12th Grade Assignment – Week #14

Group Assignments:

for Tuesday – Do the problems (from the unit *Calculus – Part I*) in the order give here:

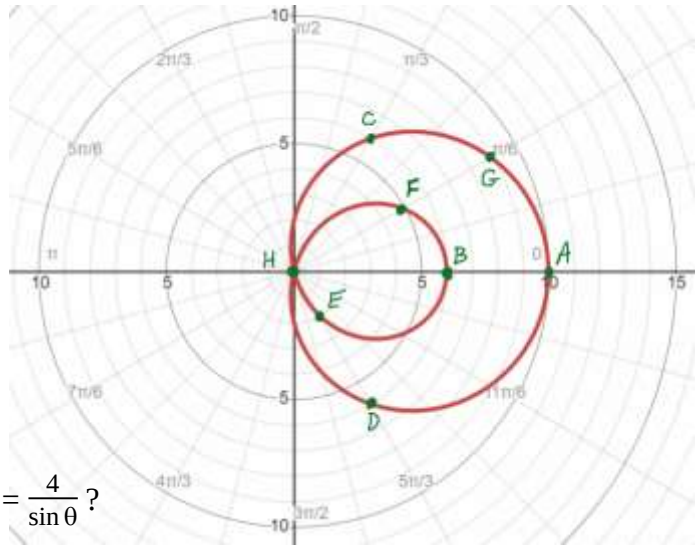
- **Important Limits.** (This problem comes from Problem Set #3, but is repeated here.)
  Use a calculator to determine the value of each limit (below), by letting the value of h get closer and closer to zero. With the first two problems, try it first with your calculator in degree mode, and then repeat it in radian mode.

$$\lim_{h\to 0}\frac{\sin(h)}{h} = \qquad \lim_{h\to 0}\frac{\cos(h)-1}{h} = \qquad \lim_{h\to 0}\frac{e^h-1}{h} =$$

- Do *Calculus Part I,* Problem Set #2, pr #19-23 (Note: You have to figure out the quotient rule!);
- Do Problem Set #2, pr #5-18 (Note: We are doing Problem Sets #1 and #2 in reverse order.)

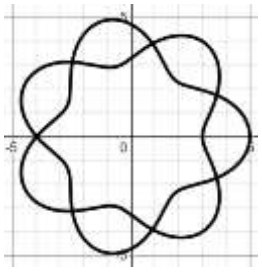
for Thursday – **Polar Graphing**

1. Last week, you investigated the polar equation in the form $r = n + m\cos\theta$.
   This curve is called a limaçon, which can appear in three general ways, as shown above.
   (You may remember drawing a limaçon in 6th grade geometric drawing.)
   a) Give the equation of the limaçon shown on the right.
   b) Give the coordinates of each of the labeled points that come from the above equation.
   (Hint: point E is not $(2, {}^{5\pi}/_{3})$)



   What does the graph of each of the below equations look like?
   c) $r = 6 + 4\sin\theta$
   d) $r = 6 + 4\sin(\theta + {}^{\pi}/_{3})$
   e) $r = -(6 + 4\sin(\theta + {}^{\pi}/_{3}))$

2. What is the graph of the polar equation $r = \dfrac{4}{\sin\theta}$?
3. Give the equation of the polar graph that is a horizontal line passing through the point $(6, {}^{7\pi}/_{6})$
4. Give the equation of the polar graph that is a line passing through the points $(6, {}^{\pi}/_{2})$ and $(6, \pi)$.
5. Given a polar equation in this form: $r = \dfrac{5}{a + b\sin\theta}$, where a, b are positive integers, what do the values of a and b indicate about the graph?
6. Try to guess what the graph of $r = 4 + \cos(5\theta)$ looks like, then check your guess with a graphing application.
7. Given a polar equation in this form: $r = a + \cos(b\,\theta)$, where a, b are positive integers, what do the values of a and b indicate about the graph?
8. *Challenge!* Try to determine the equation of the curve shown here.



9. *Challenge!* Given a polar equation in this form:
   $r = 4\cos(\frac{a}{b}\theta)$, where a, b are positive integers,
   what do the values of a and b indicate about the graph?

Individual Work (This is mostly a review of the Calculus main lesson.)

- If you need more practice before taking the test, then choose some problems from **Problem Set #6** of the *Cartesian Geometry – Part IV* unit.
- Take *Cartesian Geometry – Part IV* test, which is found at the end of this document.

Calculus – Part I

Problem Set #2

1) Find f '(x) and F(x)

a) $f(x) = 4x^3$

b) $f(x) = 2x$

c) $f(x) = 5$

d) $f(x) = 7x^2 + 6x - 2$

e) $f(x) = \frac{2}{x^3}$

2) Find the area under the curve.

a) $\int_2^6 (x^2 - 8x + 12) \, dx$

b) $\int_0^3 (x^3 - 3x^2) \, dx$

3) Use the definition of the derivative to determine the derivative of :

$f(x) = x^3 - 7x$

4) Given $f(x) = \frac{1}{3}x^3 + x^2 - 3x$

a) Find f '(x).

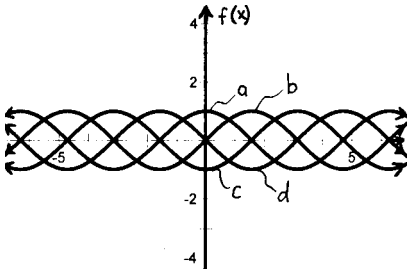
b) Graph f(x) and f '(x) on the same graph.

c) Where is the slope of f(x) positive? Where is it negative? How is this reflected in the graph of f '(x)?

d) Give the coordinates of the local max and min points of f(x).

e) Give the coordinates of the local max and min points of f '(x). How is this reflected in the graph of f(x)?

With each given function, state whether it corresponds to the graph a, b, c, or d, as shown below.



5) $f(x) = \cos(x)$

6) $f(x) = \sin(x)$

7) $f(x) = -\cos(x)$

8) $f(x) = -\sin(x)$

9) $f(x) = \sin(x + 2\pi)$

10) $f(x) = -\sin(-x)$

11) $f(x) = \sin(x - \frac{\pi}{2})$

12) $f(x) = -\sin(x - \frac{\pi}{2})$

13) $f(x) = -\cos(-x)$

14) $f(x) = \sin(x + \pi)$

15) $f(x) = \sin(x - \pi)$

16) $f(x) = \sin(x + 5\pi)$

17) Given $f(x) = \sin(x)$, make a rough sketch of f '(x). What do you think f '(x) is equal to?

18) Given $f(x) = \cos(x)$, make a rough sketch of f '(x). What do you think f '(x) is equal to?

Rules for Derivatives

Given functions, u and v…

Power Rule $\frac{d}{dx}(a\,x^n) = (a \cdot n)x^{n-1}$

Sum Rule $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

or $(u + v)'(x) = u'(x) + v'(x)$

Product Rule $\frac{d}{dx}(u \cdot v) = v\frac{du}{dx} + u\frac{dv}{dx}$

or $(u \cdot v)'(x) = v(x) \cdot u'(x) + u(x) \cdot v'(x)$

Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

19) State the meaning of each of the above rules.

20) Derivatives

a) Find $\frac{d}{dx}(x^5)$

b) Find $\frac{d}{dx}(3x)$

c) Find $\frac{d}{dx}(2x^4 - x^2 + 8)$

21) Find f '(x).

a) $f(x) = 9$

b) $f(x) = (x^2 - 4x - 3)(x^2 + 5)$

c) $f(x) = \frac{x^2 + 5x}{3x - 2}$

22) Find $\frac{dy}{dx}$.

a) $y = (x^3 - 7x + 1)(x^4 - 8)$

b) $y = \frac{x^5 + x^3}{4x^2 + 5}$

23) What could f(x) be such that f '(x) = f(x) and F(x) = f(x)?

(No, f(x) = 0 doesn't count!)

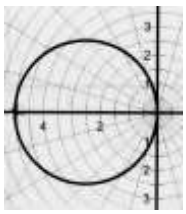
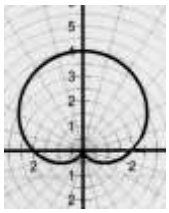
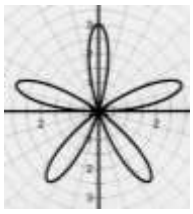
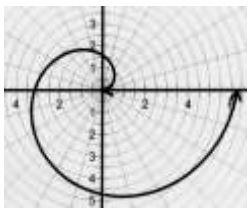
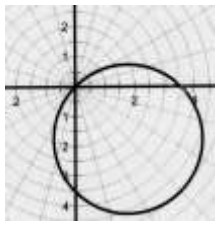
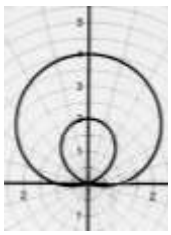
Cartesian Geometry – Part IV

Problem Set #6

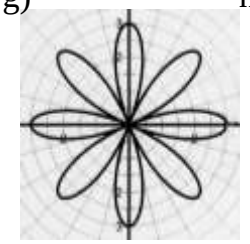
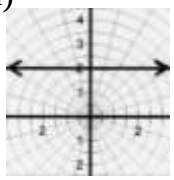
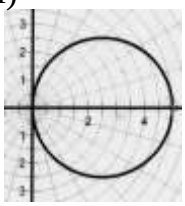
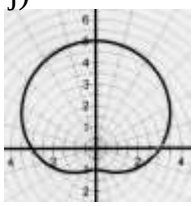
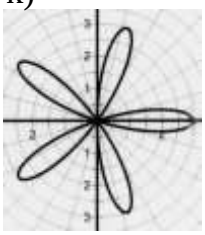
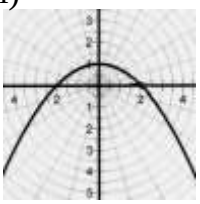
Graphing Polar Equations. Match the equation with its graph, given below.

1) $r = 3\cos(4\theta)$
2) $r = 3\sin(5\theta)$
3) $r = 3\cos(5\theta)$
4) $r = 5\cos(\theta)$
5) $r = -5\cos(\theta)$
6) $r = 5\cos(\theta + \pi/4)$
7) $r = \theta$
8) $r = \frac{2}{1+\sin\theta}$
9) $r = \frac{2}{\sin\theta}$
10) $r = 1 + 3\sin(\theta)$
11) $r = 2 + 2\sin(\theta)$
12) $r = 3 + 2\sin(\theta)$

a) b) c) d) e) f)

g) h) i) j) k) l)

Graph each function.

13) $f(x) = x(x-1)^2(x+2)^2(x-2)$

14) $f(x) = (x+1)^3(x+4)$

15) $f(x) = 4 + \sqrt{x-3}$

16) $f(x) = -e^x$

17) $f(x) = e^{-x}$

18) $f(x) = \ln(x+3)$

19) $f(x) = 7^{\frac{1}{2}x}$

20) $f(x) = \frac{x-3}{x+2}$

21) $f(x) = \frac{5}{x^2-16}$

22) $f(x) = \frac{x^3-4x}{2x^2+4}$

23) $f(x) = -\frac{1}{x}$

24) $f(x) = \frac{1}{x^2}$

25) $f(x) = \frac{x}{x^2-x-2}$

26) For each function:
- Find $f^{-1}(x)$.
- Graph $f(x)$ and $f^{-1}(x)$ on the same graph.

a) $f(x) = 3 - \ln x$

b) $f(x) = 3\log_2 x$

27) Choose any $f(x)$ given on this page.

a) What is $f(f^{-1}(2))$?

b) What is $f^{-1}(f(-5))$?

c) What is $f(f^{-1}(x))$?

28) Graph $f(x) = x^3 + 2x^2 - 3x$. Also find the local max and min coordinates (calculus is required).

29) Graph $f(x) = x^4 + x^2 - 6$. and find the complex roots.

30) Graph $f(x) = -x^3 + 2x + 4$. and find the complex roots.

Cartesian Geometry IV Test

1) Graph each function. (4 points each.) (All asymptotes should be shown as dotted lines. Be sure to label your graphs!)

a) $f(x) = -3^{-x}$

b) $f(x) = -5 + e^{x}$

c) $f(x) = \log_4 (x+5)$

d) $f(x) = 5 + \sqrt{-x}$

2) Given a function, f(x), and its inverse, $f^{-1}(x)$, what is $f(f^{-1}(x))$ always equal to? (2 points.)

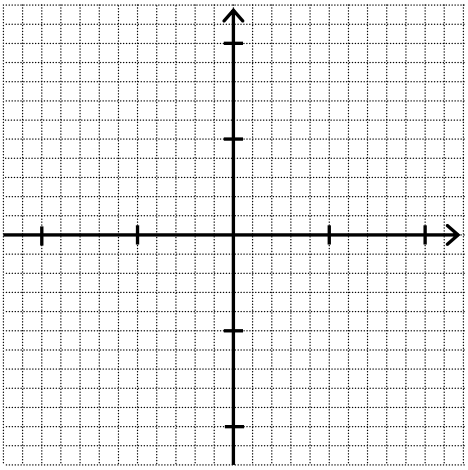
3) For each function, f(x), find the inverse function, $f^{-1}(x)$. If the function doesn't have an inverse, then write "no inverse". (2 points each.) (You don't have to graph these!)

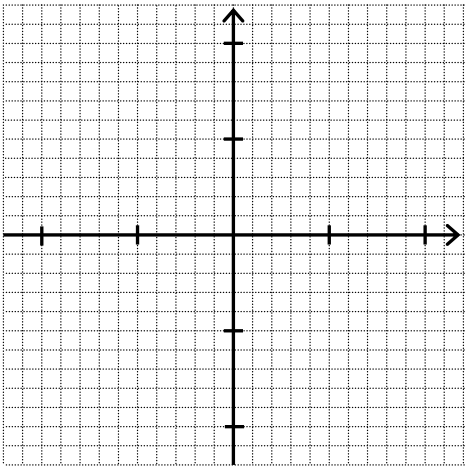
a) $f(x) = x + 8$

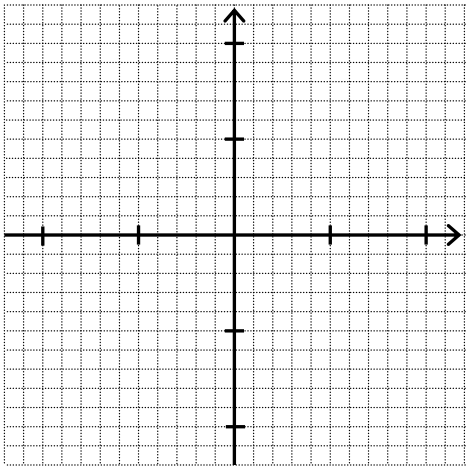
b) $f(x) = 5x$

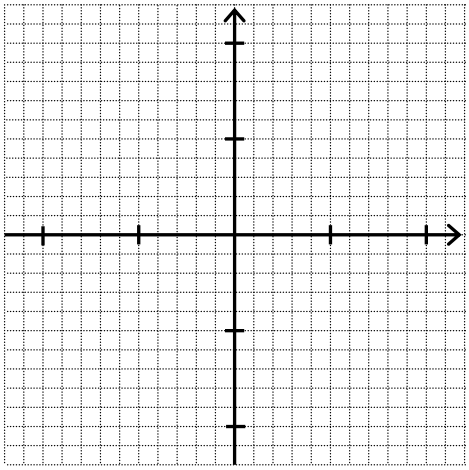
c) $f(x) = x^2 + 3$

d) $f(x) = x^3 + 4$









4) Graph each function. (4 points each.)
(All asymptotes should be shown as dotted lines. Be sure to label your graphs!)

a) $f(x) = x^3 - 4x$

b) $f(x) = -2(x+5)^3(x+3)$

c) $f(x) = \frac{8x}{x^2 - 9}$

d) $f(x) = \frac{x^2 - 4}{x^2 + 1}$

e) $f(x) = \frac{x^2 + 4}{3x}$

5) Graph each Polar Equations. (4 points each.)

a) $r = -6\cos(5\theta)$

b) $r = 4 + 4\sin\theta$