Excerpts from

Calculus Discovery Sheets

(for the WHiSTEP Mathematics Seminar)

Discovery Sheet #4

Instantaneous Speed, Part I

We will now work toward finding a general method for determining instantaneous speed.

1) If we roll a ball down an inclined plane with a steepness of about 37.8°, we get a convenient distance formula:

$$d(t) = 3 \cdot t^2$$

Use this distance formula in order to calculate the average speed…

a) from 4 seconds to 6 seconds.
b) from 4 seconds to 5 seconds.
c) from 4 seconds to 4.5 seconds.
d) from 4 seconds to 4.1 seconds.
e) from 4 seconds to 4.01 seconds.
f) from 4 seconds to 4.0001 seconds.

2) Find the instantaneous speed at 4 seconds.

3) Now use the same method to find the instantaneous speed at 7 seconds.

4) Explain each of the following *Average Speed Formulas*

$$r = \frac{\Delta d}{\Delta t} \qquad r = \frac{d(t_2) - d(t_1)}{t_2 - t_1}$$

$$r = \frac{d_2 - d_1}{t_2 - t_1} \qquad r = \frac{d(t+h) - d(t)}{h}$$

Discovery Sheet #5

Instantaneous Speed, Part II

The *Calculus Average Speed Formula* is:

$$r = \frac{d(t+h) - d(t)}{h}$$

Use this formula to do the below problems.

1) Given $d(t) = 3 \cdot t^2$, find the *average speed* from 5 seconds to 8 seconds.

2) Given $d(t) = 3 \cdot t^2$, give a formula for *average speed*, r, from 5 seconds to 5+h seconds.

3) Given $d(t) = 3 \cdot t^2$, give a formula for *average speed*, r, from t seconds to t+h seconds.

4) Given $d(t) = 3 \cdot t^2$, give a formula for *instantaneous speed*, v(t), at t seconds.

5) Given $d(t) = k \cdot t^2$, give a formula for *instantaneous speed*, v(t), at t seconds.

Discovery Sheet #6

The Derivative, Part I

We just found formulas for calculating average speed, r, and instantaneous speed, v(t). Why is this so important? Because we have managed to get around the paradox of instantaneous speed: dividing zero by zero! These formulas are:

$r = 30 + 3h$

$r = 6t + 3h$

$v(t) = 6t$

$v(t) = 2kt$

1) Explain, once again, what each of the above formulas can be used for, and what each of the variables represent.

2) Use an above formula to solve each:

a) Given $d(t) = 3 \cdot t^2$, find the *average speed* from 5 seconds to 12 seconds.
b) Given $d(t) = 3 \cdot t^2$, find the *instantaneous speed* at 3 seconds.
c) Given $d(t) = 2.2 \cdot t^2$, find the *instantaneous speed* at 4 seconds.
d) Given $d(t) = 4.9 \cdot t^2$, find the *instantaneous speed* at 6 seconds.

Discovery Sheet #7

The Derivative, Part II

The Definition of the Derivative.

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is not really a formula. It simply tells us what to do in order to find a formula for the instantaneous rate of change for f(x).

1) Use the above definition of the derivative in order to determine the derivative of:

a) $f(x) = x^3$ b) $f(x) = x^2 + 7x$

2) Given what you have learned so far, what do you think the derivative of each function is?

a) $f(x) = x^7$
b) $f(x) = 4x^3$
c) $f(x) = x^5 + 7x^4 - 2x^3 + x^2 - 3x + 8$

3) *Derive the Exponent Law for Derivatives!*
Find $f'(x)$ given $f(x) = k \cdot x^n$