12th Grade Assignment – Week #17

Group Assignments:

for Tuesday

- Do **Problem Set #6** (*Calculus* – *Part I*), pr #6-7.
- Do **Problem Set #1** (*Calculus* – *Part II*), pr #2.

for Thursday

- Do **Problem Set #2** (*Calculus* – *Part II*), pr #4-7.

Individual Work

- Do **Problem Set #6** (*Calculus* – *Part I*), pr #1-5.

— Calculus – Part I —

Problem Set #6

1) Find the slope of…

a) $f(x) = \cos x$ at $x = \pi/4$

b) $f(x) = \sin x$ at $x = 2\pi$

c) $f(x) = \sin x$ at $x = \frac{4\pi}{3}$

d) $f(x) = e^x$ at $x = ½$

e) $f(x) = \ln x$ at $x = 3$

f) $f(x) = \ln(2x)$ at $x = 3$

g) $f(x) = \ln(17x)$ at $x = ¾$

h) $f(x) = 3^x$ at $x = -2$

2) Evaluate the integrals.

a) $\int_{\pi/3}^{\pi/2} \sin x \, dx$

b) $\int_{-\pi}^{\pi} \cos x \, dx$

c) $\int_{0.1}^{1} \frac{1}{x} \, dx$

d) $\int_{-2}^{2} e^x \, dx$

3) Derivatives.

a) Find $\frac{d}{dx}\left[(\sin x + 3)^5\right]$

b) Find $\frac{d}{dx}\left[3(4x - 1)^6\right]$

4) Find $\frac{dy}{dx}$.

a) $y = \frac{\sin x}{x}$

b) $y = \frac{\sin^2 x}{x^3}$

c) $y = \frac{e^{2x}}{(x + 3)^5}$

d) $y = \cos(\pi/2 - x)$

e) $y = x^3 \sin x$

f) $y = x^3 \sin(x/3)$

5) Find f '(x).

a) $f(x) = \sec(2x)$

b) $f(x) = \ln(\sin x)$

c) $f(x) = \sin(\ln x)$

d) $f(x) = \sin^2 x - \cos^2 x$

e) $f(x) = \sin^2 x \cdot \cos^2 x$

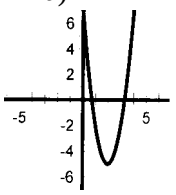
f) $f(x) = \sin\sqrt{5x+3}$

g) $f(x) = x^4 \cdot e^x$

h) $f(x) = \frac{e^x}{\sqrt{x}}$

6) Match the graph of the function, f(x), with the graph of its derivative, f '(x).

a)



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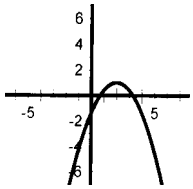

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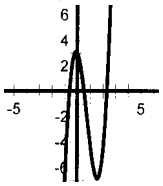

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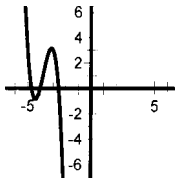

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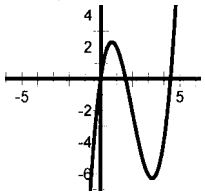

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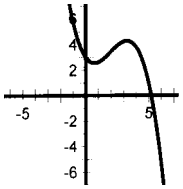

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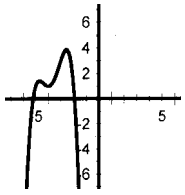

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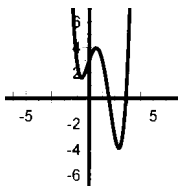
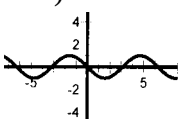


i)



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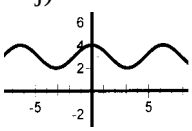

7) Give the equations of the last two above graphs.

Calculus – Part II

Problem Set #1

1) Find $\frac{dy}{dx}$.

a) $y = 4x^7 - 3x^2 + x - 5$

b) $y = 8 + x - \cos x$

c) $y = 2 \sin x - \tan x$

d) $y = x^2 \sin x$

e) $y = \cos(4x^2)$

f) $y = e^{3x}$

g) $y = \frac{4}{\cos x}$

h) $y = \frac{\cos x}{1 + \sin x}$

i) $y = \ln(\sin^2 x)$

Implicit Differentiation

With implicit differentiation, we treat differentiation as an operation and take the derivative of both sides of an equation, with respect to a chosen variable.

Keep in mind that the derivative of x (with respect to x) is $\frac{dx}{dx}$, which is 1, and the derivative of y (with respect to x) is $\frac{dy}{dx}$.

Example: Find $\frac{dy}{dx}$ given that

$$x^5 + y^2 = 4xy + 7$$

Solution:

$5x^4 \cdot \frac{dx}{dx} + 2y \cdot \frac{dy}{dx} = 4(y \cdot \frac{dx}{dx} + x \cdot \frac{dy}{dx})$

Each $\frac{dx}{dx} = 1$, and solving for $\frac{dy}{dx}$ gives us $\frac{dy}{dx} = \frac{5x^4 - 4y}{4x - 2y}$.

2) Find $\frac{dy}{dx}$ by implicit differentiation

a) $y^2 = x^2 + 3x$

b) $y^2 = 7xy + 3$

c) $x^2 = \cos y$

d) $x^2 + y^2 = 9$

e) $x^3 + 5y^2 = 3x + 2y$

f) $y^3 + y^2 = e^x$

g) $y \cdot \ln y + x \cdot \ln x = 20$

h) $x^2 - 6xy + 9y^2 - 2y + 1 = 0$ (*Challenge*: find the vertex of this "slanted" parabola.)

Related Rates

Of course, it was not necessary to use calculus to solve the above problem. Calculus is needed, however, to solve this next example on related rates.

Example: A snowball, which always has a perfectly spherical shape, is melting such that its radius is decreasing at a constant rate of 0.3cm per minute. How fast is the volume decreasing (in cm^3/minute) when the radius is 20cm?

Solution: We are given $\frac{dr}{dt} = 0.3$. Our goal is to find $\frac{dv}{dt}$.

The key is the relationship between r and v, which is given by the formula $v(r) = \frac{4}{3}\pi r^3$. Taking the derivative gives us $v'(r) = \frac{dv}{dr} = 4\pi r^2$. We can now find a formula for $\frac{dv}{dt}$.

$\frac{dv}{dt} = \frac{dr}{dt} \cdot \frac{dv}{dr} = (0.3)(4\pi r^2)$

$\frac{dv}{dt} = 1.2\pi r^2$. This formula allows us to answer our initial question (how fast is the volume decreasing when r = 20?) by putting 20 in for r:

$1.2\pi(20)^2 = \underline{480\pi}$ cm^3/minute.

3) The area of a circle is growing at a constant rate of 3 ft^2/sec. How fast is the radius growing when its length is 7 feet?

Problem Set #2

Implicit Differentiation

1) Find $\frac{dy}{dx}$.

a) $5x^3 - 7x = 3y^2$

b) $x^2y + y^2x = 12$

c) $2x = y^2 + \sin x$

d) $2y = x^2 + \sin y$

e) $y = \frac{\ln x}{x}$

f) $y = e^x \cos x$

g) $y = \frac{x^4}{\cos x}$

h) $x \sin y + y \cos x = 1$

Related Rates

2) A spherical balloon is being filled with air at a rate of 200 cm^3/sec. At what rate is the radius increasing when it is 10cm?

3) A balloon is rising vertically from the ground at a constant rate of 140 ft/minute. Bob is standing 300 feet from the spot where the balloon was released. At what rate is the distance from Bob to the balloon increasing when the balloon is 90 feet above the ground? (Assume that the ground is level.)

4) **Proving $\frac{d}{dx}(\ln x) = \frac{1}{x}$.**

Given $y = \ln x$, we can rewrite it in exponential form as $x = e^y$. Derive $\frac{dy}{dx}$ by implicit differentiation.

Speed-Distance Problems

5) The distance that a ball rolls down an inclined plane is given below by d(t), where d is in meters and t is in seconds. For each d(t), do the following:

- Give the velocity function, v(t), which equals d'(t).
- Describe how the ball has been put into motion (i.e., initial velocity, etc.).
- At the given time, t, find the distance traveled and the instantaneous speed.

a) $d(t) = 4t^2$ at $t = 4$

b) $d(t) = 3t^2 + 6t$ at $t = 2$

c) $d(t) = 2t^2 - 6t$ at $t = 3$

6) Paul throws a ball straight up in the air with an initial velocity of 25m/s.

a) Give the equation, v(t), for the velocity of the ball. (Recall that acceleration due to gravity is 9.8m/s^2.)

b) Give the equation for the height of the ball, d(t), given that it was released from 2m above the ground.

c) Calculate the ball's height and speed after 1, 3, and 6 seconds.

d) What is the maximum height of the ball?

e) Graph d(t) and v(t) on the same graph.

f) Explain how the two graphs describe the movement of the ball.

7) *Descartes' folium.*
Given $x^3 + y^3 = 9xy$

a) Find the slope of the curve at (2,4) and (4,2).

b) Where else on the curve is x = 2?

c) Where else on the curve is y = 2?

d) Where else on the curve is x = 4?

e) Where else on the curve is y = 4?

f) Where on the curve is the slope equal to −1?

g) Graph the curve.
(Hint: It is asymptotic to the line x+y + 3 = 0.)