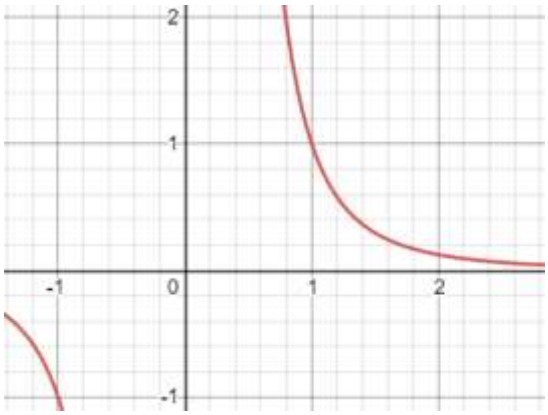
12th Grade Assignment – Week #15

Group Assignments:



for Tuesday

Derivatives and Negative Exponents

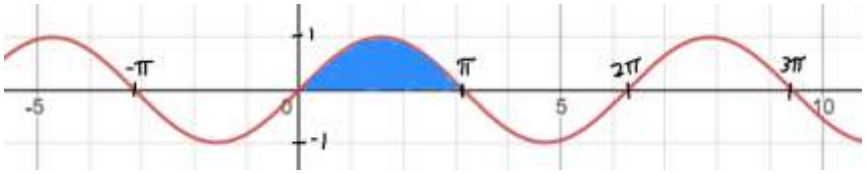
1) The graph of $f(x) = \frac{1}{x^3}$ is shown on the right.
   a) find $f'(x)$ (Hint: first rewrite it as $f(x) = x^{-3}$)
   b) What is the slope of the graph at $x = 2$?
   c) What is the slope of the graph at $x = -2$?
   d) For what value of x is the slope equal to zero?
   e) What is the anti-derivative of $f(x)$?
   f) What is the area bounded by the curve, the x-axis, and the vertical lines $x = 1$ and $x = 2$?

2) Do each of the below problems. (Important question: Which one doesn't work?)
   Find the derivative of: a) $f(x) = 5x^7$ b) $f(x) = \frac{5}{x^7}$ c) $f(x) = \frac{5}{x}$ d) $f(x) = 5$
   Find the anti-derivative of: e) $f(x) = 5x^7$ f) $f(x) = \frac{5}{x^7}$ g) $f(x) = \frac{5}{x}$ h) $f(x) = 5$

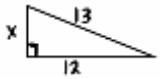
3) Prove that $\frac{d}{dx}(\cos x) = -\sin x$ (Hint: Do similarly to what I did in yesterday's lecture.)

for Thursday

4) The graph of $\sin x$ is given below. Find the area of the shaded "hump".



5) On the right is my math clock, which includes math problems from 6th grade through 12th grade. Together, go through each one to make sure you understand why it works. Since it is hard to read, I also include each one here:

1 o'clock is $-e^{i\pi}$	5 o'clock is 	9 o'clock is $27^{2/3}$
2 o'clock is $\int_0^{\pi} \sin x \, dx$	6 o'clock is $\sum_{k=0}^{\infty} \frac{4}{3^k}$	10 o'clock is $(1+3i)(1-3i)$
3 o'clock is $\tan^2 60°$	7 o'clock is 111_{binary}	11 o'clock is $\frac{682}{62}$
4 o'clock is $\log_3 81$	8 o'clock is $\sqrt{64}$	12 o'clock is ${}_4P_2$

6) In yesterday's lecture, I showed how to find the volume of an ellipsoid using integration. Use a similar method to find the volume of…
   a) a sphere with a radius of 5. b) a sphere with a radius of r.

Individual Work

- Do **Problem Set #1** (*Calculus – Part I*): Select the problems you need to work on.
  (Note: Problem Set #2 was done last week in group work, and most of Problem Set #3 was either done in the lecture or appeared in a group assignment.)
- Do **Problem Set #4** (from *Calculus – Part I*), pr #1-3.

Calculus – Part I

Problem Set #1

Derivative Practice

1) Find f '(x).
   a) $f(x) = x^5$
   b) $f(x) = 6x$
   c) $f(x) = 8$

2) Find $\frac{dy}{dx}$.
   a) $y = 4x^{-2}$
   b) $y = \frac{4}{x^2}$

3) And yet another way of expressing the derivative:
   a) Find $\frac{d}{dx}(3x^2 + 8x)$
   b) Find $\frac{d}{dx}(x^4 - 6x^3 + 4x - 5)$
   c) Find $\frac{d}{dx}(5x^7 + x^3 - \frac{1}{4x^3})$

4) *Anti-Derivatives*. Find F(x).
   a) $f(x) = x^5$
   b) $f(x) = 6x$
   c) $f(x) = 8$
   d) $f(x) = 3x^2 + 8x$
   e) $f(x) = x^4 - 6x^3 + 4x - 5$
   f) $f(x) = \frac{4}{x^2}$

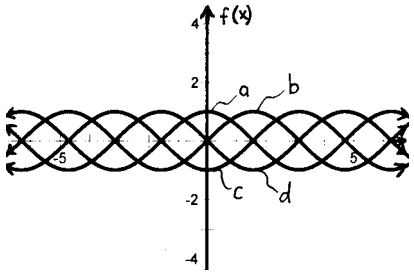
5) Draw the curve, and then find the area under the curve.
   a) $f(x) = 6x^2$, $\int_2^5 f(x)\,dx$
   b) $f(x) = \frac{1}{2}x + 3$, $\int_2^6 f(x)\,dx$
   c) $f(x) = \frac{1}{3}x^3 + x^2 - 3x$, $\int_{-4}^{-1} f(x)\,dx$

6) Given $f(x) = -x^3 + 4x^2 - 3$
   a) Fill in this table:

x	f(x)	f '(x)
5		
4		
3		
2		
1		
0		
–1		
–2		

   b) What does it mean to say f(3) = 6?
   c) What does it mean to say f '(3) = –3?
   d) Find the roots of f(x).
   e) Give the coordinates of the local max and min points.
   f) Graph f(x).
   g) Find the area under the curve from 1 to 3.

With each given function, state whether it corresponds to the graph a, b, c, or d, as shown below.



7) f(x) = sin(x)
8) f(x) = cos(x)
9) f(x) = –sin(x)
10) f(x) = –cos(x)
11) f(x) = cos(–x)
12) f(x) = cos(x+π)
13) f(x) = cos(x–π)
14) f(x) = sin(–x)
15) f(x) = cos(x+2π)
16) f(x) = cos(x–$\frac{\pi}{2}$)
17) f(x) = sin(x+$\frac{\pi}{2}$)
18) f(x) = sin(x+$\frac{3\pi}{2}$)

Problem Set #4

1) Find the slope of…

a) $f(x) = \sin x$ at $x = \pi/6$

b) $f(x) = \sin x$ at $x = \pi/2$

c) $f(x) = \cos x$ at $x = \pi/2$

d) $f(x) = \sin x$ at $x = \pi$

e) $f(x) = \cos x$ at $x = \pi$

f) $f(x) = \sin x$ at $x = 0$

g) $f(x) = e^x$ at $x = 2$

h) $f(x) = \ln x$ at $x = 3$

2) Evaluate the integrals.

a) $\int_0^{\pi} \sin x \, dx$

b) $\int_0^{\pi/2} \cos x \, dx$

c) $\int_0^{\pi} \cos x \, dx$

d) $\int_2^5 \frac{1}{x} \, dx$

e) $\int_1^e \frac{1}{x} \, dx$

f) $\int_0^2 e^x \, dx$

3) Given $f(x) = e^x$…

a) What is the height of the curve at x = 3?

b) What is the slope of the curve at x = 3?

c) What is the area of the infinitely long region which is bounded by the curve f(x), the x-axis, the line x = 3, and continues infinitely far to the left?