot \frac{\sin^2 x}{1} = \sin(2x)$

$2\sin x \cos x = \sin(2x)$

$\sin(2x) = \sin(2x)$

3) d) $\frac{\sin\theta\tan\theta + \cos\theta}{\sin\theta\sec\theta} = \csc\theta$

$\frac{\sin\theta\frac{\sin\theta}{\cos\theta} + \cos\theta}{\sin\theta\frac{1}{\cos\theta}} = \csc\theta$

$\frac{\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \csc\theta$

$\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \csc\theta$

$\frac{1}{\sin\theta} = \csc\theta$

$\csc\theta = \csc\theta$

3) e) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

$\frac{1}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta} + \frac{1}{1-\sin\theta} \cdot$

$\frac{1+\sin\theta}{1+\sin\theta} = 2\sec^2\theta$

$\frac{1-\sin\theta + 1 + \sin\theta}{1-\sin^2\theta} = 2\sec^2\theta$

$\frac{2}{\cos^2\theta} = 2\sec^2\theta$

$2\sec^2\theta = 2\sec^2\theta$

3) f) $\sin y + \sin y \cot^2 y = \csc y$

$\frac{\sin^2 y}{\sin y} + \sin y \frac{\cos^2 y}{\sin^2 y} = \csc y$

$\frac{\sin^2 y + \cos^2 y}{\sin y} = \csc y$

$\frac{1}{\sin y} = \csc y$

$\csc y = \csc y$

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4) Using the identity:

$\tan\frac{1}{2}u = \frac{\sin u}{1+\cos u}$ We get:

$\tan(\frac{1}{2}\cdot\frac{\pi}{4}) = \frac{\sin(\frac{\pi}{4})}{1+\cos(\frac{\pi}{4})} = \sqrt{2} - 1$

5)

a) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

b) $x = \frac{\pi}{4}, \frac{5\pi}{4}$

c) Surprisingly, there are 30 answers for this!

If $3\sin(5x) = 0$ then we get $5x = \sin^{-1}(0)$

$5x = 0, \pi, 2\pi, 3\pi\ldots$

$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5} \ldots \frac{9\pi}{5}$

If $2\sin^2(5x) - 1 = 0$ then we get $5x = \sin^{-1}\left(\pm\frac{\sqrt{2}}{2}\right)$

$5x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots$

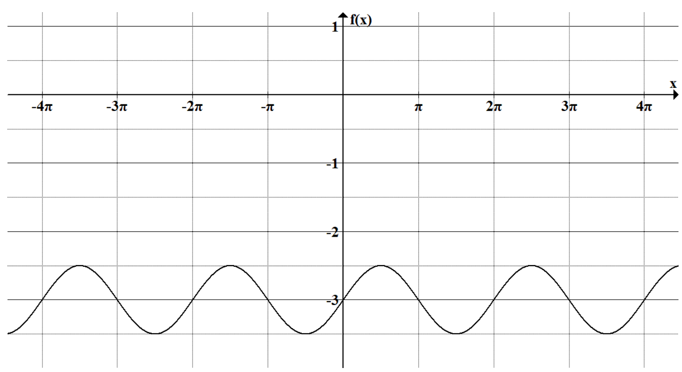
$x = \frac{\pi}{20}, \frac{3\pi}{20}, \frac{5\pi}{20}, \frac{7\pi}{20} \ldots \frac{39\pi}{20}$

d) $x = \frac{3\pi}{2}$, ≈ 0.443, ≈ 2.7

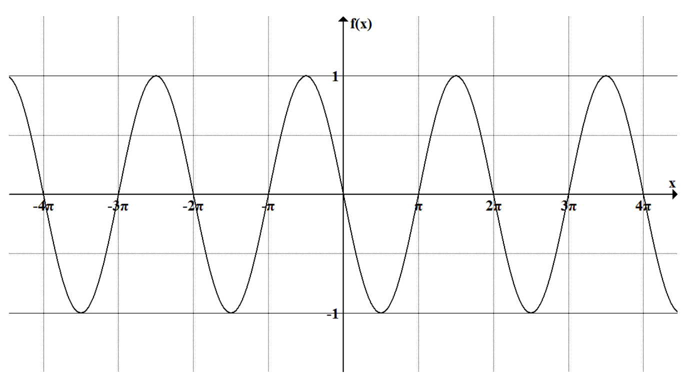
e) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3},$

f) $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

6) a)



6) b)



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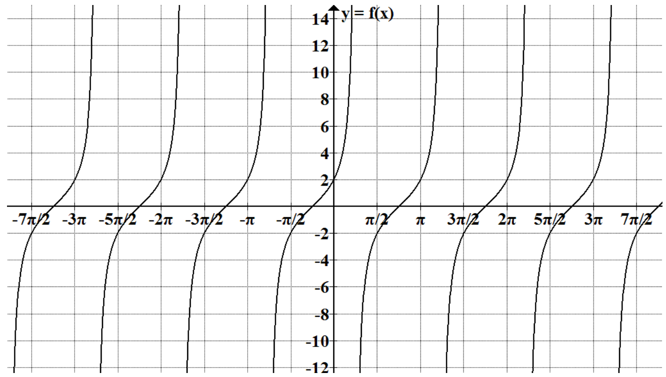
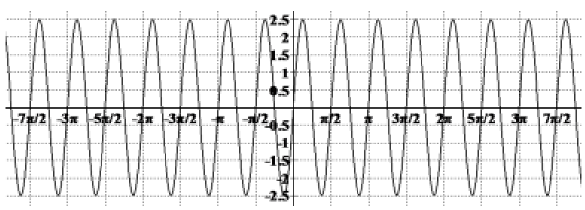
6) c)



6) d)



7)

a) After $\frac{10\pi}{3}$ or ≈ 10.472 seconds the slower bug is one-third of the way around and faster bug is two-thirds of the way around.

b) 10π or ≈ 31.4159 seconds