12^{th} Grade Assignment – Week #29

Reminder:

If your copy of **Chaos: Making a New Science** (James Gleick) is not already in your hands or on its way to you, you may need to order it with expedited shipping. Alternatively, you may be able to borrow a copy from your local library. There will be no assigned readings this week.

Before Lecture 1:

Follow this link to download a template for one of the fractals that we will be drawing during the lecture. It has three small dots (vertices of an invisible equilateral triangle), and four lines of different lengths. Kindly print that page in advance. Additionally, you will need the following:

• Compass • Ruler • Colored Pencils • Ordinary Pencil • Eraser • Blank Paper
• Pen (optional) • 30°-60°-90° Triangle (optional)

GROUP WORK – TUESDAY / After L1 and Before L2:

[I] *Fractal Analysis*

We are going to explore the perimeter and area of the Sierpiński Triangle and Koch Curve, with the ultimate goal of understanding those metrics for the complete and infinitely iterated fractals. This is challenging work. Feel welcome to begin with whichever part you expect to be easiest or most interesting. Before you dive in, though, share with each other what you think might occur.

0. YOUR BEST GUESS
If at the beginning of constructing a Sierpiński Triangle and Koch Curve (or Snowflake) the starting figure (iteration 0) for each had a perimeter of 1 and an area of 1 (do not worry about the units for these measurements), what do you think their perimeters and areas will be for the complete fractal (iteration ∞)? If you can put words to your reasoning, please share that too.

1. SIERPIŃSKI TRIANGLE – PERIMETER
In creating the Sierpiński Triangle, every line we draw is part of its perimeter. Assume the equilateral triangle with which we begin (iteration 0) has a perimeter that is exactly one. That is, when iteration number = 0, p = 1 unit of length. Or simply… **n = 0 → p = 1**.

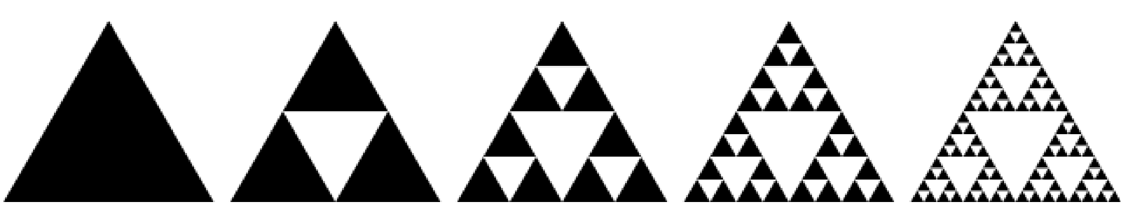
[A] What is the perimeter (**p**) of the Sierpiński Triangle at iteration number **n = 1**?

[B] What is its perimeter (**p**) when **n = 2**?

[C] What is its perimeter (**p**) when **n = 3**? (If you can answer part [D] now, you can skip [C].)

[D] What is its perimeter (**p**) for any given number of iterations (**n**)?
(Create a formula expressing perimeter (**p**) in terms of iteration number (**n**).)

[E] What is the perimeter (**p**) of the Sierpiński Triangle when **n** = ∞?



12^{th} Grade Assignment – Week #29 (continued)

<u>GROUP WORK – TUESDAY</u> <u>/ After L1 and Before L2</u>: • [I] *Fractal Analysis (continued)…*

2. SIERPIŃSKI TRIANGLE – AREA
   As we iterate, we create smaller inverted equilateral triangles and color them in. Those colored triangles are removed; they are outside the fractal's perimeter. Assume the equilateral triangle with which we begin (iteration 0) has an area that is exactly one. That is, when iteration number = 0, a = 1 unit of area. Or simply… **n = 0** ➝ **a = 1**.

   [A] What is the area (**a**) of the Sierpiński Triangle when **n = 1**?

   [B] What is its area (**a**) when **n = 2**?

   [C] What is its area (**a**) when **n = 3**? (If you can answer part [D] now, you can skip [C].)

   [D] What is its area (**a**) for any given number of iterations (**n**)?
   (Create a formula expressing area (**a**) in terms of iteration number (**n**).)

   [E] What is the area (**a**) of the Sierpiński Triangle when **n = ∞**?

3. KOCH CURVE – LENGTH
   Rather than analyzing the Koch Snowflake, let us consider here the Koch Curve. (Recall that the Snowflake is comprised of three Koch Curves. The Koch Curve begins as a single line.) Assume the line segment with which we begin (iteration 0) has a length that is exactly one. That is, when iteration number = 0, L = 1 unit of length. Or simply… **n = 0** ➝ **L = 1.**

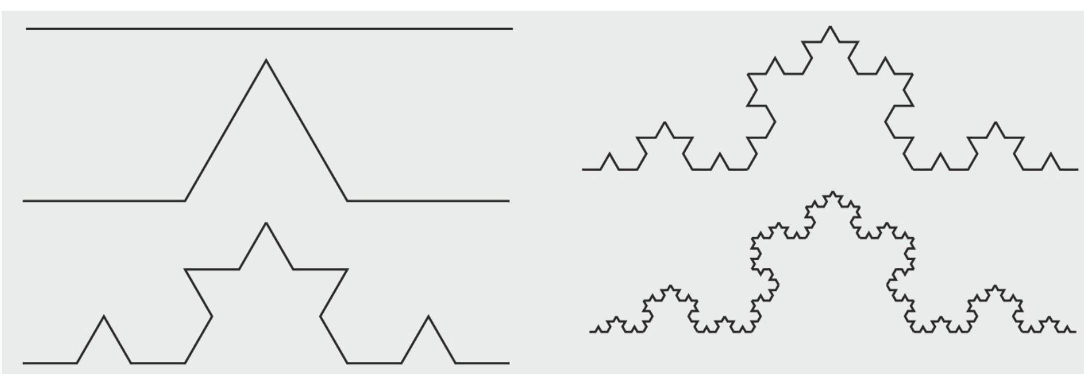
   [A] What is the length (**L**) of the Koch Curve at iteration number **n = 1**?

   [B] What is its length (**L**) when **n = 2**?

   [C] What is its length (**L**) when **n = 3**? (If you can answer part [D] now, you can skip [C].)

   [D] What is its length (**L**) for any given number of iterations (**n**)?
   (Create a formula expressing length (**L**) in terms of iteration number (**n**).)

   [E] What is the length (**L**) of the Koch Curve when **n = ∞**?



4. KOCH CURVE – AREA UNDER THE CURVE
   Consider the originating line of the Koch Curve to be lying on the ground. We can therefore say that initially (iteration 0) there is no area under the curve (**n = 0** ➝ **a = 0**). After the first iteration, there is one equilateral triangle beneath the curve. Assume this triangle has an area that is exactly one. When iteration number = 1, a = 1 unit of area. Or… **n = 1** ➝ **a = 1**.

   [A] What is the area (**a**) under the Koch Curve when **n = 2**?

   [B] What is its area (**a**) when **n = 3**?

12th Grade Assignment – Week #29 (continued)

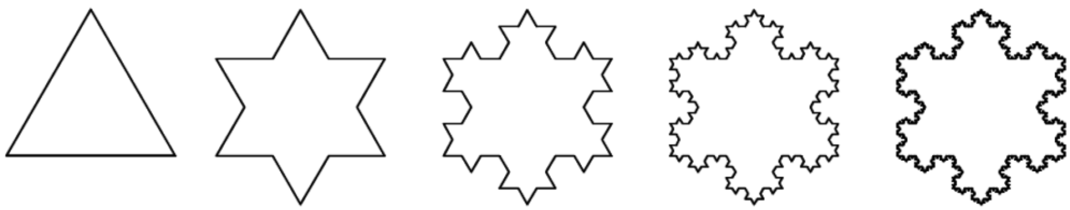
GROUP WORK – TUESDAY / After L1 and Before L2: • *Fractal Analysis (continued)…*

4. KOCH CURVE – AREA UNDER THE CURVE (CONTINUED)

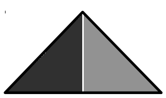
[C] What is its area (**a**) for any given number of iterations (**n**)?
(Create a formula expressing area (**a**) in terms of iteration number (**n**).)

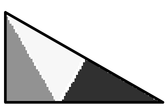
[D] What is the area (**a**) under the Koch Curve when **n = ∞**?

[E] Now let's consider the Koch Snowflake and re-scale our units such that at iteration 0, when the Snowflake is just an equilateral triangle, it has an area of 1 (**n = 0 → a = 1**). What then would be the area (**a**) of the Koch Snowflake when **n = ∞**?



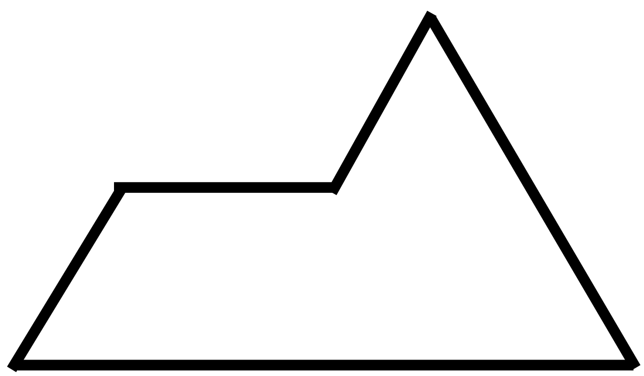
GROUP OR INDIVIDUAL WORK – Puzzles Just For Fun

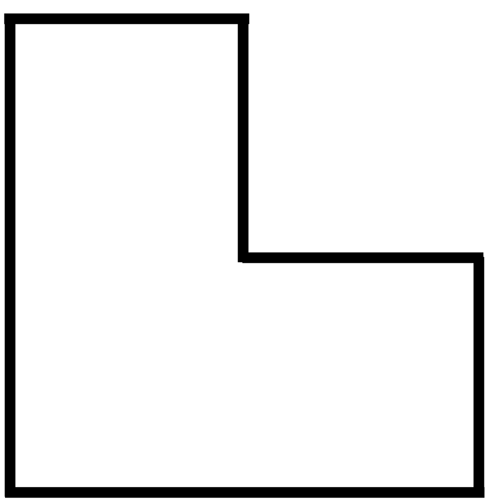


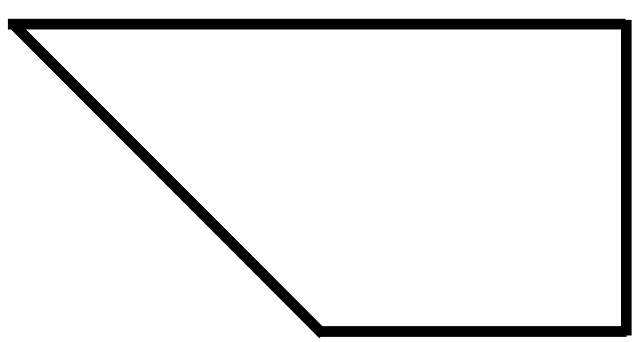


Exhibiting self-similarity is not by itself sufficient to cause a figure to be a fractal. Mathematics teacher Cynthia Lanius designed the following shapes which I have made into puzzles. She liked to call them *rep-tiles*, an abbreviation of *repeating tiles*. To the left are two examples of triangles composed of self-similar triangles.

Each of the outlined figures below can be divided into *four* self-similar figures, each of the same size, though obviously smaller than the overall outline.







12th Grade Assignment – Week #29 (continued)

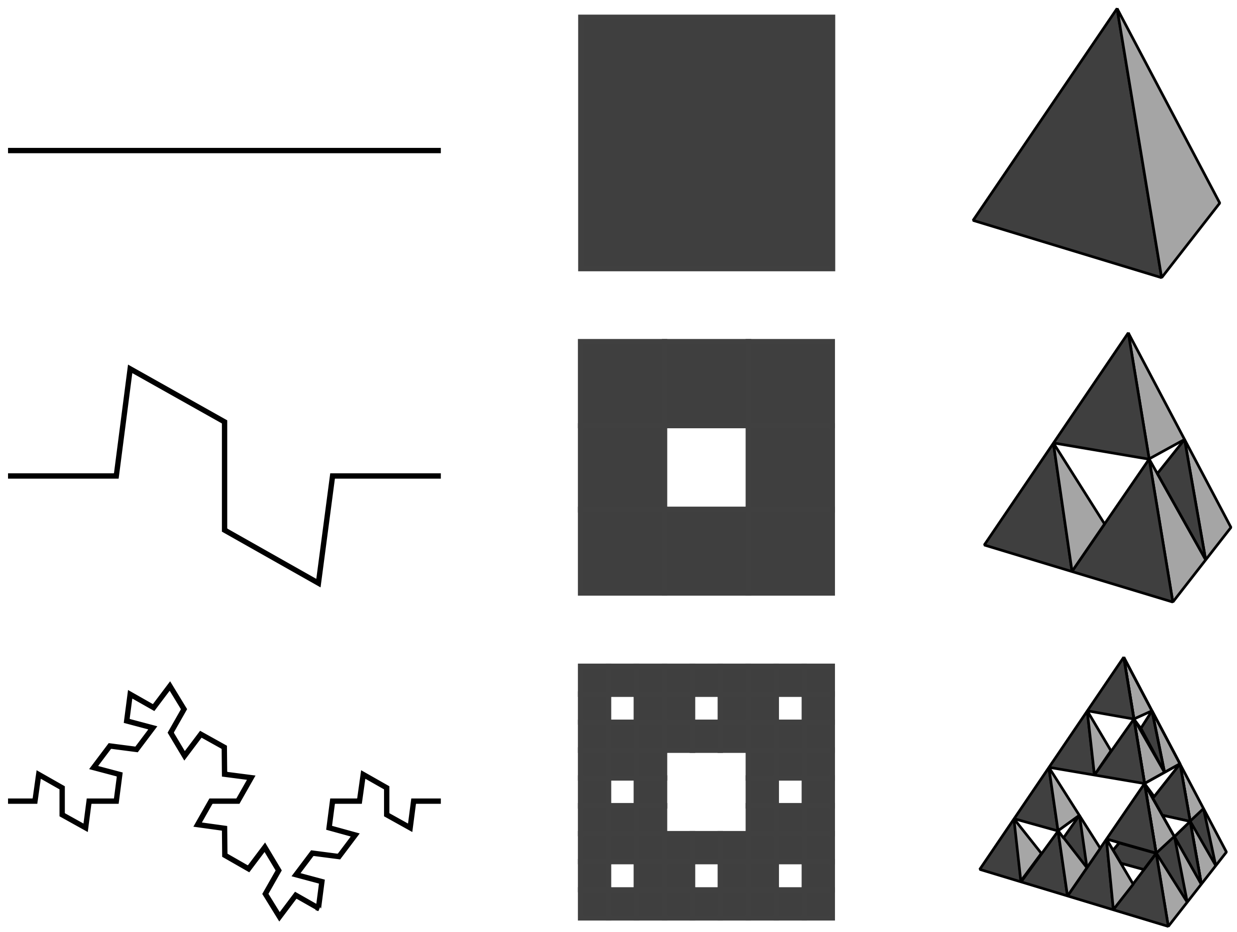
<u>GROUP WORK – THURSDAY</u> <u>/ After L2</u>:

[II] *Fractal Dimension*

Below are iterations 0, 1, and 2 for three fractals. The first fractal, an unnamed variant of the Koch Curve, begins with a line. The second fractal, the Sierpiński Carpet, begins with a square. The last fractal, the Sierpiński Tetrahedron (also called Tetrik) begins with a tetrahedron.

Calculate the fractal dimension of each.

<u>Koch Curve Variant</u> | <u>Sierpiński Carpet</u> | <u>Sierpiński Tetrahedron</u>



[III] *Complex Calculator, Part 0*

We are going to program a spreadsheet to make arithmetic calculations with complex numbers. As a first step in that direction, we need to create abstract formulae for these operations. In each case, we want to factor ***i*** from any imaginary terms so we can clearly express the real and the imaginary parts of the solution separately. The solution for addition is given as an example.

Given… **x = a + b*i*** …and… **y = c + d*i*** …determine the following:

① **x + y** ② **x − y** ③ **x · y** ④ **x ÷ y**

<u>Example</u>: x + y → (a + c) + (b + d)*i* ⇐ *Note how we express the coefficient of **i** as a group. All your answers should have **i** factored out in this way, and take the form […] + […]**i**.*

12th Grade Assignment – Week #29 (continued)

GROUP WORK – THURSDAY / After L2: • *Complex Calculator (continued)…*

[IV] *Complex Calculator, Part 1*

The following is an image of the spreadsheet we wish to complete. The white cells are where numbers are given as input. The black cells are automatically calculated based on this input. Your task is to create the formulas which go into each of those black cells.

In this example, **x = 2 + 3*i*** …and… **y = 4 − 7*i***. The real and imaginary parts are entered into separate cells. The real part of **x** is in cell **B1**. The real part of **y** is in cell **B2**. The formula for the real part of the **x + y** solution is therefore, without the quotes, "**= B1 + B2**". (All formulas in spreadsheets begin with an equal sign.)

	A	B	C	D	E
1	x =	2.000	+	3.000	i
2	y =	4.000	+	-7.000	i
3					
4	x + y =	6.000	+	-4.000	i
5					
6	x − y =	-2.000	+	10.000	i
7					
8	x × y =	29.000	+	-2.000	i
9					
10	x ÷ y =	-0.200	+	0.400	i
11					

Here is a link to a Google Sheet to get you started. You can make a copy of it and collaborate online, or you can download a copy and open it locally in your own software. Try to adapt the formulas you created in the previous section to the language of spreadsheets. You can test your results against the figures shown above.

Note that for a multiplication sign, spreadsheets use an asterisk (*), and for division they use the forward slash (/). If you want to square something, it is easiest to just multiply it by itself. You can look at the "QF" sheet (see tab at bottom) to see a complicated formula in action. That sheet solves a quadratic equation. When writing formulas, parentheses are your friends!

INDIVIDUAL WORK

- Continue any group work you would like to take further. Work on main lesson pages as suits.
- Embrace the meditative joy of constructing high precision Sierpiński and Koch fractals. ☺