12^{th} Grade Assignment – Week #10

<u>Group Assignments</u>:

for Tuesday

- Work on **Problem Set #1** from *Cartesian Geometry – Part IV*

for Thursday – **Three Numbers – Part I**

Puzzle! Find three numbers that add to 13 and multiply to 32.

Do <u>not</u> use algebra to solve this until you get to #3.

1) Find a solution such that all three numbers are positive integers.
2) Find a solution such that all three numbers are integers, and at least one of the numbers is negative.
3) Write down the two algebraic equations (in terms of x, y, z) for this problem, and then use substitution to eliminate z and end up with one equation in terms of x and y.
4) Graph the equation you got for #3 by using a graphing application (e.g., Desmos).
   (First, you should verify that your equation is equivalent to the equation found at the bottom of this page. Is it?)
   What does this graph tell you about the solutions of the original problem?

<u>Individual Work</u>

- Take the **Calculus test** found at the end of this document.
- Work on **Problem Set #2** from *Cartesian Geometry – Part IV*

$x^2y + xy^2 – 13xy + 32 = 0$

Cartesian Geometry – Part IV

Problem Set #1

Polynomial Functions

Match the function with its graph, given below.

1) $f(x) = 3x^2 + 6x$

2) $f(x) = 3x^2 - 6x$

3) $f(x) = 3x^2 - 6$

4) $f(x) = \frac{1}{3}x^2 - 6$

5) $f(x) = \frac{1}{10}x^3 + \frac{1}{10}x$

6) $f(x) = x^4 - 4x^2 + 1$

7) $f(x) = -x^4 + 4x^2 - 1$

8) $f(x) = \frac{1}{5}x^5 - 3x^3 + 10x$

9) Graph each function. Besides the roots, only plot two or three key points.

a) $f(x) = (x-2)(x-4)(x-6)$

b) $f(x) = (x-2)(x-4)^2(x-6)$

c) $f(x) = -(x-2)(x-4)^3(x-6)$

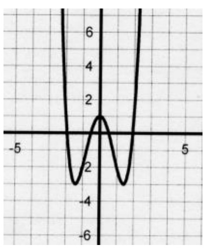
d) $f(x) = x^3 - 4x^2$

e) $f(x) = x^4 - 3x^2$

f) $f(x) = -x^4 + 3x^3$

g) $f(x) = x^4 - 3$

10) The graphs below show two types of special symmetry. The first type is shown by graphs a, b, e, and f. The second type of symmetry is shown by c and g. Describe both of these symmetries.

a)


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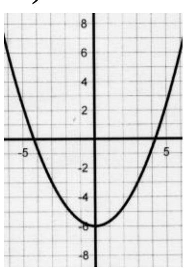

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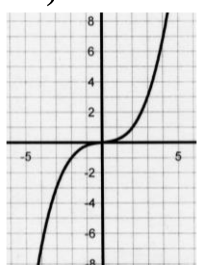

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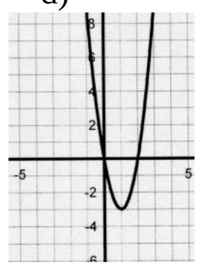

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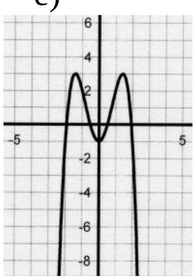

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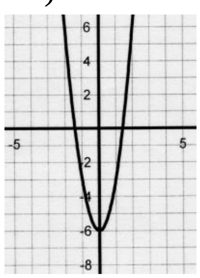

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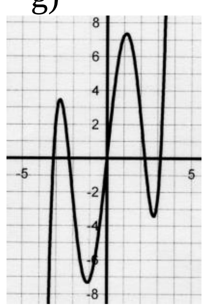

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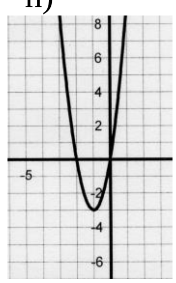
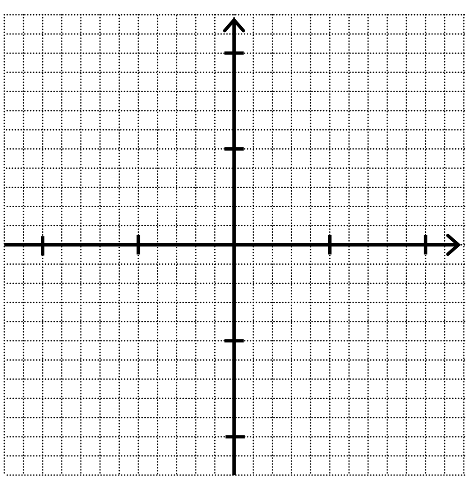
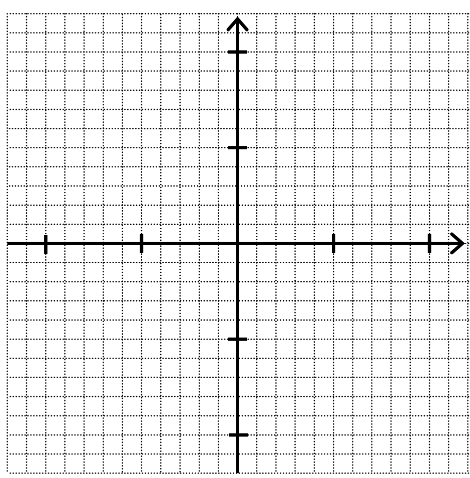
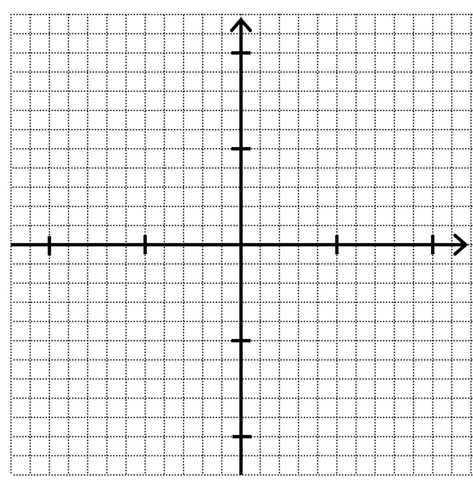






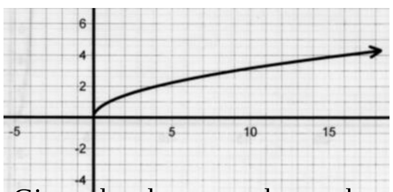


Even and Odd Functions

The two types of symmetries referred to in the last problem occur with even and odd functions. The graph of an *even function* has reflective symmetry about the y-axis. The graph of an *odd function* has 180° rotational symmetry about the origin.

11) Look at the functions, and their matching graphs, given at the start of this problem set. How can you tell by looking at a function (not looking at its graph) whether the function is even, odd, or neither?

12) Consider $f(x) = \sqrt{x}$.
Its graph is given below. Note that since the domain is only for $x \geq 0$ there are no negative x values on the graph.



Given the above graph, graph each of the below functions.

a) $f(x) = -\sqrt{x}$

b) $f(x) = \sqrt{-x}$

c) $f(x) = \sqrt{x+4} - 3$

d) $f(x) = -3\sqrt{x}$

e) $f(x) = \sqrt{2x}$

f) $f(x) = -\frac{1}{2}\sqrt{x+4} - 3$

13) Find f(x) given that it is a third degree polynomial equa-tion with roots x = 0, 6, –5, and the coefficient of
the x^3 term is 2.

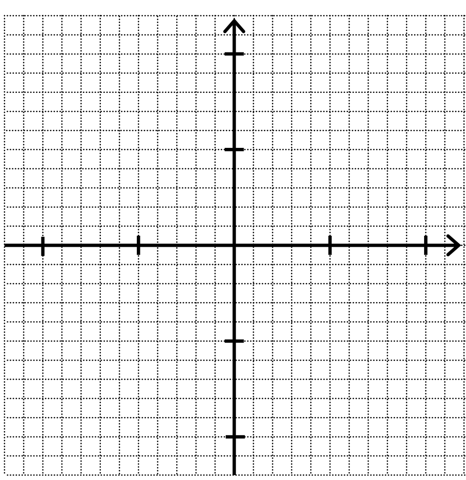
Rational Functions

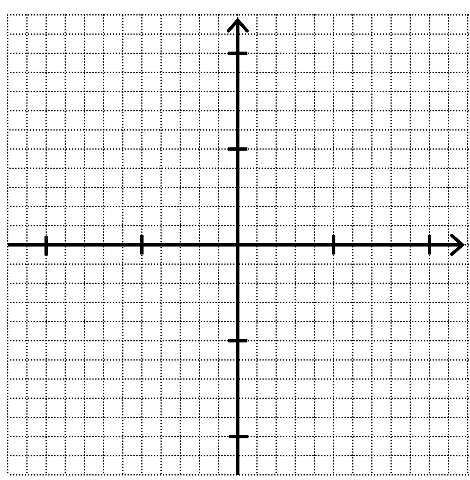
A rational function is a function with a variable in the denomin-ator. Often, the graph of a rational function has *asymptotes* where the domain of the function has a "gap".

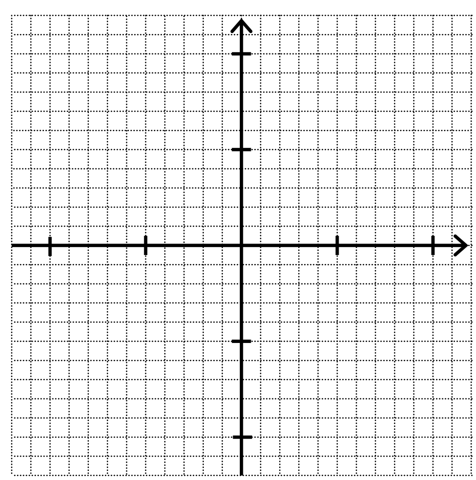
14) For each function make a table to generate points – especially choosing points *very* close to where the domain isn't defined. Then graph the function as accurately as possible. (All asymptotes should be shown as dotted lines.)

a) $f(x) = \dfrac{2}{x-3}$

b) $f(x) = \dfrac{4}{x^2-4}$







Problem Set #2

Polynomial Functions

Match the function with its graph, given below.

1) $f(x) = -x^3 + 3$

2) $f(x) = x^3 - 3x^2$

3) $f(x) = -x^3 + 4x$

4) $f(x) = \frac{1}{4}x^3 - 4x$

5) $f(x) = \frac{1}{4}x^4 + x^3$

6) $f(x) = \frac{1}{4}x^4 - x^3$

7) $f(x) = \frac{1}{4}x^4 - x^2$

8) $f(x) = \frac{1}{4}x^4 - 1$

9) With the above functions, state whether each is an even function, an odd function, or neither. (You should be able to tell by looking either at the function or at its graph.)

Even and Odd Functions

We shall now derive some basic rules for even and odd functions.

Fill in the blanks:

10) For any even function,
$f(-x) =$ ______

11) For any odd function,
$f(-x) =$ ______

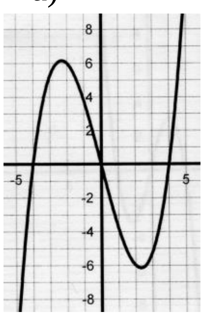
Rational Functions

Graph each function.
(All asymptotes should be shown as dotted lines.)

12) $f(x) = \frac{2x}{x+4}$

13) $f(x) = \frac{2x}{x^2-9}$

a)



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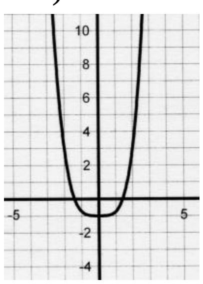

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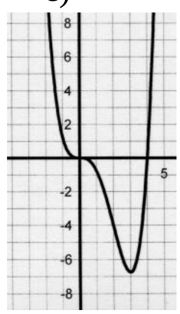

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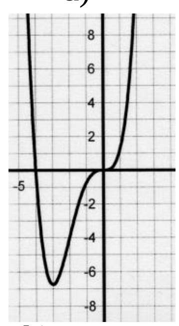

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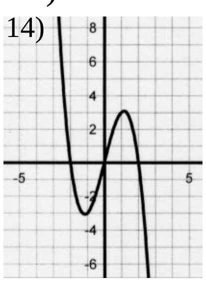

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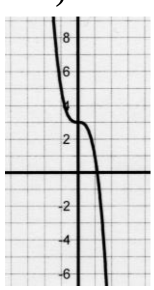

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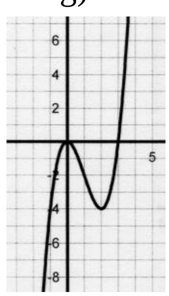

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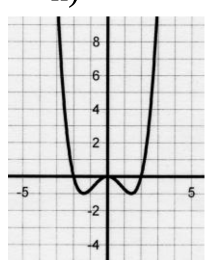
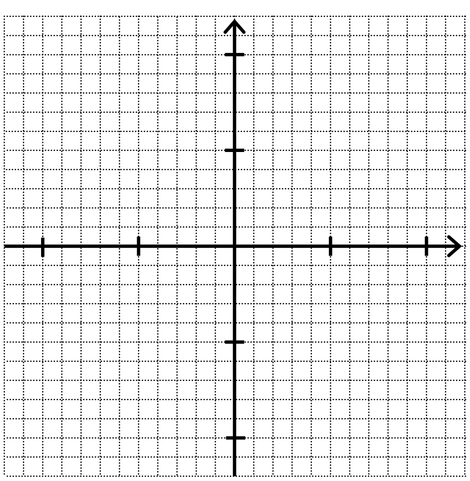
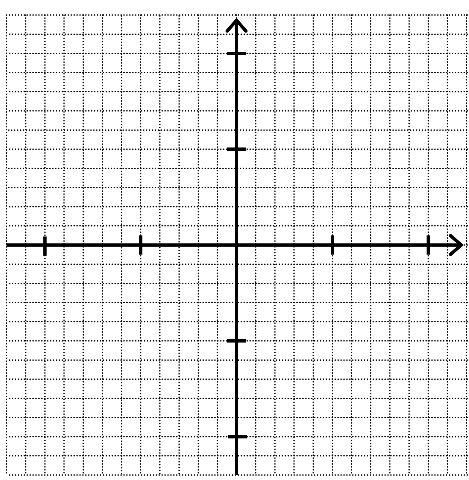
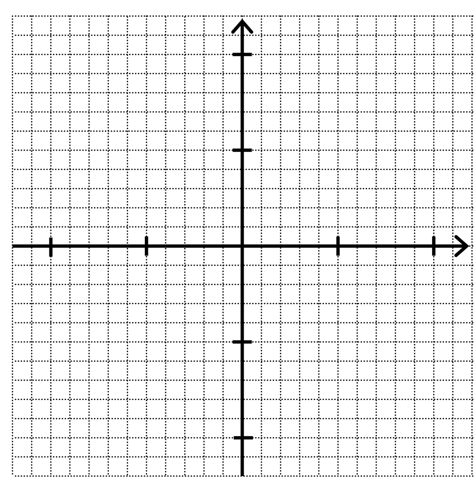








How can you determine whether a rational function is an even or odd function?

15) Graph each function. Besides the roots, only plot two or three key points. If you can't easily find the roots, then you will need to make a table in order to plot several points.

a) $f(x) = -2\,(x+5)^3(x+3)$

b) $f(x) = x^2(x-2)^2(x+3)$

c) $f(x) = x^2 + 3x - 4$

d) $f(x) = x^3 + 3x^2 - 4x$

e) $f(x) = x^3 + 3x^2 - 4$

16) Find f(x) given that it is a third degree polynomial equa-tion with $x = -2$ as one root, $x = 3$ as a double root, and the coefficient of the x^3 term is –5.

Inverse Functions

Loosely speaking, two functions are inverses of each other if they are opposite procedures. Subtraction is the inverse operation of addition. Division is the inverse of multiplication. Cuberooting is the inverse of cubing, and logarithms are the inverse of exponentiation.

But what is the inverse of a function like $f(x) = \frac{1}{3}x + 2$?

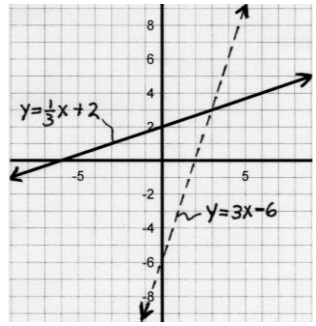
Here is one way to look at it. The opposite of first dividing x by 3 (or multiplying by ⅓), and then adding 2, is to first subtract 2 from x, and then to multiply by 3. We can then say that the inverse of f(x) is:

$$f^{-1}(x) = 3(x-2)$$

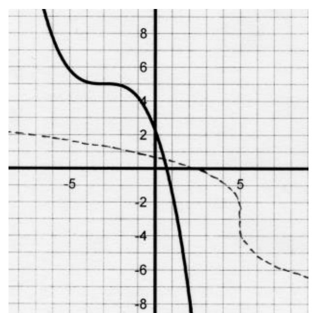
Which can be simplified to:

$$f^{-1}(x) = 3x - 6.$$

The graphs of these two inverse functions are below.



We can follow the same process for more complicated functions and still get the inverse function. Here are the graphs of two more inverse functions:



17) Look at the above graphs. What can be said about the graphs of a function, f(x), and its inverse, $f^{-1}(x)$?

18) Find $f^{-1}(x)$.

a) $f(x) = 7x$

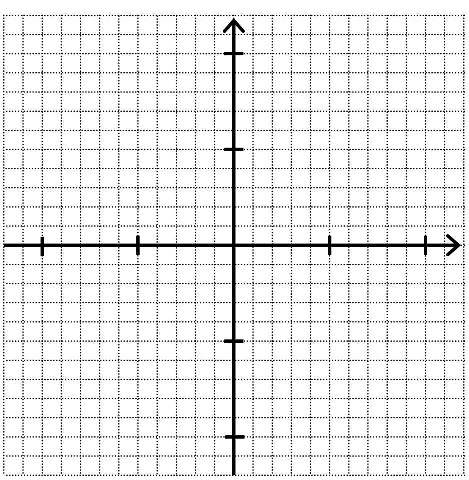
b) $f(x) = x - 8$

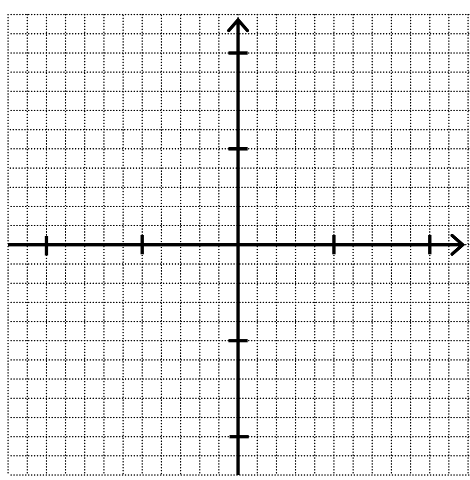
c) $f(x) = 2x + 7$

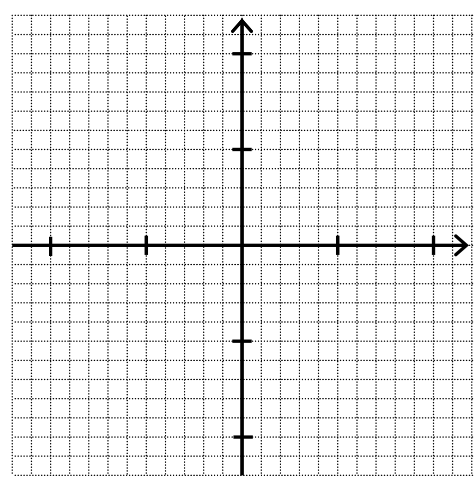
d) $f(x) = x^3 - 4$

e) $f(x) = x^2 + 5$

19) Why was the last one problematic?







Introduction to Calculus

Test

The Quadratic Formula is: $X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Each question is worth 4 points unless otherwise stated.

1) Find the formula for instantaneous speed given the distance formula $d(t) = 4 \cdot t^2$.

2) Find the average speed between $t = 3$ and $t = 8$ given the distance formula $d(t) = 4 \cdot t^2$.

3) Find the derivative, $f'(x)$, of…

a) $f(x) = x^6$

b) $f(x) = 2x$

c) $f(x) = 7$

d) $f(x) = 2x^5 + 2x^3 + x$

4) Find the anti-derivative, $F(x)$, of…

a) $f(x) = x^9$

b) $f(x) = 2$

c) $f(x) = 3x$

d) $f(x) = 2x^4 - x^3 + 7$

5) Find the area under the curve given… **(8 points)**

a) $f(x) = 3x^2$, $\int_2^5 f(x)\,dx$

6) Given $f(x) = x^2 - 3x - 4$

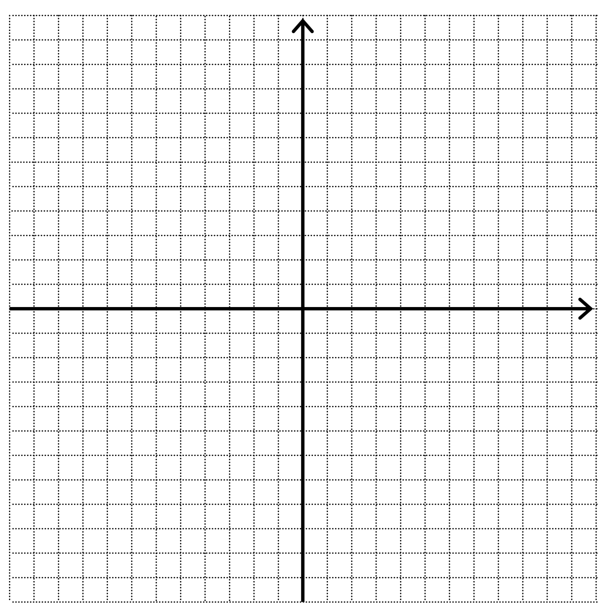
a) Determine $f'(x)$.

b) Fill in the below table of values. **(8 points)**

x	f(x)	f '(x)
5		
4		
3		
2		
1		
0		
–1		
–2		

c) Give the coordinates of the points where $f'(x) = 0$.

d) Make a very accurate graph of $f(x)$. **(8 points)**



7) **(6 points)**
Use the definition of the derivative to find the derivative of:
$f(x) = 4x^2 - 5$

Short Answer (6 points each)

8) Why was finding a method for calculating instantaneous speed so challenging?

9) For a given function f(x), what does it mean to say
$$\int_2^5 f(x)\,dx = -4\tfrac{1}{2}$$

10) For a given function f(x), what does it mean to say $f'(2) = -5$?