12th Grade Assignment – Week #3

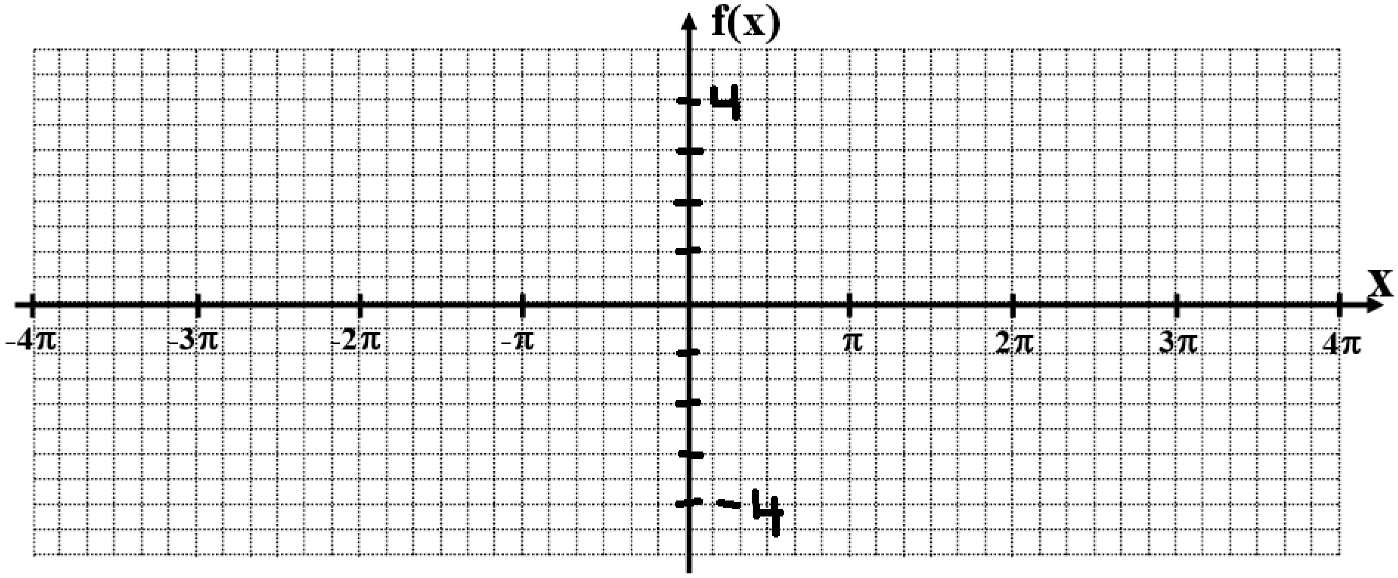
Individual Work

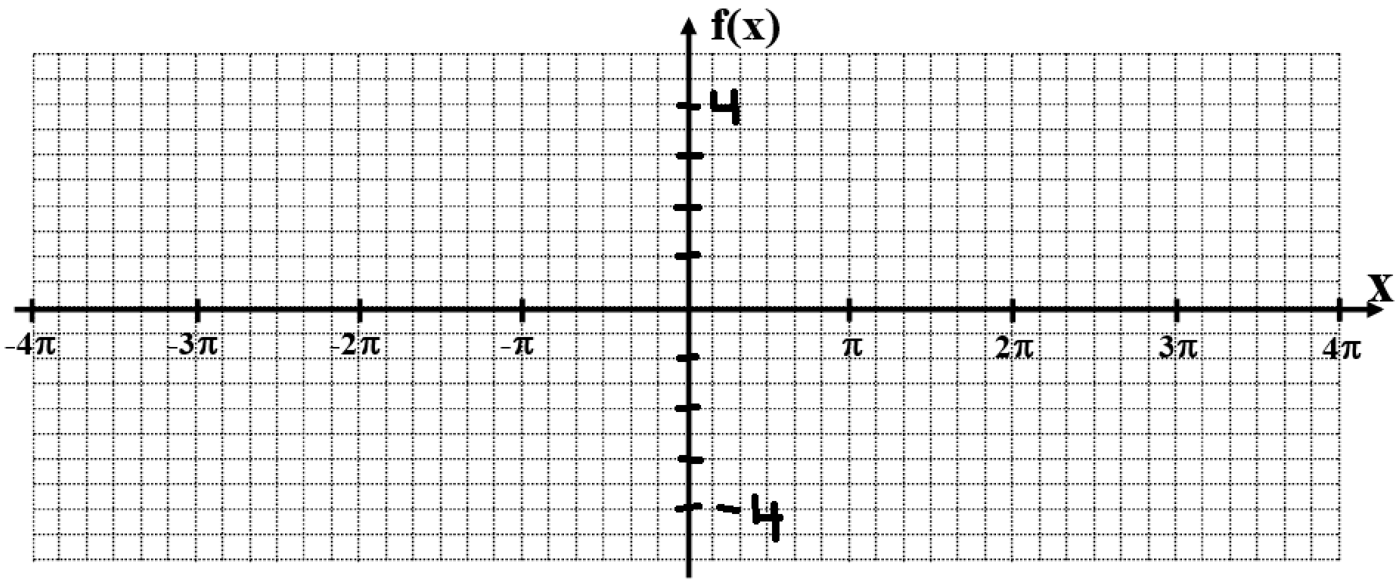
- Did you already take the *11th Grade Review Test* ? (See last week's assignment.)
- Work on the problems from the *Trigonometry – Part IV* unit, **Problem Set #2 and Problem Set #3** (except don't do problem #6).

Group Assignments:

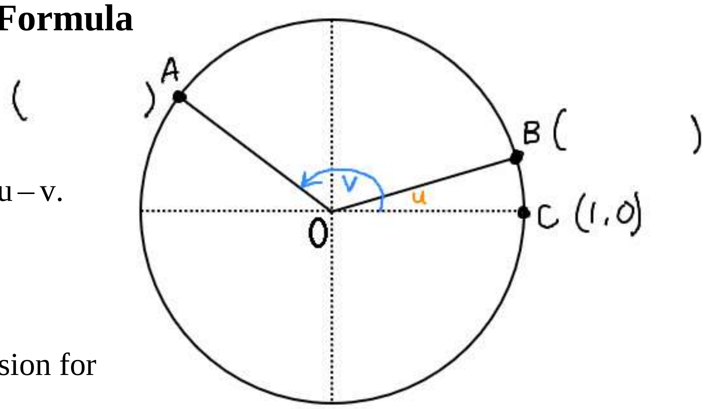
for Tuesday

1) On the first graph (below), make a table similar to what I did in the lecture, and then graph **f (x) = cos (x)**
2) On the other graph (below), graph **f (x) = tan (x)**
3) On the same graph as #1, graph **f (x) = sec (x)**
4) On the same graph as #2, graph **f (x) = cot (x)**





for Thursday. **Proving the Cosine Difference Formula**



- With the trig unit circle (on the right), angle **u** brings us to point B, and angle **v** brings us to point A. Follow these instructions:
  1. Mark point D on the circle such that its angle is $u - v$.
  2. Fill in the **x** and **y** coordinates in terms of sine and cosine for points A, B, D.
  3. Draw lines connecting A to B, and C to D.
  4. In terms of the coordinates, determine an expression for the lengths of lines AB and CD.
  5. How do know that triangles AOB and DOC must be congruent?
  6. This tells us that $AB = CD$. Therefore, you should now set the two expressions (from step #4) equal to each other, square both sides, and eventually solve for **$\cos(u-v)$**. (<u>Hint</u>: The *Pythagorean Trigonometric Identity* $\sin^2 x + \cos^2 x = 1$ will help you.)

- *Challenge!* Once you have the above cosine difference formula **$\cos(u-v) = \cos u \cdot \cos v + \sin u \cdot \sin v$**, try to help each other understand the below steps in order to prove many of the other trig identities.

Let $V = -X$, and use the *Opposite Angle Identities* to get: $\boxed{\cos(u+x) = \cos u \cdot \cos x - \sin u \cdot \sin x}$

Let $U = \pi/2 - Y$ to get: $\cos(\pi/2 - y + x) = \cos(\pi/2 - y) \cdot \cos x - \sin(\pi/2 - y) \cdot \sin x$

$$\cos(\pi/2 - (y - x)) = \cos(\pi/2 - y) \cdot \cos x - \sin(\pi/2 - y) \cdot \sin x$$

Then use the identity $\cos(\pi/2 - \theta) = \sin\theta$ to get: $\boxed{\sin(y - x) = \sin y \cdot \cos x - \cos y \cdot \sin x}$

And then letting $X = -W$ gives us $\boxed{\sin(y + w) = \sin y \cdot \cos w + \cos y \cdot \sin w}$

Furthermore, $\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin u \cdot \cos v + \cos u \cdot \sin v}{\cos u \cdot \cos v - \sin u \cdot \sin v}$ If we divide the numerator and the denominator by $\cos u \cdot \cos v$, then we get the identity $\boxed{\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \cdot \tan v}}$ and then $\boxed{\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \cdot \tan v}}$

From this, come the <u>**Double-Angle Identities**</u>. We think of $\sin(2u)$ as $\sin(u+u)$, and use the sum identity for sine given above to get $\boxed{\sin(2u) = 2\sin u \cdot \cos u}$

Similarly, $\boxed{\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u}$

And with tangent we use the identity $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \cdot \tan v}$ to get $\boxed{\tan 2u = \frac{2\tan u}{1 - \tan^2 u}}$

From there, we can derive the <u>**Power Reducing Identities**</u> from the double-angle identities:

From $\cos 2u = 2\cos^2 u - 1$ we get $\boxed{\cos^2 u = \tfrac{1}{2}(1 + \cos 2u)}$

From $\cos 2u = 1 - 2\sin^2 u$ we get $\boxed{\sin^2 u = \tfrac{1}{2}(1 - \cos 2u)}$

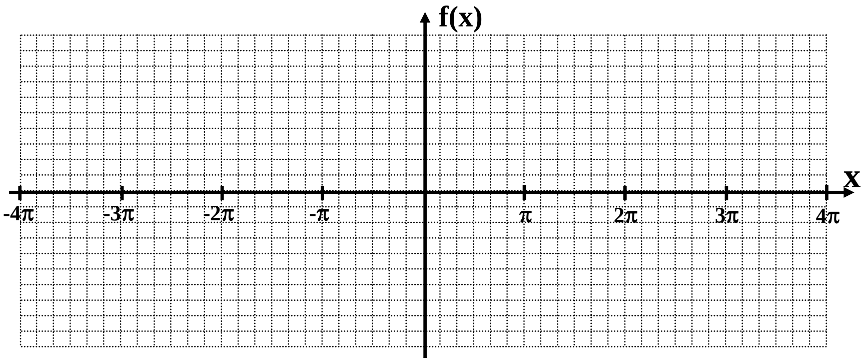
And therefore $\boxed{\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}}$

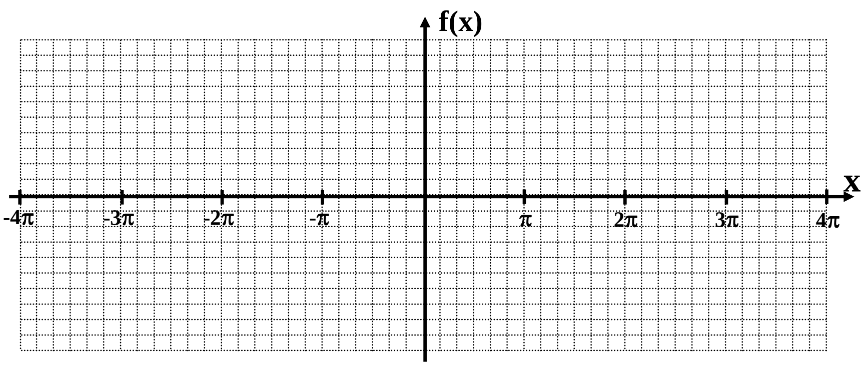
The <u>**Half-Angle Identities**</u>. We use $\cos^2 u = \tfrac{1}{2}(1 + \cos 2u)$ and let $U = \tfrac{1}{2}X$ to get $\boxed{\cos(\tfrac{1}{2}x) = \sqrt{\tfrac{1}{2}(1 + \cos x)}}$

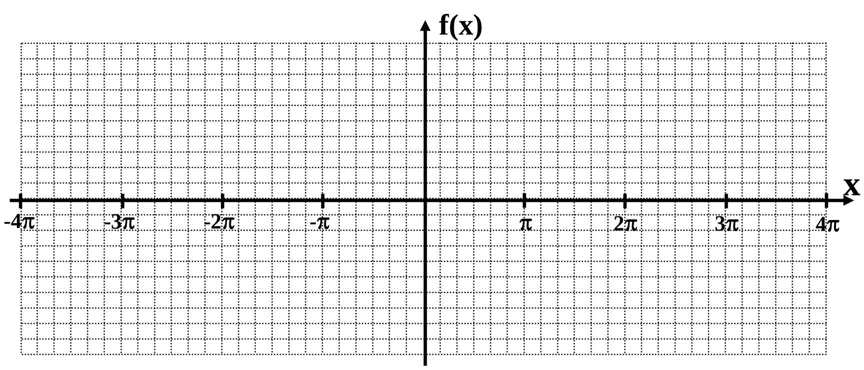
And using $\sin^2 u = \tfrac{1}{2}(1 - \cos 2u)$ and letting $U = \tfrac{1}{2}X$, we get $\boxed{\sin(\tfrac{1}{2}x) = \sqrt{\tfrac{1}{2}(1 - \cos x)}}$

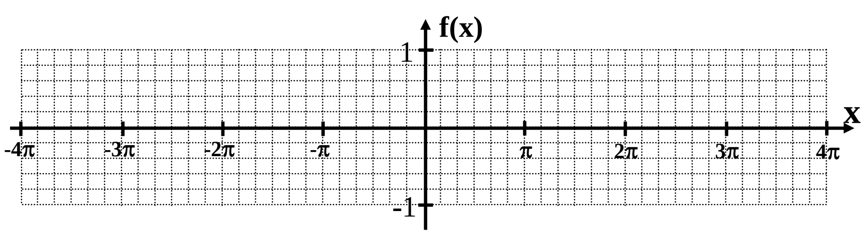
It follows then that $\tan(\tfrac{1}{2}x) = \frac{\sqrt{\tfrac{1}{2}(1 - \cos x)}}{\sqrt{\tfrac{1}{2}(1 + \cos x)}} \rightarrow \sqrt{\frac{\tfrac{1}{2}(1 - \cos x)}{\tfrac{1}{2}(1 + \cos x)}}$ and canceling the ½'s and multiplying top and bottom by $1 - \cos x$ gives: $\sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}} \rightarrow \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} \rightarrow \boxed{\tan(\tfrac{1}{2}x) = \frac{1 - \cos x}{\sin x}}$

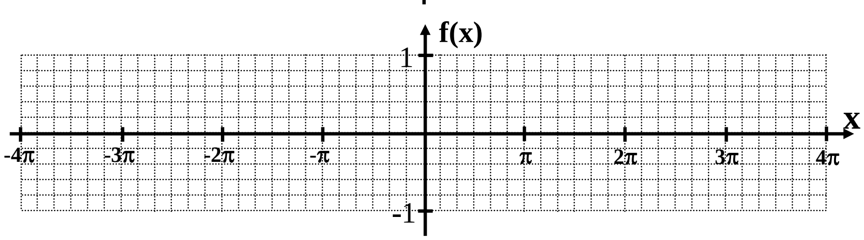
Graph Paper for Trig Functions

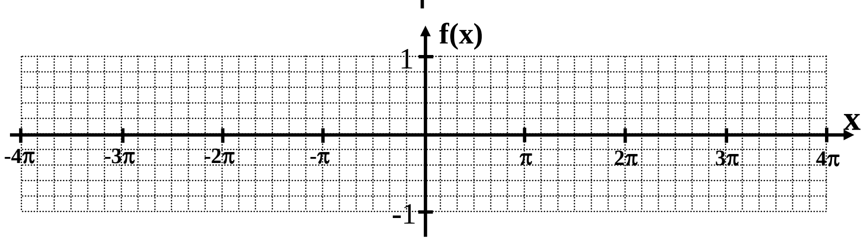












Trigonometric Identities and Laws

Opposite Angle Identities
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos(\theta)$
$\tan(-\theta) = -\tan\theta$

Supplementary Identities
$\sin(\pi-\theta) = \sin\theta$
$\cos(\pi-\theta) = -\cos\theta$
$\tan(\pi-\theta) = -\tan\theta$

Co-Function Identities
$\sin(\pi/2 - x) = \cos x$
$\cos(\pi/2 - x) = \sin x$
$\tan(\pi/2 - x) = \cot x$
$\cot(\pi/2 - x) = \tan x$
$\sec(\pi/2 - x) = \csc x$
$\csc(\pi/2 - x) = \sec x$

Pythagorean Identities
$\cos^2\theta + \sin^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$\cot^2\theta + 1 = \csc^2\theta$

Law of Sines
$a:b = \sin A : \sin B$ or $\frac{\sin A}{a} = \frac{\sin B}{b}$

Law of Cosines
$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$

Law of Tangents
$$\frac{a+b}{a-b} = \frac{\tan[½(A+B)]}{\tan[½(A-B)]}$$

Sum/Difference Identities
$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$$

Double-Angle Identities
$\sin 2u = 2 \sin u \cdot \cos u$
$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$
$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$

Half-Angle Identities
$\sin ½u = \pm\sqrt{½(1 - \cos u)}$
$\cos ½u = \pm\sqrt{½(1 + \cos u)}$
(Signs depend on quadrant of ∠½u)
$\tan ½u = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$

Power Reducing Identities
$\sin^2 u = ½(1 - \cos 2u)$
$\cos^2 u = ½(1 + \cos 2u)$
$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$

Sum-to-Product Identities
$\sin u + \sin v = 2\left(\sin\frac{u+v}{2} \cdot \cos\frac{u-v}{2}\right)$
$\sin u - \sin v = 2\left(\cos\frac{u+v}{2} \cdot \sin\frac{u-v}{2}\right)$
$\cos u + \cos v = 2\left(\cos\frac{u+v}{2} \cdot \cos\frac{u-v}{2}\right)$
$\cos u - \cos v = -2\left(\sin\frac{u+v}{2} \cdot \sin\frac{u-v}{2}\right)$

Product-to-Sum Identities
$\sin u \sin v = ½[\cos(u-v) - \cos(u+v)]$
$\cos u \cos v = ½[\cos(u-v) + \cos(u+v)]$
$\sin u \cos v = ½[\sin(u+v) + \sin(u-v)]$

Problem Set #2

1) *Trigonometric Identities*.
Look over the sheet titled *Trigonometric Identities and Laws* (see previous page). In groups, explain to one another what each of the trigonometic identities means, and how it can be used.

2) Evaluate

a) $\cos(^{7\pi}/_{4})$ d) $\sec(^{3\pi}/_{4})$

b) $\sin(^{2\pi}/_{3})$ e) $\csc(^{5\pi}/_{6})$

c) $\tan(\pi)$ f) $\cot(^{3\pi}/_{4})$

3) Evaluate. Give all possible answers between 0 and 2π.

a) $\cos^{-1}(-0.5)$ c) $\tan^{-1}(\sqrt{3})$

b) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ d) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Note: For graphing trig functions in the form $f(x) = a\cdot\cos(bx-c) + d$

- amplitude is "a": It says how far up and down it goes.
- The period is given by $\frac{2\pi}{b}$.
  It defines the length(in terms of x) of a complete cycle.
- c and d define the shift.

4) Graph each trigonometric function for $-4\pi \le x \le 4\pi$.

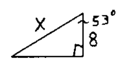
a) $f(x) = 3\cos(x)$

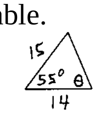
b) $f(x) = \sin(3x)$

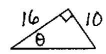
c) $f(x) = -3\tan(\frac{1}{2}x)$

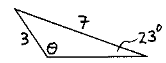
d) $f(x) = -2 + 5\cos(x + {}^{\pi}/_{6})$

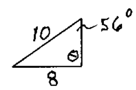
5) Find the variable.

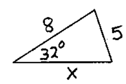
a) 

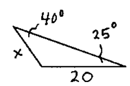
e) 

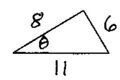
b) 

f) 

c) 

g) 

d) 

h) 

6) Factor each:

a) $\sec^2\theta - 1$

b) $\tan^2\theta + 3\tan\theta - 4$

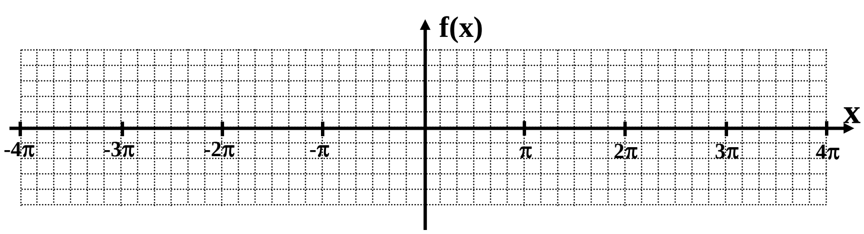
c) $4\sin x\cos^2 x - 8\cos^3 x$

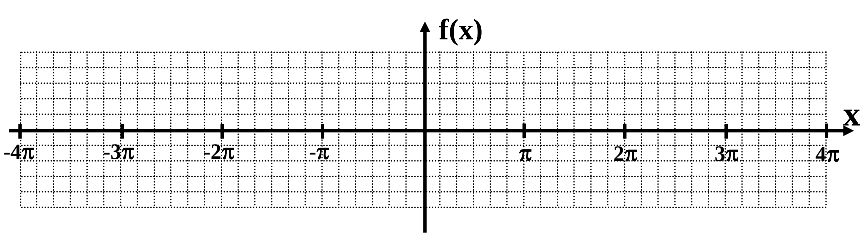
7) Simplify.

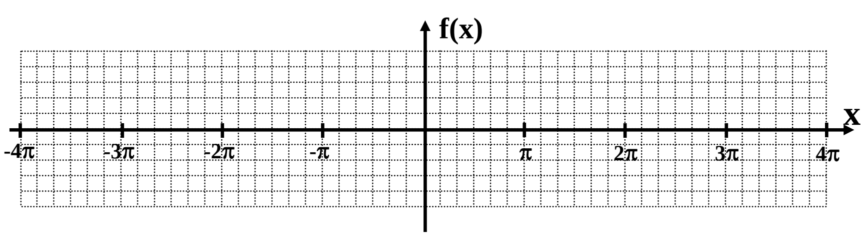
a) $\sin(x + 3\pi)$

b) $(1 - \sin^2 x)(\sec x)$

c) $\cot x \sec x$

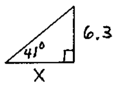


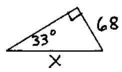


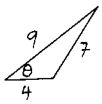


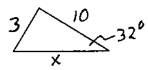
Problem Set #3

1) Find the variable.

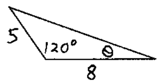
a) 

d) 

b) 

e) 

c) 

f) 

Guidleines for simplifying trig expressions. In general, a trig expression is consider most simplified when…

- Only the variable is "inside".
  <u>ex</u>: $\sin^3(x)$, not $\tan(4x)$
- It is in terms of one trig function.
  <u>ex</u>: $2\sin^2(x)$, not $\tan x + \sin x$
- There are no trig functions in the denominator.

2) Simplify each.

a) $\tan(\theta + 3\pi)$

b) $\sin\left(x - \frac{\pi}{2}\right)$

c) $\cos\left(x - \frac{\pi}{2}\right)$

d) $\cos\left(x + \frac{\pi}{2}\right)$

e) $\frac{\sin(-x)}{\cos(-x)}$

f) $\sin\theta + \cot\theta\cos\theta$

g) $\cos x\,(1 + \tan^2 x)$

h) $\cot^2 x - \cot^2 x\cos^2 x$

3) Prove each identity.

<u>Notes for proving identities</u>:

- It is considered best to work only with one side of the equation until it becomes equal to the other side.
- You may not manipulate the equation (e.g., multiply both sides by the same thing).

a) $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

b) $\frac{2\cot x}{1 + \cot^2 x} = 2\sin x\cos x$

c) $\cot^2 x = (\csc x - 1)(\csc x + 1)$

d) $\frac{\csc x}{\cot^2 x} = (\tan x)(\sec x)$

e) $\tan^2 A - \sin^2 A = \sin^2 A\tan^2 A$

f) $\sec\theta - \cos\theta = \sin\theta\tan\theta$

g) $\tan x + \cot x = \sec x\csc x$

h) $\frac{1 + \csc x}{\sec x} = \cos x + \cot x$

4) Rewrite $\sin(3x)$ in terms of $\sin x$ only.

5) Rewrite $\frac{1}{1 + \sin\theta}$ so that there are no fractions in the expression.

6) Solve for all values of x such that $0 \le x < 2\pi$.

a) $8\cos x - 4 = 0$

b) $5\tan x + 7 = 2$