12^{th} Grade Assignment – Week #18

Group Assignments:

for Tuesday – Related Rate Practice

- *The Rising Balloon*. Do **Problem Set #2** (*Calculus – Part II*), pr #3
  (Note: This problem is found in the problem sets from last week's assignment.)
- *A Growing Shadow*. On a clear day, close to sunset somewhere near the equator, a 3m-tall pole cast a shadow onto flat ground. At what rate is the length of the shadow growing when the sun is 16 degrees above the horizon? (Note: I will go over this problem in the next lecture.)

for Thursday

- Do **Problem Set #3** (*Calculus – Part II*), pr #3 and #4. (This problem set is found in this document, below.)

- Extra challenge: Last week, we encountered this equation of a slanted/tilted parabola:
  $$x^2 - 6xy + 9y^2 - 2y + 1 = 0$$
  You may have been curious about where this equation comes from. One way to approach it is to consider the loci definition of a parabola (from 8^{th} grade geometry):
  A parabola is the locus of points equidistant from a directrix line a focal point.
  Here is your task: Find the equation of the parabola that has the directrix line 3y+2x=21 and has the focal point (7,11).

Individual Work

- If you would like more practice for the test, do **Problem Set #1** (*Calculus – Part II*), pr #1.
  (Note: These problems are found in the problem sets from last week's assignment.)
- Take the *Calculus (Part I) Test* found at the end of this document.
- Practice implicit differentiation by doing **Problem Set #2** (*Calculus – Part II*), pr #1
  (Note: These problems are found in the problem sets from last week's assignment.)

Problem Set #3

Max-Min Problems

1) Pam is designing a rectangular garden where one of the sides is against the wall of her house. She has 30 feet of fence to put around the garden, which will go around three sides of the garden.

a) y is the length of the side of the garden that runs along the house, and x is the width of the rectangle. Give an equation for y in terms of x.

b) Give an equation for the area, A, of the rectangle in terms of x.

c) Find $\frac{dA}{dx}$. What does it mean?

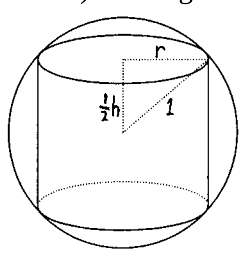
d) Graph both the equation for the area (A in terms of x) and the equation just found for its derivative. Explain the relationship between these two graphs.

e) What are the dimensions of the garden such that it has maximum area?

2) In this guided example, our goal is to find the cylinder with the greatest possible volume that can be inscribed in a sphere with a radius of 1.

a) Give the formula for the volume of a cylinder with a radius r and height h.

We now need to find a way to relate r, h and the radius of the sphere. We can do this by using a right triangle inside the cylinder (see below). This gives us $(\frac{1}{2}h)^2 + r^2 = 1^2$.



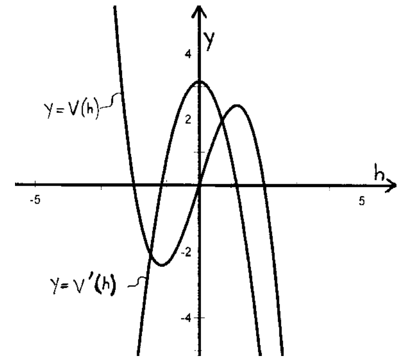
b) Derive a formula for the volume of the cylinder, v, in terms of just the variable h. (Hint: Sub for r^2.)

Now it is helpful to imagine that the cylinder is changing shape. We start with a short, wide cylinder, and then imagine that the height, h, increases (which means that r must decrease in order for the cylinder to remain inscribed in the sphere). During this whole transformation, where the height has gone from 0 to 2, there is an instant where the cylinder reaches a maximum volume, and then it starts to shrink again. Our task is to find this instant – the special value for h – which produces the maximum volume. First, we need a formula for the rate of change of the volume as the height changes.

c) Derive $\frac{dV}{dh}$ or v'(h).

d) What does the above formula mean?

Do you know how we can now find the maximum volume of the cylinder? Perhaps looking at the graphs of v(h) and v'(h) (which is $\frac{dV}{dh}$) will be helpful. These graphs should be consistent with the formulas you found. Are they?



In reality, we are only interested in the portions of the graphs for h between 0 and 2, because h can't be negative or greater than 2.

e) Calculate v(0.5). What does this mean?

f) Calculate v'(0.5). What does this mean?

g) Calculate v(1.5). What does this mean?

h) Calculate v'(1.5). What does this mean?

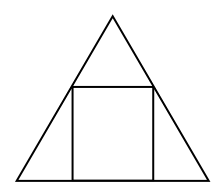
Now, the key is to realize that the volume reaches a maximum at the instant that the volume stops increasing – an instant earlier the volume, v(h), was increasing, an instant later it begins to decrease. In terms of $\frac{dV}{dh}$ or v'(h), we are looking for the instant that the rate of change of the volume is zero – an instant earlier the rate of change was positive, an instant later it was negative.

i) (And now, the big question!) Find the height, radius and volume of the cylinder when its volume is maximized.

j) How are these three answers reflected in the graphs?

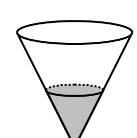
Now it is time for you to try a max-min problem on your own.

3) What are the dimensions and area of the rectangle with the largest possible area that fits inside an equilateral triangle (with edge length of 1) such that the rectangle lies on the base of the triangle?



Now, we return to a problem on related rates…

4) A cone (with a height of 12 cm and a radius of 8 cm) is being filled with water at a rate of 30 cm^3/sec. At what rate is the depth of the water increasing when it is…



a) 2 cm deep?

b) 10 cm deep?

c) How long does it take the cone to fill up?

Calculus – Part I Test

Simplify when reasonable.
All problems are worth 4 points!

1) Find the slope of…

a) $f(x) = \cos x$ at $x = {}^{2\pi}/_{3}$

b) $f(x) = \ln x$ at $x = 9$

c) $f(x) = \ln(\cos x)$ at $x = {}^{\pi}/_{3}$

2) Find $f'(x)$.

a) $f(x) = (5x - 4)^7$

b) $f(x) = \sec(\pi - 3x)$

c) $f(x) = \frac{\sin x}{7x}$

d) $f(x) = \sqrt{\cos^3 x + 1}$

e) $f(x) = (\ln x)(\cos x)$

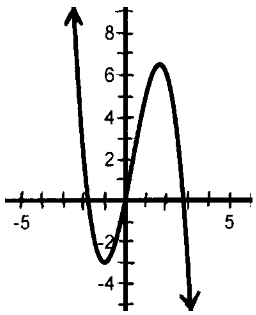
f) $f(x) = x^2 + \cos x$

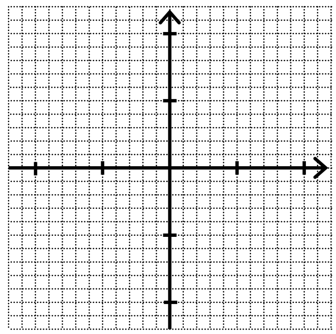
3) Evaluate the integrals.

a) $\int_{\pi/4}^{\pi/3} \cos x \, dx$

b) $\int_{-\infty}^{2} e^x \, dx$

4) Given the graph of f(x), shown below, draw a rough sketch of f '(x).





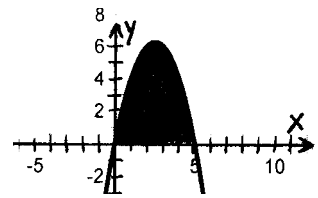
5) Find f '(x) of each…

a) $f(x) = 8\cos x + 5\tan x$

b) $f(x) = \ln(13x)$

c) $f(x) = \cos^2(3x) \cdot \sin x$

6) Below is the graph of $f(x) = -x^2 + 5x$. Find the area of the region bounded by the x-axis and the curve. (This region is shaded.)



7) *Challenge!* (Only do if you have extra time.) Find f(x) given that it is a cubic polynomial function in the form $f(x) = ax^3 + bx^2 + cx + d$, where $a = 1$, $d = 0$, and at the point $(-2, 3)$ the slope is $^3/_2$. Find f(x).