12th Grade Assignment – Week #11

Group Assignments:

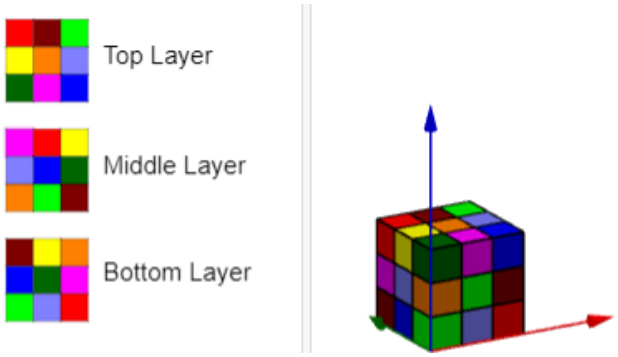
for Tuesday – **Three Numbers – Part II**

Puzzle! Find three numbers that add to 13 and multiply to 32.
Yes, you can use algebra for all of these.

1) Find a solution such that one of the three numbers is 2.
2) Find a solution such that one of the three numbers is 10.
3) Find a solution such that two of the numbers are equal.
4) Find a solution such that all three numbers are rational (fractions), and at least one is not an integer. See if you can find multiple such solutions.

for Thursday **A 9-Colored Cube**

- You are given 27 small cubes – three each of nine colors. Your task is to make one large cube such that each of its faces contain one of every color.
- The cube shown on the right (which shows an incorrect solution), also gives a diagram of the three layers so that you know the colors of all six faces. If you study the layers, you can tell that four the faces correctly have nine different colors, and two faces (the front and back) are incorrect. Make sure that everyone understands this before trying to find a correct solution.



- Now, try to find your own solution. In order to work on this together, it will be best to share your screen and use this interactive version: https://nrich.maths.org/ninecolours

Individual Work

- Work on **Problem Set #3** from *Cartesian Geometry – Part IV*, but don't do Problem #9.

Problem Set #3

Inverse Functions

On the previous problem set, you hopefully noticed that two inverse functions have graphs that are reflections about the line $y = x$.

1) If (2,5) is one point on the graph of f(x), then what is one point that must be on the graph of $f^{-1}(x)$?

This gives us an insight into how to find the inverse of a more complicated function. We simply need to switch the x and y values around.

Example: Find the inverse of

$$f(x) = \frac{5}{2x+8}$$

Solution: $y = \frac{5}{2x+8}$

Switch x and y: $x = \frac{5}{2y+8}$

Solve for y: $y = \frac{5}{2x} - 4$

Answer: $f^{-1}(x) = \frac{5}{2x} - 4$

2) For each function, f(x), find the inverse function, $f^{-1}(x)$. Also graph both f(x) and $f^{-1}(x)$ on the same graph.

a) $f(x) = x^3$

b) $f(x) = 2x - 6$

c) $f(x) = \frac{12}{x}$

Graphing Practice

3) Graph each function.

a) $f(x) = x^3 - 4x$

b) $f(x) = \frac{1}{4}x^3 - x$

c) $f(x) = \frac{1}{3}x^5 - 2x^4 + 3x^3$

d) $f(x) = \frac{3x}{x^2-9}$

e) $f(x) = \frac{3x^2}{x^2-9}$

f) $f(x) = \frac{10}{x^2+9}$

g) $f(x) = \frac{10x}{x^2+9}$

h) $f(x) = \frac{10x^2}{x^2+9}$

i) $f(x) = \sqrt[3]{x} - 5$

Function Properties

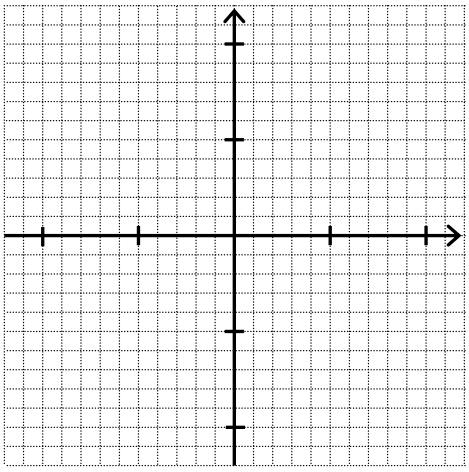
One of the basic properties of functions is that any given x value can produce only one value for the function.

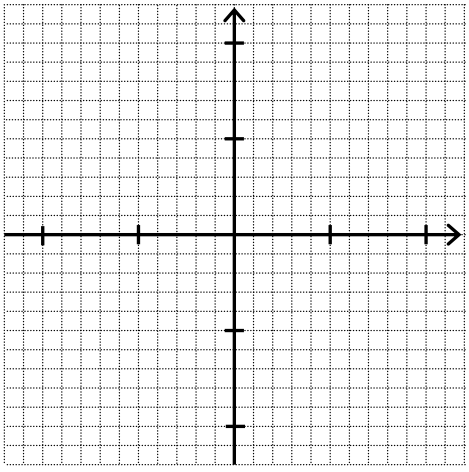
4) What is the equation of a circle that has a radius of 5 and its center is at the origin?

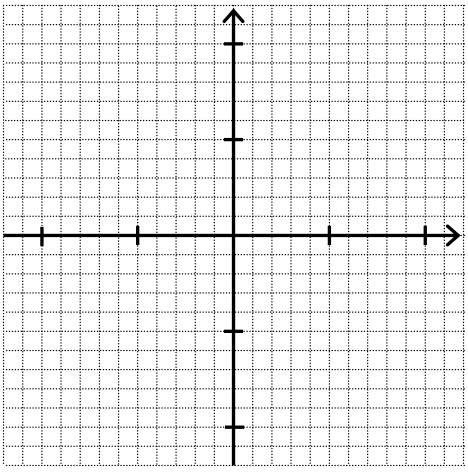
5) Express the above equation as a function.

6) The answer to the above problem is: "Not possible". How can you tell by looking at a graph that it can't be expressed as a function?

7) How can you tell by looking at a graph that it doesn't have an inverse that can be expressed as a function?





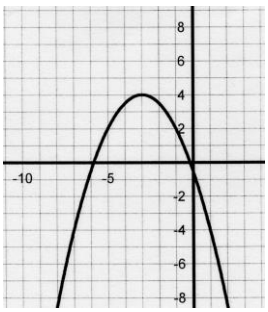


8) Categorize each graph shown below as either:

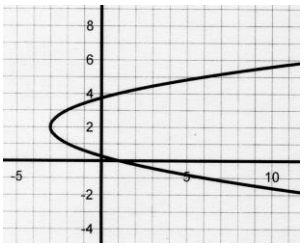
- A function that has an inverse.
- A function that doesn't have an inverse.
- Not a function.

Additionally, if the graph does have an inverse, then draw (freehand) its inverse.

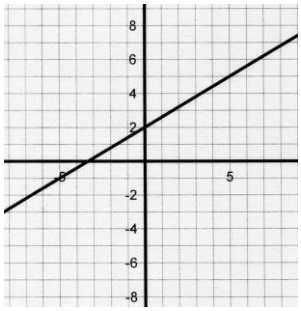
a)



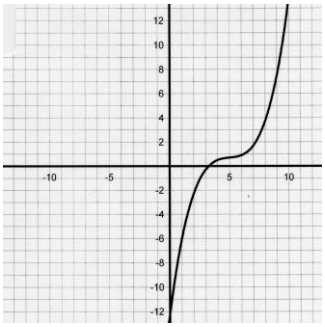
b)



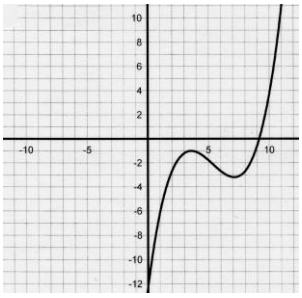
c)



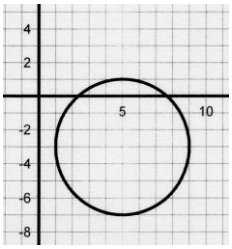
d)



e)



f)



Exponential and Logarithmic Functions

9) Graph each function.

a) $f(x) = 2^x$

b) $f(x) = \log_2 x$

c) $f(x) = 5^x$

d) $f(x) = \log_5 x$

10) Find f(x) given that it is a third degree polynomial equation with roots x = 0, 1, 3, and the curve passes through the point (2,–4).