12th Grade Assignment – Week #5

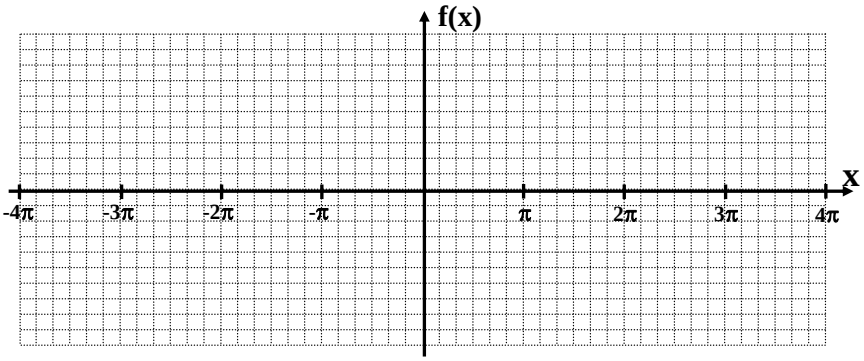
Group Assignments: *for either Tuesday or Thursday*

1) Together, work on problem #7 from **Problem Set #6** (*Trigonometry – Part IV* unit).
2) Help each other out preparing for the test (on *Trigonometry – Part IV*) by going over selected problems from **Problem Set #6**.
3) *Puzzle!* **The Thirsty Cow**. A cow is standing 360m from a long, straight river. The farmhouse, which is on the same side of the river as the cow, is 60m from the river. If the cow were to walk directly to the farmhouse, the distance would be 500m. However, the cow wishes to get a drink from the river before walking to the farmhouse. What is the shortest total distance for the cow to walk first to the river and then to the farmhouse?
4) *Puzzle!* **Tennis Tournament**.
   Background: A single-elimination tournament is when the players (or teams) play in a "bracket" and keep advancing in the tournament until losing. You lose once, and you are finished! Often times, single-elimination tournaments are arranged so that every player/team would have to win the same number of matches to win the tournament, such as the tournament bracket shown below on the left. But with some tournaments, it is different. Sometimes a player (or team) gets a "bye" (which means they get to advance without playing the first round), as shown below (on the right) with the 2012 NFL (football) playoff backet. You will notice that Baltimore had to play one more game than San Francisco in order to get to the final.

   As an extreme example, the Wimbledon tennis tournament used to be arranged so that the previous year's champion automatically played in the next year's final. The reality is that – perhaps surprisingly – no matter how a single-elimination tournament is structured for a given number of participants, the total number of matches played will be the same.
   Question: If there are 100 players that enter a (single-elimination) tennis tournament, how many total matches must be played?

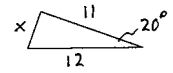
Individual Work

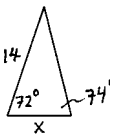
- Work on the problems from **Problem Set #6** (*Trigonometry – Part IV* unit).
- Take the *Trigonometry – Part IV Test*, which is found at the end of this document.

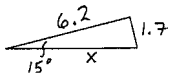


Problem Set #6

1) Find the variable.

a) 

c) 

b) 

d) 

2) Simplify.

a) $\tan(\theta + 2\pi)$

b) $\dfrac{1}{1+\cot^2 x}$

c) $\sec y \tan(\frac{\pi}{2} - y)$

d) $\sin^3\theta + \sin\theta\cos^2\theta$

e) $\dfrac{(\sin^2 x - 1)(\tan^2 x + 1)}{\csc x}$

f) $\dfrac{1+\sec(-x)}{\sin(-x)+\tan(-x)}$

3) Prove each identity.

a) $\sin x \cos x \tan x = 1 - \cos^2 x$

b) $\dfrac{\cos 3x}{6} + \dfrac{\cos x}{2} = \dfrac{2\cos^3 x}{3}$

c) $\dfrac{2\cot x}{1+\cot^2 x} = \sin(2x)$

d) $\dfrac{\sin\theta\tan\theta + \cos\theta}{\sin\theta\sec\theta} = \csc\theta$

e) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

f) $\sin y + \sin y \cot^2 y = \csc y$

4) Find the value of $\tan(\pi/8)$ without using a calculator. (Give an exact answer in radical form.)

5) Solve for all values of x such that $0 \le x < 2\pi$.

a) $\frac{1}{2}\sec x + 1 = 0$

b) $5\tan x - 1 = 3\tan x + 1$

c) $6\sin^3(5x) - 3\sin(5x) = 0$

d) $7\cos^2 x - 4\sin x = 4$

e) $6\csc^2 x = 8$

f) $2 - \sin^2 x = 2\cos^2(\frac{1}{2}x)$

6) Graph each trigonometric function, for all x such that $-4\pi \le x \le 4\pi$.

a) $f(x) = \frac{1}{2}\sin(x) - 3$

b) $f(x) = -\cos(x - \pi/2)$

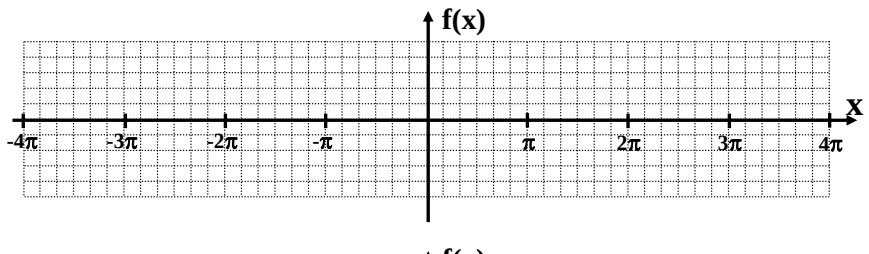
c) $f(x) = 2\tan(x + \pi/4)$

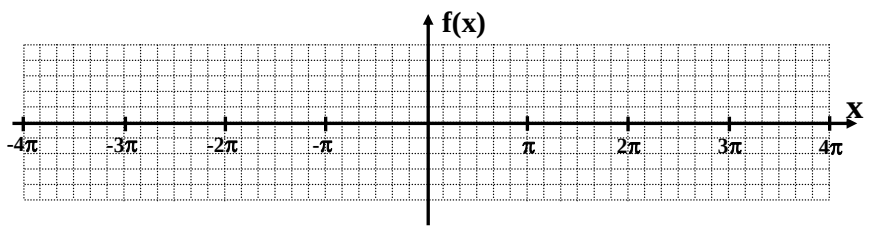
d) $f(x) = 5/2\sin(4x)$

7) Two bugs crawl along a circular path inside a stationary hampster wheel. Answer the following questions given that the wheel has a radius of 5 cm, the slower bug is crawling at a rate of 1 cm/sec, the faster bug at a rate of 2 cm/sec, and both bugs start out together in the same direction at the bottom of the wheel.

a) After how much time are both of the bugs at the same elevation (for the first time)?

b) How long does it take the faster bug to lap the slower bug?





Trig IV Test

1) Simplify each. (3 points each.)

a) $\sin\left(x + \frac{\pi}{2}\right)$

b) $\cos\left(x - \frac{3\pi}{2}\right)$

c) $\sec x \left(\frac{\sin x}{\tan x}\right)$

d) $\sin^2 x \left(\csc^2 x - 1\right)$

e) $\cos x \cot\left(\frac{\pi}{2} - x\right)$

2) Prove each identity. (6 points each.)

a) $\frac{1}{1 + \tan^2 x} = \cos^2 x$

b) $\sin^4 x - \cos^4 x + 1 = 2\sin^2 x$

c) $\cot^2 x - \cot^2 x \cos^2 x = \sin^2 x \csc^2 x - \sin^2 x$

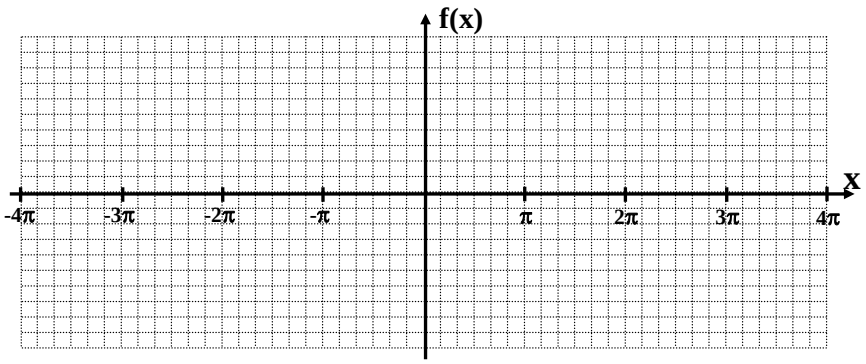
3) Solve, giving all possible values for x. (4 points)

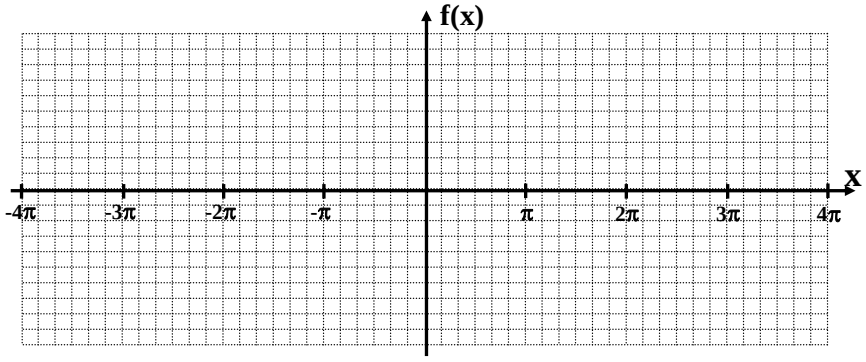
$2\cos x - \sqrt{3} = 0$

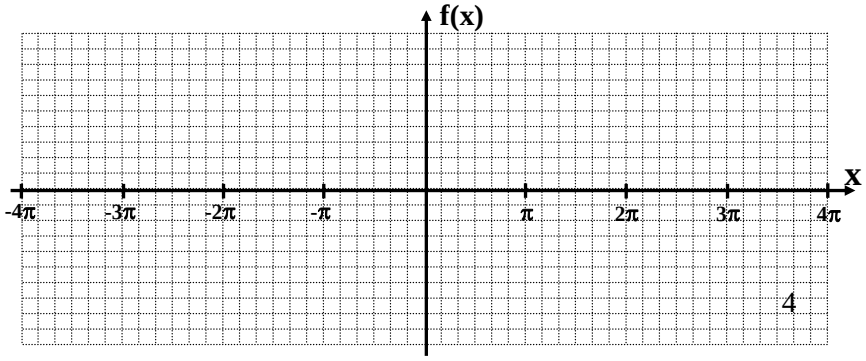
4) Graph each trigonometric function, on the below graph, for $-4\pi \le x \le 4\pi$. (6 points each)

a) $f(x) = 3 + \cos x$

b) $f(x) = \sin(x - \frac{\pi}{2})$







5) Solve for x such that $-\pi < x \le \pi$. (4 points)

$\sin(2x) = \cos x$

6) Solve for x such that $0 \le x < 2\pi$. (4 points)

$4\sec^2 x - \tan^4 x = 7$