# 12th Grade Assignment – Week #12

## Group Assignments:

*for Tuesday* – **The Prisoners' Dilemma**

One afternoon, the prison warden tells the twenty prisoners in the men's ward that the following morning they will be blindfolded and marched out to the courtyard. Once there, they will be lined up with everyone facing forward (towards the prisoner directly in front of him) and a red or blue hat will randomly be placed on each head. The blindfolds will then be removed so that each prisoner can see all the hats in front of him. Then, beginning at the back of the line, each prisoner, in turn, will be allowed to say only the word "red" or "blue". If he correctly states the color of his own hat, he will be set free, otherwise he will not be freed – and this decision will be announced by the guard clearly for everyone to hear.

After hearing about this plan, the prisoners requested to the warden that they be allowed tonight to discuss and come up with a strategy for how they will do this tomorrow morning. What is the best strategy that will free the most prisoners?

(There is no certain number of red or blue hats. Once the prisoners are placed in line, they may not communicate with one another in any way, other than by saying "red" or "blue" when their turn comes, and no "extra" information may be conveyed by saying the word in a different manner (e.g., tone, loudness, etc.) We shall also assume that all of the prisoners are highly intelligent, and therefore can follow any coherent plan.)

**Important:** After you have come up with your best strategy, you should email your instructor to tell your strategy.

*for Thursday* – **The Prisoners' Dilemma (continued)**

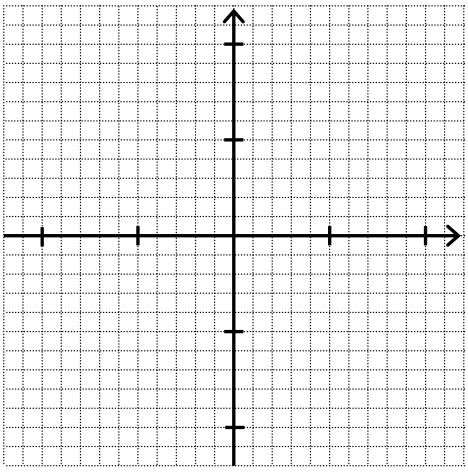
After hearing from you (after your Tuesday group meeting) about what your best strategy is for the Prisoners' Dilemma, your instructor will let you know if you have given the most optimal solution. If not, then you may receive a hint so that you can continue to work on it. Otherwise, you can spend your time working on the Individual Work.

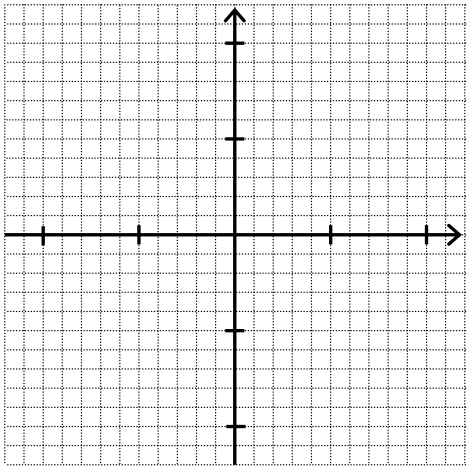
## Individual Work

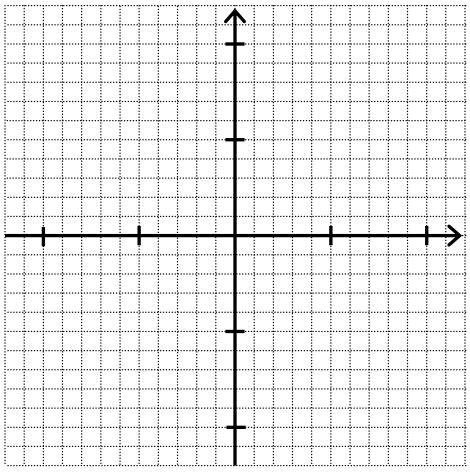
- Work on **Problem Set #4** from *Cartesian Geometry – Part IV* (but not problem #17)

## Announcement

- You will need the polar graph paper, found on the last page, for your tutorial session.







# Problem Set #4

## Exponential and Logarithmic Functions

Match the function with its graph, given below.
(Note: Two of the functions have the same graph.)

1) $f(x) = 3^x$

2) $f(x) = 3^{x-5}$

3) $f(x) = 3^x - 5$

4) $f(x) = 3^{-x}$

5) $f(x) = -3^x$

6) $f(x) = (\frac{1}{3})^x$

7) $f(x) = \log_3 x$

8) $f(x) = -\log_3 x$

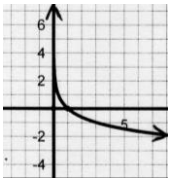
9) $f(x) = \log_3 (-x)$

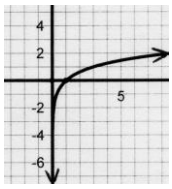
10) $f(x) = -\log_3 (-x)$

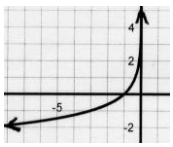
11) $f(x) = 2\log_3 x$

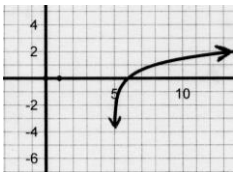
12) $f(x) = \log_3 (x-5)$

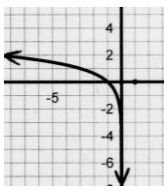
13) $f(x) = -5 + \log_3 x$

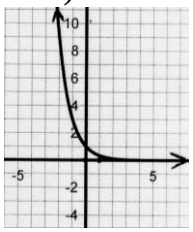
a) 

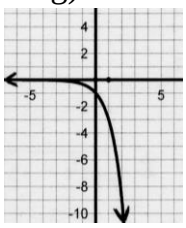
b) 

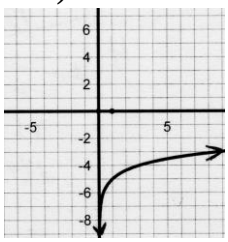
c) 

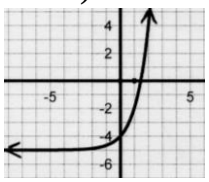
d) 

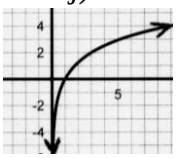
e) 

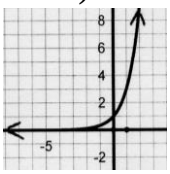
f) 

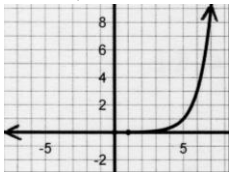
g) 

h) 

i) 

j) 

k) 

l) 

## Inverse Functions

14) Given $f(x) = 4x + 3$
and $g(x) = x^3 - 2$

a) Find $f^{-1}(x)$.

b) Find $g^{-1}(x)$.

Evaluate:

c) $f(5)$

d) $f^{-1}(5)$

e) $g(6)$

f) $g^{-1}(6)$

g) $g(f(0))$

h) $f(g(0))$

i) $f(g(x))$

j) $g(f(x))$

k) $f(f^{-1}(15))$

l) $f^{-1}(f(73))$

m) $f(f^{-1}(x))$

n) $f^{-1}(f(x))$

o) $g(g^{-1}(25))$

p) $g^{-1}(g(11))$

q) $g(g^{-1}(x))$

r) $g^{-1}(g(x))$

15) For each given function, first find $f^{-1}(x)$, then graph both $f(x)$ and $f^{-1}(x)$ on the same graph.

a) $f(x) = 2x+3$

b) $f(x) = 5 - x$

c) $f(x) = \sqrt[3]{8 - x^3}$

## Rational Functions

16) Graph each function.

a) $f(x) = \frac{x}{4x^2 - 36}$

b) $f(x) = \frac{x^2}{4x^2 - 36}$

c) $f(x) = \frac{x^3}{4x^2 - 36}$