

12th Grade Assignment – Week #23

Announcement:

- This week, we will begin reading chapters from *Logicomix*. The first discussion on this book will take place during this Thursday's group meeting. After this week, for the next four weeks, you will read the next chapter of *Logicomix* in preparation for your Tuesday group meeting. Other readings (from the reading packet) will be discussed in the Thursday group meetings.
- Once again, I implore you to watch the lectures and do the reading assignments before the group meetings so that the discussions can be fruitful.
- Once again, please be sure that you have a scribe from your group email notes to the tutor after the meeting concludes.

Individual Work

- Read Poincaré's paper before Tuesday's group meeting.
- Read the Intro ("Overture") and First Chapter ("Pembroke Lodge") of *Logicomix* before Thursday's group meeting.

Group Assignments:

for Tuesday. Discuss the following questions:

- Discussion: Poincaré's article (Try to limit this to 25 minutes.)
 1. What are the five steps that Poincaré outlines as the typical process for mathematical discovery?
 2. Poincaré speaks of a "sudden illumination" (or inspiration or intuition). Where do you think such things come from?
 3. Can you think of one instance where you experienced a "sudden illumination" to either solve a problem, or it came to you while working on some sort of creative endeavor?
- Hilbert's Infinite Hotel. Here is what you need to know about the hotel:
 - The hotel has (countably) infinitely many rooms. There are no locks on the doors, so no keys are needed.
 - Whenever anyone arrives, the receptionist always says this standard line: "I am sorry, but the hotel is already full. However, I can make room for you."
 - All guests at the Infinite Hotel have agreed to the following arrangement (which is part of the reason that a room in the hotel costs so little money):
You must be in your room at all times during your stay. At any point during your stay, there may be an announcement made for you to move to a different room. You must then accurately calculate your new room number and then quickly move to that new room. Everyone will move rooms at the same time.
 - The announcement for the current guests always includes a method (perhaps a formula) for determining their new room number. And then the new guests are also given a method (perhaps a formula) for determining their new room number.
 - Example: As a single guest, I arrive at the hotel. The receptionist gives me the standard line, and then announces: "Attention all guests, you must now move to the room with a number one greater than your current room number." The receptionist then tells me: "You may now have room #1."

Solve the following problems:

4. A bus arrives with 1000 new guests, each with a seat number from the bus. In order to accommodate all of these new guests, what should the receptionist announce to the current guests, and to the new guests?
5. A bus arrives with (countably) infinitely many new guests (each with a seat number from the bus). In order to accommodate all of these new guests, what should the receptionist announce to the current and new guests?
6. Ten buses arrive, each with (countably) infinitely many new guests. Each guest has a bus number and a seat number. In order to accommodate all of these new guests, what should the receptionist announce to the current guests, and to the new guests?

for Thursday.

- **Logicomix reading**
Discussion for *Logicomix* reading, chapter #1 (Limit this to 15 minutes.)
 7. What was BR's (Bertrand Russell's) grandmother trying to hide from him?
 8. What is the "Quest" that the authors keep referring to?
 9. What role did geometric proof (Euclid) play in BR's development?
 10. What was BR's "terrible disappointment"?
- **Worksheet on Sets.** Work through the problems found on the next page.

Set Worksheet

Some sets to start with

- Set Name: **Novels** Description: Novels that Mr. York has read
Members: {*War and Peace, Shikasta, Moby Dick, Tale of Two Cities, Men of Mathematics...*}
- Set Name: **Cities** Description: Cities with a population greater than 5 million.
Members: {Bombay, Sao Paulo, New York, Paris, Cairo, Tokyo...}
- Set Name: **Prime** Description: Prime numbers
Members: {2, 3, 5, 7...}
- Set Name: **Non-Prime** Description: Natural numbers that aren't prime
Members: {1, 4, 6, 8, 9...}
- Set Name: **Two-Digit** Description: All two-digit whole numbers
Members: {10, 11, 12...}

Instructions: First of all, be sure that you understand the above sets. Each set below has the unusual characteristic that its members are themselves sets, i.e., sets of sets. You need to fill in each set's members. To simplify matters somewhat, you are only required to consider all of the sets that are defined on this page. The first one has been done for you.

Example: Set Name: **Number-Sets** Description: Sets that have numbers as members
Members: {Prime, Non-Prime, Two-Digit}

- 1) Set Name: **23-Sets** Description: Sets that include the number 23
Members: { }
- 2) Set Name: **X-Sets** Description: Sets that aren't a member of Number-Sets
Members: { }
- 3) Set Name: **N-Sets** Description: Sets having names beginning with "N"
Members: { }
- 4) Set Name: **A-Sets** Description: Sets having names beginning with "C"
Members: { }
- 5) Set Name: **S-Sets** Description: Sets that *are* members of themselves
Members: { }
- 6) Set Name: **R-Sets** (Russell's set!) Description: Sets that *are not* members of themselves
Members: { }
- 7) Is S-Sets a member of itself?
- 8) Is R-Sets a member of itself? (This is the question Russell sent to Frege in his letter.)
- 9) *The Liar's Paradox.* Is the below statement true or false?
"This statement is false."
- 10) *The Barber's Paradox.* Suppose there is a town with just one male barber; and that every man in the town (who has facial hair) keeps himself clean-shaven: some by shaving themselves, some by attending the barber. It seems reasonable to imagine that the barber obeys the following rule: He shaves all and only those men who do not shave themselves.
Does the barber shave himself?