

## 12<sup>th</sup> Grade Assignment – Week #26

### Individual Work

- Read Chapter Four of *Logicomix* before Tuesday's group meeting.
- Before Thursday's group meeting, read the excerpt from Rebecca Goldstein's book *Incompleteness*, which is about the Vienna Circle & Logical Positivists.

### Group Assignments:

for Tuesday.

- Discuss the *Logicomix* reading (Chapter #4)
  - (p168-171) How did Cantor, Peano, Hilbert and Frege react to Russell's paradox?
  - (p174-175) What was the main purpose of BR's invention of his "theory of types"?
  - How long did Russell and Whitehead work on PM (*Principia Mathematica*)?
  - Why was it so frustrating?
  - What did the publishers think of PM?
  - Who was the only person ever to read the whole of PM?
  - What was the authors' main purpose of the *Entracté*?
  - What is the "logic from madness" theme?
- Wittgenstein's Tractatus. Read through "*A Very Brief Summary of Wittgenstein's Tractatus*" (which is found at the end of this document). This reading should be done during the group meeting (not beforehand). You can either take turns having one person at a time read a proposition, or you can take 5 or 10 minutes to have everyone (fairly quickly) read through it silently on their own.  
Then do the following:

- Each person in the group should choose two propositions that most stood out as interesting or notable, and then announce to the group these two propositions, perhaps briefly stating why you found them to be interesting.
- Discuss the meaning, significance, and your opinions of these propositions in *Tractatus*: 4.003; 4.11; 4.112; 6.42; 6.54
- If you still have extra time, start working on *Sum of Two Squares – Part II* (see Thursday's assignment).

for Thursday.

- Discussion: Goldstein's essay on the Vienna Circle (from her book, *Incompleteness*)
  - (p29) What does it mean to say that Logical Positivism (Vienna Circle) was "a program for intellectual hygiene"?
  - (p31) What does it mean to say that "Since the limits of knowability are congruent with the limits of meaning, no meaningful matter can escape our grasp. We are cognitively complete."
  - (p32) Discuss the following: In the second-to-last paragraph, the author says that Wittgenstein was "paradox friendly". Just before this, she- says that "the entire *Tractatus* constitutes a self-avowed paradox."
  - What do you think Godel was thinking while he was sitting in the Vienna Circle meetings?
- Puzzle: Sum of Two Squares – Part II.
  - The following is a theorem:  
*If a number is prime and has a remainder of 1 after dividing it by 4, then it can be expressed as the sum of two square numbers in one and only one way. If a number is prime and has a remainder of 3 after dividing it by 4, then it is not possible to express it as the sum of two square numbers.*  
Example: The number 73 is prime and has a remainder of 1 when divided by 4. This theorem tells us that there must be exactly one way to express 73 as the sum of two squares, which is  $8^2 + 3^2$ .  
Example: The number 43 is prime and has a remainder of 3 when divided by 4. Therefore, according to the theorem, it must be impossible to express 43 as the sum of two squares.  
**Your task** is to answer this question. How can we use the above theorem to determine if composite numbers (non-prime numbers) can or cannot be expressed as the sum of two squares?

A “Very Brief Summary” of  
**Ludwig Wittgenstein’s *Tractatus Logico-Philosophicus***

(From <http://www.btinternet.com/~glynhughes/squashed/wittgenstein.htm>.)

- 1 The world is all that is the case.
- 1.1 The world is the totality of facts, not of things.
- 1.2 The world divides into facts.
- 2 What is the case (a fact) is the existence of atomic facts.
- 2.1 We picture facts to ourselves.
- 2.12 A picture is a model of reality.
- 2.141 A picture is a fact.
- 2.172 A picture cannot depict its pictorial form: it displays it.
- 2.19 Logical pictures can depict the world.
- 2.223 In order to tell whether a picture is true or false we must compare it with reality.
- 2.224 It is impossible to tell from the picture alone whether it is true or false.
- 3 A logical picture of the facts is the thought.
- 3.01 The totality of true thoughts is a picture of the world.
- 3.1 In a proposition, a thought finds an expression that can be perceived by the senses.
- 3.332 No proposition can make a statement about itself, because a propositional sign cannot be contained in itself.
- 4 The thought is the significant proposition.
- 4.001 The totality of propositions is language.
- 4.003 Most of the propositions and questions to be found in philosophical works are not false but nonsensical.
- 4.0031 All philosophy is a critique of language.
- 4.11 The totality of true propositions is the whole of natural science
- 4.112 Philosophy aims at the logical clarification of thoughts. Philosophy is not a doctrine, but an activity.
- 4.461 Propositions show what they say; tautologies and contradictions show that they say nothing.
- 4.464 A tautology's truth is certain; a proposition's truth is possible; a contradiction's truth is impossible.
- 5 A proposition is a truth-function of elementary propositions. (An elementary proposition is a truth-function of itself.)
- 5.5302 Russell's definition of '=' is inadequate, because according to it we cannot say that two objects have all their properties in common. (Even if this proposition is never correct, it still has sense .)
- 5.6 The limits of my language mean the limits of my world.
- 5.61 We cannot think what we cannot think; so what we cannot think, we cannot say either.
- 5.621 The world and life are one.
- 5.63 I am my world. (The microcosm.)
- 6 The general form of a truth-function is  $[p, E, N(E)]$ . This is the general form of a proposition.
- 6.1 The propositions of logic are tautologies.
- 6.13 Logic is not a doctrine, but a mirror-image of the world. Logic is transcendental.
- 6.2 Mathematics is a logical method. The propositions of mathematics are equations, and therefore pseudo-propositions.
- 6.21 A proposition of mathematics does not express a thought.
- 6.22 The logic of the world, which with logic is shown in tautologies, is shown in equations by mathematics.
- 6.41 The sense of the world must lie outside the world.
- 6.431 At death the world does not alter, but comes to an end.
- 6.4311 Death is not an event in life: we do not live to experience death.
- 6.4 All propositions are of equal value.
- 6.42 There can be no propositions of ethics.
- 6.44 It is not how things are in the world that is mystical, but that it exists.
- 6.5 For an answer which cannot be expressed, the question too cannot be expressed. The riddle does not exist. If a question can be framed at all, it is also possible to answer it.
- 6.54 He who understands my propositions recognizes them as senseless. (He must so to speak throw away the ladder, after he has climbed up it.)
- 7 Whereof one cannot speak, thereof one must remain silence.