12th Grade Assignment – Week #4

Group Assignments: *for either Tuesday or Thursday*

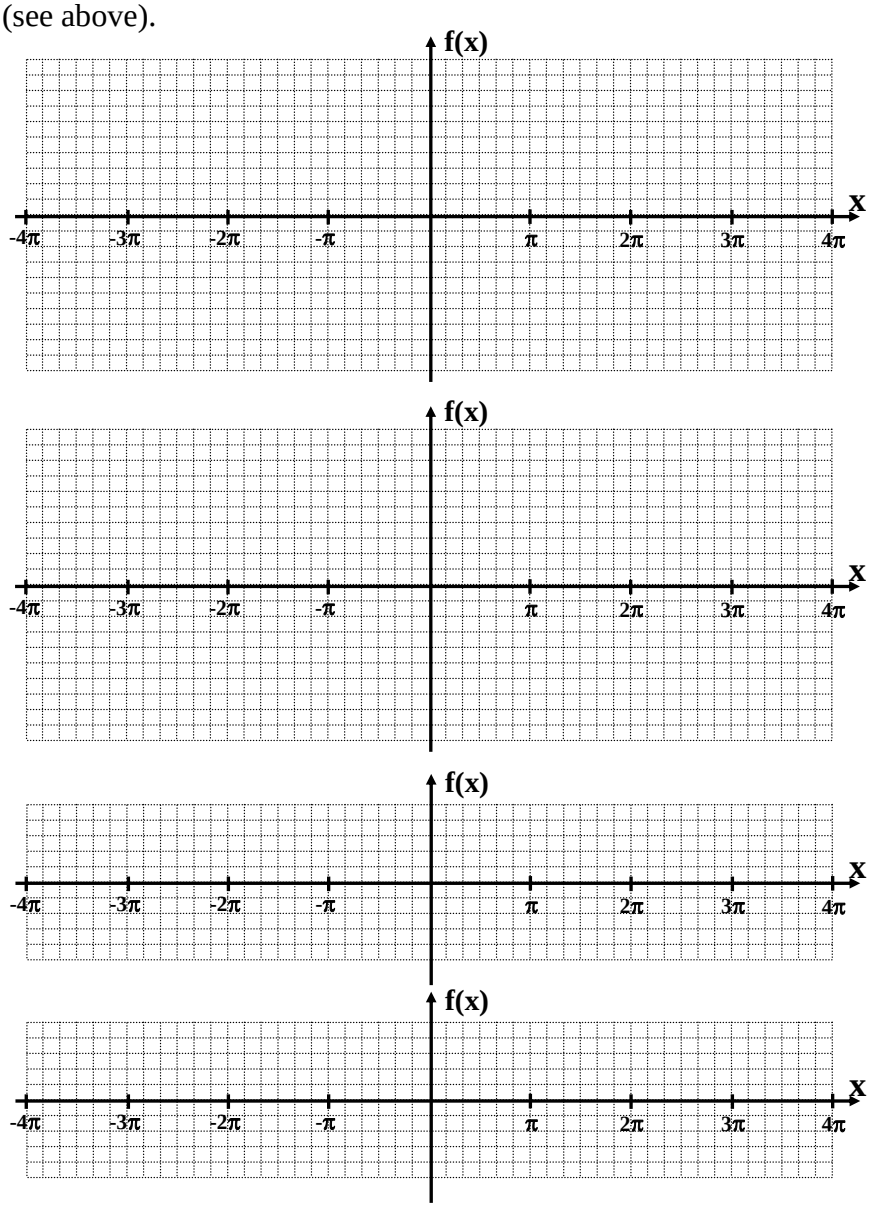
1) Graph these functions:
   a) $f(x) = 3 \sin (x - \pi/6)$
   b) $f(x) = \cos (\frac{1}{2} x) - 3$
   c) $f(x) = -\tan (3x)$

2) Together, work on these problems from the *Trigonometry – Part IV* unit:
   Problem Set #4: Problems #1b, 2e, 3e, 4, 6.
   Problem Set #5: Problems #3b, 6, 7

3) *Puzzle!* **Four Sons**
   An old man had four sons who asked for some money before they set forth into the world. The old man decided to distribute the money in the following manner. He took his entire life savings of gold coins and gave the first son 4 coins, and then gave him one-fourth of what remained. Then he went to the second son and gave him 4 coins, and one-fourth of what remained. He then he did the same with the third and the fourth son. What is the fewest number of coins that the old man could have started with? (Assume that every time he took one-fourth of what remained (i.e., divided by 4) there was no remainder?)

Individual Work

- Work on the problems from Problem Set #4 and Problem Set #5, except for those assigned for group work (see above).



Problem Set #4

1) Simplify each.

a) $\sin(x + \pi)$

b) $\cos(x - \frac{3\pi}{2})$

c) $\frac{\tan^2 x}{\sec^2 x}$

d) $\frac{\sin(\pi/2 - x)}{\cos(\pi/2 - x)}$

e) $\tan(x + \frac{\pi}{4})$

(You may leave "tan x" in the denominator.)

f) $\frac{\sec^2 x - 1}{\sin^2 x}$

g) $\sec^2 x \tan^2 x + \sec^2 x$

h) $\sin x \cos^2 x - \sin x$

2) Prove each identity.

a) $\frac{\tan x \cot x}{\cos x} = \sec x$

b) $(1 - \tan\theta)^2 = \frac{1 - \sin 2\theta}{\cos^2 \theta}$

c) $\frac{1 + \tan\theta}{1 + \cot\theta} = \tan\theta$

d) $\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$

e) $\csc^2 x - \csc^2 x \cdot \sin^2 x = \cot^2 x$

f) $\frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$

3) Solve for all values of x such that $0 \le x < 2\pi$.

a) $2 \sin x - \sqrt{3} = 0$

b) $2 \sin x + \sqrt{3} = 0$

c) $4 \cos x - 6 = \cos x$

d) $3 \sec x + 5 = 6 \sec x - 1$

e) $2 \cot x \cos^2 x = \cot x$

f) $2 \sin^2 x + 3 \cos x = 0$

4) Rewrite $\cos(3x)$ in terms of $\cos x$ only.

5) Rewrite $\cos^4 x$ without exponents.

6) A right triangle has a horizontal leg of 55m, a vertical leg of x. The angle between the hypotenuse and the horizontal leg is θ. Write an expression for θ in terms of x.

Problem Set #5

1) Simplify.

a) $\tan^2 x - \sec^2 x$

b) $\frac{\cos(\pi/2 - x)}{\cos x}$

c) $\sin x\,(\csc x - \sin x)$

d) $\sin(\frac{\pi}{2} - x)\csc x$

e) $\frac{1 + \sin x}{1 + \csc x}$

f) $\tan(\pi - x)$

g) $\tan^4\theta + 2\tan^2\theta + 1$

2) Prove each identity.

a) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

b) $(\cot x + \tan x)^2 - \cot^2 x = \tan^2 x + 2$

c) $\frac{\cos^2 x + \cot x}{\cos^2 x - \cot x} = \frac{\cos^2 x \tan x + 1}{\cos^2 x \tan x - 1}$

d) $\frac{\sin\theta}{1+\cos\theta} + \cot\theta = \csc\theta$

e) $\cos^4 x - \sin^4 x = \cos(2x)$

f) $\sec x - \tan x \sin x = \cos x$

3) Solve for all values of x such that $-\pi < x \le \pi$.

a) $3\cos x + 1 = \cos x$

b) $5\sqrt{3}\cot x - 3 = 4\sqrt{3}\cot x - 2$

c) $16\sin^3 x = 12\sin x$

d) $\cos x + 1 = \sin x$

e) $5\cos^2 x = 2 - 9\cos x$

f) $2\cos(3x) + 1 = 0$

g) $\sec^2 x - 2\tan x = 4$

4) Find the roots of f(x), given $f(x) = 3\tan^2 x - 1$. (Give all possible answers.)

5) Rewrite tan(3x) in terms of tan x only.

6) Graph $f(x) = 4\cos^2 x - 2$
(Hint: First rewrite it without exponents.)

7) In a normal rest state, Henry takes 15 breathes per minute. After inhaling, his lungs contain 3.0 liters of air. After exhaling, his lungs contain 2.4 liters of air. Give the function, V(t), where t is time, and V is the volume of air in his lungs. Assume that at time zero, he has just fully inhaled. Also graph V(t).