Logarithms – Part II- ANSWERS

Problem Set #1

1) $\frac{3}{2} = 1\frac{1}{2}$
2) 1
3) $\frac{8}{27}$
4) $\frac{27}{8} = 3\frac{3}{8}$
5) 160,000
6) $\frac{1}{160,000}$
7) 20
8) $\frac{1}{20}$
9) 1,000
10) 100
11) $\frac{1}{100}$
12) $\frac{1}{10,000}$
13) $\frac{1}{8}$
14) 32,768
15) $\frac{1}{256}$
16) $\frac{1}{32}$
17) $\frac{1}{16}$
18) 2
19) 3
20) 4
21) $\frac{1}{2}$
22) 2
23) $\frac{1}{2}$
24) 3
25) $\frac{1}{3}$
26) 2
27) 512
28) $\frac{1}{2}$
29) $\frac{1}{512}$
30) 2
31) 4
32) -1
33) -2
34) -1
35) 2
36) 10
37) 3
38) -3
39) $\frac{1}{3}$
40) $-\frac{1}{3}$
41) Undefined.
42) 2
43) -2
44) $\frac{1}{2}$
45) $-\frac{1}{2}$
46) $\frac{3}{2} = 1\frac{1}{2}$
47) No real solution.
48)
   a) $4^3 = 64$
   b) $10^{-1} = 0.1$
   c) $16^{-\frac{1}{2}} = \frac{1}{4}$

Logarithms – Part II- ANSWERS

49)

a) $\log_6 36 = 2$

b) $\log_6(\frac{1}{36}) = -2$

c) $\log_{16} 8 = \frac{3}{4}$

Problem Set #2

1) 81
2) 3
3) $\frac{1}{81}$
4) $\frac{1}{3}$
5) 200
6) $\frac{1}{200}$
7) 40,000
8) $\frac{1}{40000}$
9) 1000
10) 2
11) 4
12) -1
13) 0
14) $\frac{1}{2}$
15) -2
16) No real solution.
17) $-\frac{1}{2}$
18) $\frac{1}{2}$
19)

a) $10^5 = 100,000$

b) $4^{-3} = \frac{1}{64}$

c) $3^5 = 4x$

20)

a) $\log_7 343 = 3$

b) $\log_8(\frac{1}{512}) = -3$

c) $\log_9 285 = 4x + 7$

21) 3
22) $\frac{1}{2}$
23) $\frac{3}{2} = 1\frac{1}{2}$
24) $-\frac{1}{2}$
25) -1
26) $-\frac{3}{2} = -1\frac{1}{2}$
27) 3
28) -3
29) -6
30) $-\frac{1}{2}$
31) -2
32) 1296
33) 6
34) $\frac{1}{1296}$
35) $\frac{1}{6}$
36) $\frac{4}{3} = 1\frac{1}{3}$
37) $\frac{2}{3}$
38) Undefined.
39) 0
40) -1
41) $\frac{1}{4}$
42) $-\frac{8}{3} = -2\frac{2}{3}$
43) No real solution.

Logarithms – Part II- ANSWERS

44) $-\frac{3}{2} = -1\frac{1}{2}$
45) $\frac{4}{3} = 1\frac{1}{3}$
46) $-\frac{3}{4}$
47) $x = 4$
48) $x = \pm 2$ (Only positive 2 if written in log form).
49) $x = -3$
50) $x = 2$
51) $x = 2$
52) $x = \frac{1}{16}$
53) $x = 27$
54) $x = \frac{3}{8}$
55) $x = \frac{5}{2} = 2\frac{1}{2}$

Problem Set #3

1)
   a) 4
   b) 6
   c) 10
2)
   a) 3
   b) 5
   c) 8
3) 5
4) $\log_b M + \log_b N$
5)
   a) 5
   b) 3
   c) 2
6)
   a) 7
   b) 5
   c) 2
7) 4
8) $\log_b M - \log_b N$
9)
   a) 3
   b) 9
10)
   a) 3
   b) 15
11) 14
12) $k \log_b N$
13)
   a) 3
   b) -3
14)
   a) 5
   b) -5
15) $\log_b(\frac{1}{N}) = -\log_b N$
16)
   a) 4
   b) $\frac{1}{4}$
17)
   a) 2
   b) $\frac{1}{2}$
18) $\log_a b = \frac{1}{\log_b a}$
19) 7
20) 6
21) k
22) 625
23) 1000
24) N
25) For rewriting a logarithm problem in a different base. It can be used to solve problems like $4^x = 7$ or $\log_4 7 = x$.

Logarithms – Part II- ANSWERS

Problem Set #4

1)
   a) 243
   b) 81
   c) 27
   d) 9
   e) 3
   f) 1
   g) $\frac{1}{3}$
   h) $\frac{1}{9}$
   i) $\frac{1}{27}$
   j) $\frac{1}{81}$
   k) $\frac{1}{243}$
   l) 2
   m) 3
   n) 2
   o) -2
   p) -4
   q) -2
   r) $-\frac{1}{2}$
   s) No real solution.
   t) -1
   u) $\frac{5}{3} = 1\frac{2}{3}$
   v) $-\frac{5}{3} = -1\frac{2}{3}$

2) Answers may vary.

3)
   a) $\log_2 16 + \log_2 32 =$
   $4 + 5 = 9$
   b) $\log_4 16384 - \log_4 256 = 3$
   c) $4\log_5 125 = 12$
   d) $\frac{1}{\log_5 125} = \frac{1}{3}$
   e) $-\log_3 27 = -3$
   f) 8
   g) 64

4)
   a) $\frac{\log_3 81}{\log_3 27} = \frac{4}{3} = 1\frac{1}{3}$
   b) $\frac{\log_2 4}{\log_2 8} = \frac{2}{3}$
   c) $\frac{\log_2 \frac{1}{8}}{\log_2 16} = \frac{-3}{4} = -\frac{3}{4}$

5)
   a) ≈ 3.91
   b) ≈ 4.11
   c) ≈ 0.631
   d) ≈ -0.834
   e) ≈ 312.86
   f) ≈ 0.018

Problem Set #5

1)
   a) ≈ 3.95
   b) ≈ 5.94
   c) ≈ -2.25
   d) ≈ 563
   e) ≈ 12,600,000
   f) ≈ 0.00055

2)
   a) $4 + 2\log_2 x$
   b) $3 + \log_5 x - \log_5 y$
   c) $4 + \log_5 x + \log_5 y - 6\log_5 z$
   d) $-1 + \log x - 3\log y$

3)
   a) $\log_3(xa)$
   g) $\log_7(\frac{d}{8})$
   b) $\log_2 (64x^5)$
   c) $\log_3\left(\frac{x}{y^2 z^5}\right)$

Logarithms – Part II- ANSWERS

4)

a) $x \approx 1.81$
b) $x \approx 0.651$
c) $x = -\frac{1}{3}$
d) $x = \log_z w = \frac{\log_b w}{\log_b z}$

$b, w, z > 0$

e) $x = 243$
f) $x \approx 3.36$
g) $x \approx 2.1$
h) $x \approx 2.71$
i) $x = -\frac{7}{2} = -3\frac{1}{2}$
j) $x = \frac{1}{125}$
k) $x \approx -0.051$

Problem Set #6

1)

a) 50
b) $\frac{1}{50}$
c) 2500
d) $\frac{1}{2500}$
e) 4
f) $\frac{1}{16}$
g) -1
h) 2
i) 0
j) $\frac{1}{4}$
k) -2
l) No real solution.
m) $\frac{5}{3} = 1\frac{2}{3}$
n) $\frac{1}{3}$
o) Undefined.
p) 0
q) $-\frac{7}{2} = -3\frac{1}{2}$
r) 0
s) $\frac{4}{3} = 1\frac{1}{3}$
t) $-\frac{3}{4}$

2)

a) $\log_3 81 + \log_3 27 = 7$
b) $\log_7 16807 - \log_7 343 = 2$
c) $8 \log_6 7776 = 40$
d) Answers may vary:
$\frac{1}{\log_8 64} = \frac{1}{2}$ or
$\frac{\log_2 8}{\log_2 64} = \frac{1}{2}$
e) $-\log_{10} 1000000 = -6$
f) 30
g) 7

3)

a) $-3 + \log_2 x - \log_2 y$
b) $-4 + 2\log_3 c - \log_3 z$
c) $2 + 5\log y$
d) $-2 + 2\log_4 x + \log_4 z - \log_4 y$

4)

a) $\log_5(4a)$
b) $\log_a\left(\frac{5}{x}\right)$
c) $\log(x^3 y)$
d) $\log_2\left(\frac{xy^4}{2z}\right)$

Logarithms – Part II- ANSWERS

5)

a) ≈ 2.83
b) ≈ 4.9
c) ≈ -1.21
d) ≈ 6310
e) ≈ 692
f) ≈ 0.00252

6)

a) $\frac{4}{3} = 1\frac{1}{3}$

b) $\frac{3}{5}$

c) $-\frac{3}{2} = -1\frac{1}{2}$

7)

a) ≈ 3.153
b) ≈ 2.75
c) ≈ 0.774
d) ≈ -1.096
e) ≈ 28.443
f) ≈ 0.01

8)

a) $x \approx 2.86$
b) $x \approx -2.031$
c) $x = \log_a c = \frac{\log_b c}{\log_b a}$

$a, b, c > 0$

d) $x = \pm \sqrt[y]{c}$ if y is even.

<u>or</u> $x = \sqrt[y]{c}$ if y is odd.

c > 0.

e) $x = 216$
f) $x \approx 1.774$
g) $x \approx 17.321$
h) $x \approx 9.183$
i) $x = 10$

Problem Set #7

1) ≈ 5350
2) ≈ 7060
3) $\approx 562{,}000$
4) ≈ 17
5) $\approx 87{,}600$
6) $\approx 1.9 \cdot 10^{14}$
7) $\approx 33{,}300{,}000$
8) ≈ 42.8
9) ≈ 5.66
10) ≈ 2.42

Trigonometry – Part I - ANSWERS

Problem Set #1

1)
   a) ≈ 7.7 cm
   b) ≈ 11.8 cm
   c) ≈ 10.9 cm
   d) ≈ 10.9 cm
   e) c & d. 130° + 230° = 360°

2)
   a) ≈ 7.1 cm
   b) $5\sqrt{2}$ cm ≈ 7.07 cm
   c) Yes!

3)
   a) 2.736
   b) 1.71
   c) 3.762
   d) Not possible…yet.

4)
   a) If the arc is 100°, then the length of the resulting chord would be approximately 0.776 times as long as the circle's diameter.
   b) If the arc is 28°, then the length of the resulting chord would be approximately 0.242 times as long as the circle's diameter.
   c) If the arc is 97.2°, then the length of the resulting chord would be approximately 0.75 times as long as the circle's diameter.

5)
   a) 0.8192
   b) 0.342
   c) ≈ 0.643
   d) 0.64275
   e) 0.48475
   f) ≈ 0.4848

6)

Arc, θ	Crd(θ)
40°	0.342
60°	0.5
80°	0.643
90°	0.707
100°	0.766
110°	0.8192
120°	0.866
180°	1
240°	0.866
260°	0.766
360°	0

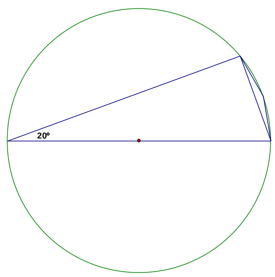
Problem Set #2

Arcs	Chords	Decimal
1) ½°	0, 31,25	0.5236
2) 1°	1, 2, 50	1.047
3) 1½°	1, 34, 15	1.571
4) 2°	2, 5, 40	2.094
5) 2½°	2, 37, 4	2.618
6) 3°	3, 8, 28	3.141
7) 3½°	3, 39, 52	3.664
8) 4°	4, 11,16	4.188
9) 4½°	4, 42, 40	4.711
10) 5°	5, 14, 4	5.234
11) 5½°	5, 45, 27	5.758
12) 6°	6, 16, 49	6.280
13) 6½°	6, 48, 11	6.803
14) 7°	7, 19, 33	7.326
15) 7½°	7, 50,54	7.8483

Trigonometry – Part I - ANSWERS

Problem Set #3

1)
   a) ≈ 9.2 cm
   b) ≈ 10.9 cm
   c) ≈ 4.1 cm
   d) ≈ 4.1 cm
   e) c & d. A 20° inscribed angle subtends a 40° arc. A 160° inscribed angle subtends a 320° arc. 320° + 40° = 360° .



   f) Problem 1c from Problem Set #1 is equal to problem 1b from this set. Recall *Inscribed Angle Theorem.*

2)
   a) ≈ 16.91 cm
   b) ≈ 4.70 cm
   c) ≈ 5.64 cm
   d) Not possible…yet!

3) Answers may vary. See example in problem.

4)
   a) sin(70°) ≈ 0.940
   b) sin(10°) ≈ 0.174
   c) sin(50°) ≈ 0.766
   d) sin(50°) ≈ 0.766
   e) sin(130°) ≈ 0.766

5) $\sin(\propto) = \frac{C}{D}$

6) $\sin(\propto) = \sin(\beta)$

Problem Set #4

1)
   a) sin(34°) ≈ 0.559
   b) sin(102°) ≈ 0.978
   c) sin(78°) ≈ 0.978
   d) sin(50°) ≈ 0.766

2) b) sin(40°) ≈ 0.643

3)

∝	sin(∝)	cos(∝)
0°	0	1
10°	0.174	0.985
20°	0.342	0.939
30°	0.5	0.866
40°	0.643	0.766
45°	0.7071	0.7071
50°	0.766	0.643
60°	0.866	0.5
70°	0.939	0.342
80°	0.985	0.174
90°	1	0
100°	0.985	
110°	0.939	
120°	0.866	
130°	0.766	
135°	0.7071	
140°	0.643	
150°	0.5	
160°	0.342	
170°	0.174	
180°	0	

4)
   a) $x = 3.42$
   b) $x = 2.088$
   c) $x = 3.94$
   d) $x \approx 4.061$
   e) $x = 0.28044$
   f) $x = 7.482$
   g) $x = 9.192$
   h) $x = 4.7492$
   i) $x = 14.095$
   j) $x = 6.894$

Trigonometry – Part I - ANSWERS

Problem Set #5

1)
   a) In a right triangle with a 41° angle, the adjacent leg is approximately 0.755 times as long as the hypotenuse.
   b) In a right triangle with an 83° angle, the adjacent leg is approximately 0.122 times as long as the hypotenuse.
   c) In a right triangle with an 18° angle, the adjacent leg is approximately 0.951 times as long as the hypotenuse.

2)
   a) $\cos(34°) \approx 0.829$
   b) $\cos(56°) \approx 0.559$
   c) $\cos(36°) \approx 0.809$
   d) $\cos(36°) \approx 0.809$

3) See the answer to Problem 3 in Problem Set #4.

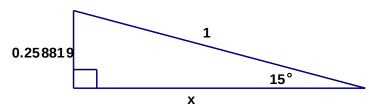
4)
   a) $x = 3.9438$
   b) $x = 1.3746$
   c) $x = 55.298$
   d) $x = 55.298$
   e) $x = 12.207$
   f) $x \approx 22.989$

5) $\sin(\propto) = \cos(\beta)$

6) $\sin^2(\propto) + \cos^2(\propto) = 1$

Problem Set #6

1) 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°
2) Answers may vary.
3) $\sin(15°) \approx 0.258819$
4) Use the Pythagorean Theorem to find cos(15°), cos(7.5°) etc. Example:



Because $\sin(15°) \approx 0.258819$, we can find
$x = \cos(15°) \approx 0.965926$

   a) $\sin(7.5°) \approx 0.130526$
   b) $\cos(7.5°) \approx 0.991445$
      $\sin(3.75°) \approx 0.065403$
   c) $\cos(3.75°) \approx 0.997859$
      $\sin(1.875°) \approx 0.032718$
   d) $\cos(1.875°) \approx 0.999465$
      $\sin(0.9375°) \approx 0.016355$
   Note that this value is slightly off because we rounded our answers during each step of the process. The correct answer (rounded to 6 significant digits) is $\sin(0.9375°) \approx 0.0163617$

5)
   a) $\approx 1.931852:1$
   b) $\approx 1.982892:1$
   c) $\approx 1.995719:1$
   d) $\approx 1.998930:1$
   e) $\approx 1.999694:1$

6) As angle α approaches zero, $\sin(2\alpha)$: $\sin(\alpha)$ approaches 2:1.

7)

a)

i) The sine of a times angle α is approx. equal to a times the sine of angle α.

ii) The ratio of two angles is approximately equal to the ratio of the sines of these two angles.

b) For very small angles.

8) Given the angles are small, we use the approximation $\alpha{:}\beta \approx \sin(\alpha){:}\sin(\beta)$

a) $\sin(2°) \approx \frac{\sin(8°)}{4} \approx 0.034793$

b) $\sin(0.7°) \approx 0.0122161$

c) $\sin(3.3°) \approx 0.0575922$

d) $\sin(2.4°) \approx 0.0418603$

e) $\sin(3.7°) \approx 0.0645496$

f) $\sin(1°) \approx 0.0174525$

This value is very accurate, but the correct answer (rounded to 6 significant digits) is $\sin(1°) \approx 0.0174524$

9) Using ratios:

a) $\sin(0.5°) \approx 0.00872626$

b) $\sin(0.25°) \approx 0.00436313$

Problem Set #7

1)

a) $\sin(6.5°) \approx 0.113203$

b) $\cos(6.5°) \approx 0.993572$

c) $\sin(4°) \approx 0.0697572$

This value is very accurate, but the correct answer (rounded to 6 significant digits) is $\sin(4°) \approx 0.0697565$

d) $\cos(4°) \approx 0.997564$

2) $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$
<u>or</u> $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$
<u>or</u> $\cos(2\alpha) = 2\cos^2(\alpha) - 1$

3) All previously unknown values in this table have been found using the *Sum Formula*, $\cos(2\propto)$ formula and the *Pythagorean Theorem*. Note that the answers given here are the correct values rounded to 6 significant figures. Your answers may vary slightly.

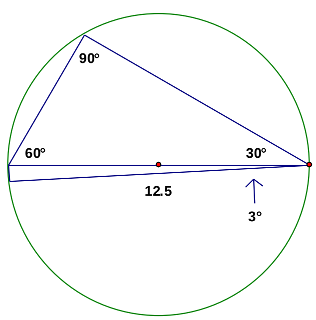
∝	sin(∝)	cos(∝)
0°	0	1
¼°	0.00436331	0.999991
½°	0.00872654	0.999962
1°	0.0174524	0.999848
1½°	0.0261769	0.999657
2°	0.0348995	0.999391
2½°	0.0436194	0.999048
3°	0.0523360	0.998630
3½°	0.0610485	0.998135
3¾°	0.0654031	0.997859
4°	0.0697565	0.997564
4½°	0.0784591	0.996917
5°	0.0871557	0.996195
5½°	0.0958458	0.995396
6°	0.104529	0.994522
∝	sin(∝)	cos(∝)
6½°	0.113203	0.993572
7°	0.121869	0.992546
7½°	0.130526	0.991445
8°	0.139173	0.990268

4) $\frac{4.188}{120} = 0.0349 = \sin(2°)$. If 4° is the arc, then 2° would be the inscribed angle that intercepts this arc.

5)
 a) $\sin(87°) = \cos(3°) \approx 0.998630$
 b) $\cos(85°) = \sin(5°) \approx 0.0871557$

 c) $\cos(14°) = 2\cos^2(7°) - 1 \approx 0.970296$
 d) $\sin(172°) = \sin(8°) \approx 0.139173$
 e) $\sin(92°) = \sin(88°) = \cos(2°) \approx 0.999391$

6) Recall that the sine of an inscribed angle is equal to the ratio of the chord this angle subtends to the diameter of the circle. First find the diameter of each circle.
 a) $x \approx 10.8$
 Use the Theorem of Thales and sin(87°) to find the diameter.



 b) $x \approx 7.35$
 c) $x \approx 12.3$
 d) $x \approx 3.13$

7) See the explanation from problem 6. If d is the diameter of the circle, then

$\sin A = \frac{a}{d} \rightarrow d = \frac{a}{\sin A}$

$\sin B = \frac{b}{d} \rightarrow d = \frac{b}{\sin B}$

thus $\frac{a}{\sin A} = \frac{b}{\sin B}$ or

$\frac{\sin A}{a} = \frac{\sin B}{b}$ which is also

a : b = sinA : sinB

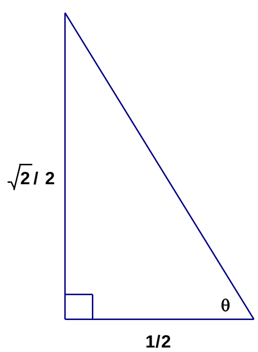
8)
 a) $x \approx 49.2$
 b) $x \approx 6.40$

9)
 a) $\sin(\theta) = \frac{2.07}{3.04} \approx 0.6809$ thus $\theta \approx 43°$
 b) $\theta \approx 13.5°$
 c) $\theta \approx 59°$
 d) $\theta \approx 63°$

10)
 a) $x \approx 3.71$
 b) $x \approx 12.34$
 c) $x \approx 4.19$
 d) $x \approx 6.64$

11) 180/96 = 1.875. Find the length of the side opposite the 1.875 angle in a right-triangle. $\sin(1.875) = \frac{x}{0.5}$ thus $x \approx 0.016359$. This is half the length of the side of our regular 96-gon. Multiply by 192 and we get our answer: ≈ 3.14103 which was how Archimedes found his approximation for π.

12) Split the octahedron into two pyramids. The angle of one of the faces of a pyramid to the base can be found by creating a right triangle using the height of the pyramid and half length of an edge of the pyramid. Assume the edges of the octahedron are of length one. Thus we can use the triangle:



Finding the hypotenuse and using sine gives us $\theta \approx 54.74°$. Double this to get our answer $\approx 109.5°$

Problem Set #8

1) See previous problem sets. Answers may vary.

2)
   a) $x \approx 39.7$
   b) $x \approx 19.3$
   c) $x \approx 184$
   d) $x \approx 1.95$
   e) $x \approx 3.47$
   f) $\theta \approx 53°$
   g) $D \approx 11.8$
   h) $x \approx 9.58$
   i) $x \approx 11.2$
   j) $x \approx 2.92$
   k) $x \approx 4.54$
   l) $x \approx 1.30$
      $y \approx 1.47$
   m) $D \approx 15.0$
   n) $\theta \approx 21.5°$

3) ≈ 5.56 ft.

4) ≈ 495 ft

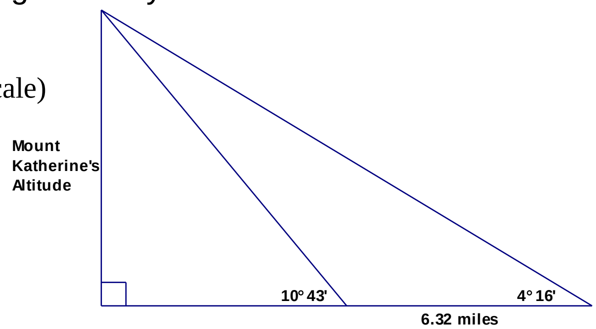
5)
   a) Create 4 right triangles using the diagonals of the rhombuses to get the larger angle $\approx 109.5°$ and the smaller angle $\approx 70.5°$
   b) $\sqrt{2} : 1$

6) $\frac{1}{4}$

Trigonometry – Part I - ANSWERS

7)

a) (Not to scale)



Mount Katherine's Elevation is approximately 5833 feet

b) ≈ 21,715 feet or ≈ 4.11 miles

Exponential Growth

Problem Set #1

1)
- a) $6739.24
- b) $9056.81
- c) $16216.99
- d) $50,313.28

2) $5000

3) ≈ 2.1%

4) 18 years

5) ≈ 40.3%

6)
- a) Because the interest is compounded annually.
- b) ≈ 3.71%

7) 37,126

8) ≈ 7.18%

9)
- a) ≈ 17.7 years
- b) ≈ 28 years
- c) ≈ 12.4 years
- d) ≈ 12.4 years

10) APR ≈ 10.40895137%

After 1 year	$485.80
2 years	$536.37
3 years	$592.20
4 years	$653.84
5 years	$721.90
6 years	$797.04
7 years	$880.00

11) Goes quickly to infinity.
12) Goes slowly to infinity.
13) Always equals 1.
14) Goes slowly to 0.
15) Goes quickly to 0.
16) Same as problem # 11.
17) Same as problem #14.

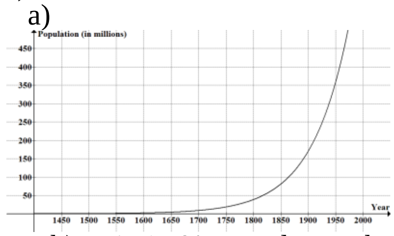
Exponential Growth ANSWERS

Problem Set #2

1) Impossible to say.

2)

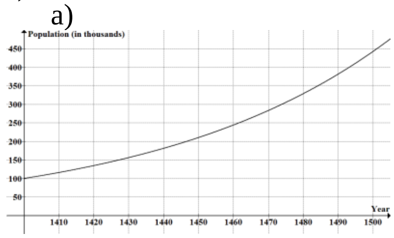
a)



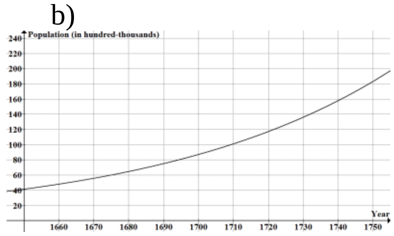
b) It is 1.5% annual growth everywhere!

3)

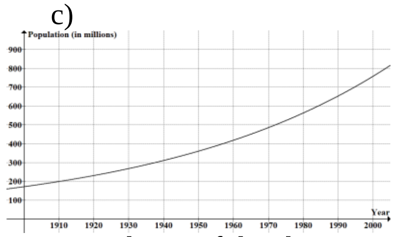
a)



b)



c)



Note: Each one of the above graphs is simply a zoomed in section of the graph in #2. It may seem surprising that they all look the same, since these sections look quite different in #2, but each graph is over a 100-year interval, and the growth rate is the same throughout.

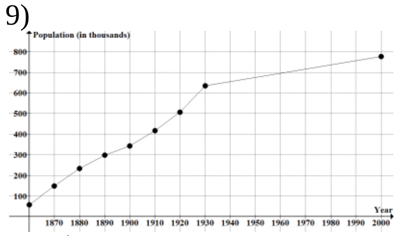
3 – 5) We can't really guess what the growth rate is by looking at a graph. Any rate can be made to look like any other rate by "zooming in" or "zooming out".

6) ≈ 131 years

7) Decade: ≈ 16.1%
Century: ≈ 343.2%

8) 1.96%

9)



a) ≈ 10.1%
b) ≈ 2.26%
c) ≈ 4.59%
d) ≈ 0.291%

Problem Set #3

1) $1628.00

2)

a) ≈ 61.6 years
b) ≈ 61.6 years
c) ≈ 61.17 years
d) ≈ 61.08 years

3)

a) $788.1299348
b) $788.9648491
c) $789.1532181
d) $789.2446826
e) $789.2476654
f) $789.247793
g) $789.2477961
h) $789.2477961

4) $2046.24

5)

a) 1000 years.
b) $2^{40} \approx 1.1 \cdot 10^{12}$
c) Answers may vary.

Exponential Growth ANSWERS

Problem Set #4

1) \$8549.82
2) \$2.72
3) 2.718281828
4)
   a) $Q = \frac{n}{r}$ so $P = P_0(1 + \frac{1}{Q})^{Qrt}$
   b) $P = P_o e^{rt}$
5)
   a) \$8549.82
   b) \$1207.69
   c) \$61,532.26

Problem Set #5

1) $\frac{1}{8}$
2) 4
3) 2
4) $\frac{1}{4096}$
5) 32
6) $\frac{1}{256}$
7) $\frac{4}{25}$
8) $\frac{16}{9} = 1\frac{7}{9}$
9) $\frac{16}{49}$
10) 3
11) $\frac{1}{2}$
12) -2
13) No real solution.
14) -4
15) $-\frac{1}{2}$
16) $\frac{3}{2} = 1\frac{1}{2}$
17) $-\frac{4}{3}$
18) $-\frac{1}{2}$
19) 0
20) Undefined.
21) -1
22) 4
23) 3 + 4 = 7
24) 10 – 6 = 4
25) 7·2 = 14
26) 6
27) 17
28) $\log_9(\frac{1}{27}) = -\frac{3}{2} = -1\frac{1}{2}$
29) $7^{-4} = \frac{1}{2401}$
30)
   a) $x = 3$
   b) $x = 81$
   c) $x = 5$
   d) $x = -1$
   e) $x = \frac{193}{128} = 1\frac{65}{128}$
   f) $x = -4$
   g) $x = \frac{1}{6}$
31) $-3 + 3\log_4 x - \log_4 y$
32) Answers may vary.
   a) ≈ 3.867
   b) ≈ 0.315
   c) ≈ 24.251
33)
   a) ≈ 4.587711
   b) ≈ -4.2660007
   c) ≈ 69,200
   d) ≈ 0.00002515
   e) ≈ 2,245,000
   f) ≈ 1.37
34) \$650.00
35) ≈ 3.74%

36) ≈ 6.29%
37)
  a) ≈ 18.6 years
  b) ≈ 18.6 years
  c) ≈ 18.33 years
  d) ≈ 18.27 years
  e) ≈ 18.24 years
38)
  a) $5361.90
  b) $5376.99
  c) $5380.41
  d) $5382.08
  e) $5382.13
39) $1602.68
40)
  a) ≈ 61 years
  b) ≈ 24.8 years
41)
  a) $e \approx 2.718281828$
  b) $e \approx 2.718281828$

Problem Set #6

1) $\frac{1}{1296}$
2) 6
3) $\frac{1}{6}$
4) $\frac{1}{125}$
5) $\frac{4}{3} = 1\frac{1}{3}$
6) $\frac{5}{6}$
7) $\frac{81}{16} = 5\frac{1}{16}$
8) 2
9) $\frac{1}{2}$
10) No real solution.
11) 0
12) 3
13) -1
14) $\frac{1}{3}$
15) $-\frac{4}{3} = -1\frac{1}{3}$
16) $\frac{4}{3} = 1\frac{1}{3}$
17) 3
18) -3
19) 7 + 5 = 12
20) 6 – 4 = 2
21) 8·4 = 32
22) 4
23) 3
24) $\log_5\left(\frac{1}{125}\right) = -3$
25) $4^{\frac{5}{2}} = 32$
26)
  a) $x = 4$
  b) $x = -2$
  c) $x = 16$
  d) $x = 5$
  e) $x = -\frac{19}{12} = -1\frac{7}{12}$
  f) $x = 30$
  g) $x = 10$
27) $\log_6 \frac{x^5}{yz}$
28)
  a) ≈ 4.153
  b) ≈ -0.748
  c) ≈ 0.006
29)
  a) ≈ 3.922
  b) ≈ -2.1135
  c) ≈ 2,000,000
  d) ≈ 0.0007945
  e) ≈ 11,600
  f) ≈ 1,550,000
30) ≈ 23 years

Exponential Growth ANSWERS

31) ≈ 110.7%

32)
- a) 1681
- b) 1,658,712

33)
- a) ≈ 23.45 years
- b) ≈ 8.88 years
- c) ≈ 69 years

34) Decade: ≈ 12.7%
Century: ≈ 230%

35) $3762.75

36) ≈ 2.2%

37)
- a) ≈ 23.4%
- b) ≈ 13.42 lbs
- c) ≈ 31.12 lbs
- d) ≈ 109.92 lbs
- e) ≈ 1,371 lbs
- f) ≈ 213,278 lbs
- g) ≈ 33,179,428 lbs
- h) ≈ 5,161,692,761 lbs
- i) ≈ $8 \cdot 10^{11}$ lbs
- j) ≈ $2.42 \cdot 10^{17}$ lbs
- k) ≈ $7.32 \cdot 10^{22}$ lbs
- l) ≈ $2.21 \cdot 10^{28}$ lbs

Sequences & Series ANSWERS

Problem Set #1

1) A = $\frac{x+y}{2}$

2) If x is the smaller number and y is the larger number:

➔ $\frac{G - x}{x} = \frac{y - G}{G}$

➔ $\frac{G}{x} - 1 = \frac{y}{G} - 1$

➔ G = $\sqrt{xy}$

3) $\frac{H - x}{x} = \frac{y - H}{y}$

$\frac{H}{x} - 1 = 1 - \frac{H}{y}$ thus

H = $\frac{2xy}{x+y}$

4) A = 25
G = 20
H = 16

5) A = 25
G = 15
H = 9

6) A = 25
G = $3\sqrt{69} \approx 24.92$
H = 24.84

7) Let H be the harmonic mean of x and y. Then $\frac{H - y}{x - H} = \frac{y}{x}$.

Solving for H yields

H = $\frac{2xy}{x+y}$

8) $\frac{1}{H} = \frac{\frac{1}{x} + \frac{1}{y}}{2}$ thus H = $\frac{2xy}{x+y}$

9) Note that an increase by a factor of 40 (a.k.a. 40-fold increase) means that it becomes 40 times larger. This is a geometric mean problem. G = $\sqrt{40 \cdot 10}$ = 20. This means an average annual increase by a factor of 20.

10) \$25/hr. Arithmetic Mean.

11) 16 mph. Harmonic Mean.

12) $20(1 + r)^5 = 30$ yields an annual pay raise of $r \approx 8.45\%$. Thus Beth should be making \$21.69/hr in her first year. Note that the common mistake is to think that a 10% annual increase results in a 5-year increase of 50%. Because of the compounding effect, a 10% annual increase amounts to a 61.05% increase over the 5-year period.

13)
a) ≈ 14.0%
b) ≈ 2.66%

14)
a) \$15
b) \$3

15) ≈ 830.6 Hz

Problem Set #2

1)
   a) Geometric.
   b) 468.75, 585.9375
   c) 153.6, 192

2)
   a) Arithmetic.
   b) 420, 480
   c) 120, 180

3)
   a) Harmonic.
   b) 600 , 1200
   c) $171\frac{3}{7}$, 200

4)
   a) Arithmetic.
   b) 90, 105
   c) 15, 30

5)
   a) Geometric
   b) $106\frac{2}{3}$, $142\frac{2}{9}$
   c) 25.3125, 33.75

6)
   a) Harmonic.
   b) 180, ∞
   c) 30, 36

7) $x_1 = \frac{x_0+x_2}{2} \rightarrow x_2 = 2x_1 - x_0$ (1)

$x_2 = \frac{x_1+x_3}{2} \rightarrow x_3 = 2x_2 - x_1$ (2)

Substituting (1) into (2) gives us $x_3 = 3x_1 - 2x_0$. Continuing in this fashion gives us:

$x_n = nx_1 - (n-1)x_0$

$\therefore x_n = x_0 + (x_1 - x_0)n$

8) $x_1 = \sqrt{x_0 x_2} \rightarrow x_2 = \frac{x_1^2}{x_0}$ (1)

$x_2 = \sqrt{x_1 x_3} \rightarrow x_3 = \frac{x_2^2}{x_1}$ (2)

Substituting (1) into (2) gives us $x_3 = x_0\left(\frac{x_1}{x_0}\right)^3$.

Continuing in this fashion gives us:

$x_n = x_0\left(\frac{x_1}{x_0}\right)^n$

9) $x_1 = \frac{2x_0x_2}{x_0 + x_2} \rightarrow x_2 = \frac{x_0x_1}{2x_0 - x_1}$ (1)

$x_2 = \frac{2x_1x_3}{x_1 + x_3} \rightarrow x_3 = \frac{x_1x_2}{2x_1 - x_2}$ (2)

Substituting (1) into (2) gives us:

$x_3 = \frac{x_0x_1}{x_1 - 3(x_1 - x_0)}$. Continuing in this fashion gives us:

$x_n = \frac{x_0x_1}{x_1 - n(x_1 - x_0)}$

10) 155
11) ≈ 41.2
12) 52.8
13) -32
14) ≈ 0.139
15) -1.5
16) As the sequence goes to the right the terms slowly go toward infinity. As the sequence goes toward the left the terms slowly go toward “negative infinity”.
17) As the sequence goes to the right the terms go increasingly quickly toward infinity, but never pass through infinity. As the sequence goes toward the left the terms slowly approach zero.
18) As the sequence goes to the right the terms move quickly past infinity, become negative, and then slowly approach zero. As the sequence goes towards the left the terms slowly approach zero.

Problem Set #3

1) 15, 21, 28, 36
Recursive formula:
$x_n = 2x_{n-1} - x_{n-2} + 1$
General formula (starting at $x_1 = 1$):
$x_n = \frac{n(n+1)}{2}$

2) 25, 36, 49, 64
Recursive formula:
$x_n = 2x_{n-1} - x_{n-2} + 2$
General formula (starting at $x_1 = 1$):
$x_n = n^2$

3) 35, 51, 70, 92
Recursive formula:
$x_n = 2x_{n-1} - x_{n-2} + 3$
General formula (starting at $x_1 = 1$):
$x_n = \frac{1}{2}n(3n - 1)$

4) The first 8 hexagonal numbers are:
1, 6, 15, 28, 45, 66, 91, 120
Recursive formula:
$x_n = 2x_{n-1} - x_{n-2} + 4$
General formula (starting at x_1):
$x_n = n(2n - 1)$

5) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233

6) There are many possible formulas including:
$x_{n+1}^2 + x_n^2 = x_{2n+1}$
$-x_{n+2}^2 + x_n^2 = x_{2n+2}$
$x_{n+3}^2 + x_n^2 = 2 \cdot x_{2n+3}$
$-x_{n+4}^2 + x_n^2 = 3 \cdot x_{2n+4}$
$x_{n+5}^2 + x_n^2 = 5 \cdot x_{2n+5}$
$-x_{n+6}^2 + x_n^2 = 8 \cdot x_{2n+6}$

The above formulas can be combined into the more general formula:

$(-1)^{r+1} \cdot x_{n+r}^2 + x_n^2 = (x_r)(x_{2n+r})$

Other formulas include:

$x_n + x_{n+1} + x_{n+2} = 2x_{n+2}$

$x_n^2 + (-1)^n = (x_{n+1})(x_{n-1})$

$\sum_{i=1}^{n} x_n = x_{n+2} - 1$

7) $x_n = x_{n-1} + x_{n-2}$

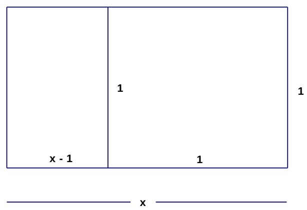
8)
a) 3, 6, 9, 15, 24, 39, 63, 102, 165, 267
b) 11, 18, 29, 47, 76, 123, 199, 322, 521, 843
c) 14, 26, 40, 66, 106, 172, 278, 450, 728, 1178

9) 1, 2, 1.5, 1.67, 1.6, 1.625, 1.61538, 1.6190476, 1.617647, 1.618181, 1.61797753, 1.61805555. This is approaching a common number. See the next problem set.

10) No matter what the first two terms in the sequence are, the ratio of consecutive terms always approach the same number – $\Phi \approx 1.61803$

Problem Set #4

1) Assume the height of the original rectangle is of length one. Then the length of the original rectangle is the *Golden Ratio* (labeled x in the figure below):



Set up the ratio $\frac{x}{1} = \frac{1}{x-1}$

➔ $x(x - 1) = 1$

➔ $x^2 - x - 1 = 0$

Using the quadratic formula we get $x = \Phi = \frac{1+\sqrt{5}}{2} \approx 1.618$

2) Problem Set #3 – Problems #9, 10. Answers may vary.
3) Goes to Φ.
4) Goes to Φ.
5) ≈ 2.618.
$\Phi^2 = \Phi + 1$
6) ≈ 0.618.
$\Phi^{-1} = \Phi - 1$
7) 1
$\Phi \approx 1.618$
$1 + \Phi \approx 2.618$
$1 + 2\Phi \approx 4.236$
$2 + 3\Phi \approx 6.854$
$3 + 5\Phi \approx 11.09$
$5 + 8\Phi \approx 17.944$
$8 + 13\Phi \approx 29.034$
8) $\Phi^0 = 1$
$\Phi^1 \approx 1.618$
$\Phi^2 \approx 2.618$
$\Phi^3 \approx 4.236$
$\Phi^4 \approx 6.854$
$\Phi^5 \approx 11.09$
$\Phi^6 \approx 17.944$
$\Phi^7 \approx 29.034$
9) Both of the sequences are the same. This is a unique sequence that follows both the Fibonacci additive property, and is also geometric.
10)
a) $\Phi^3 = \Phi \cdot \Phi^2$
Given that $\Phi^2 = \Phi + 1$, we then get:

➔ $\Phi^3 = \Phi(\Phi + 1)$

➔ $\Phi^3 = \Phi^2 + \Phi$

➔ $\Phi^3 = (\Phi + 1) + \Phi$

➔ $\Phi^3 = \Phi + 1 + \Phi$

➔ $\Phi^3 = 2\,\Phi + 1$

b) $\Phi^4 = \Phi^2 \cdot \Phi^2$

➔ $\Phi^4 = \Phi^3 \cdot \Phi$

➔ $\Phi^4 = (2\,\Phi + 1)\,\Phi$

➔ $\Phi^4 = 2\,\Phi^2 + \Phi$

➔ $\Phi^4 = 2\,(\Phi + 1) + \Phi$

➔ $\Phi^4 = 2\,\Phi + 2 + \Phi$

➔ $\Phi^4 = 3\Phi + 2$

c) $\Phi^5 = \Phi^4 \cdot \Phi$

➔ $\Phi^5 = (3\Phi + 2)\,\Phi$

➔ $\Phi^5 = 3\Phi^2 + 2\Phi$

➔ $\Phi^5 = 3(\Phi + 1) + 2\Phi$

➔ $\Phi^5 = 3\Phi + 3 + 2\Phi$

➔ $\Phi^5 = 5\Phi + 3$

11) $\sum_{i=1}^{6} 5i = 105$
12) $\sum_{i=0}^{5}(40 + 5i) = 315$
13) $\sum_{i=0}^{5}(23 + 5i) = 213$
14) $\sum_{i=0}^{10}(20 + 3i) = 385$
15) $\sum_{i=1}^{100} i = 5050$

16) $\sum_{i=1}^{5} 7 = 35$

17) $\sum_{i=1}^{4} a = 4a$

18) 15
19) 15
20) 40
21) 5050
22) 91
23) 18
24) 18
25) 3n
26) bn

Problem Set #5

1) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2) $\sum_{i=1}^{n}(ai + b) = (a + b) + (2a + b) + \ldots + (an + b) = \frac{an^2 + an + 2bn}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Problem Set #6

1) 136
2) 1,496
3) 18,496
4) 312
5) 348
6) 1,656
7) 2,856
8) $x^2 - 1$
9) $x^3 - 1$
10) $x^4 - 1$
11) $x^7 - 1$
12) $x^{13} - 1$
13) $x^7 - 1$
14) $x^{23} - 1$
15) $x^{n+1} - 1$

16) $\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$

17) $\frac{10^{14} - 1}{9} \approx 1.11 \cdot 10^{13}$
18) 1093
19) 1093
20) 524,287
21) 524,287
22)
   a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$
   b) 2
23) $y \approx 1.13 \cdot 10^{15}$
24) $y \approx 5.8 \cdot 10^{12}$
25) $y \approx 1.61 \cdot 10^{10}$
26) $y = 9100$
27) $y = 117$
28) $y = 11.5$
29) $y = 1.11$
30) $y = 1$
31) $y \approx 0.605$
32) $y \approx 0.0769$
33) $y \approx 0.00515$
34) $y \approx 1.80 \cdot 10^{-8}$
35) $y \approx 8.88 \cdot 10^{-16}$
36) $y \approx 7.18 \cdot 10^{-27}$
37) $1 \cdot 10^{-100}$
38) Answers may vary.
39) x^{n+1} goes to zero.

40) $\sum_{i=0}^{\infty} x^i = \frac{1}{1 - x}$

Problem Set #7

1) 2
2) $\frac{3}{2} = 1\frac{1}{2}$
3) $\frac{4}{3} = 1\frac{1}{3}$
4) $\frac{10}{9} = 1\frac{1}{9}$
5) 4
6) $\frac{8}{5} = 1\frac{3}{5}$
7) $\frac{1}{1-\frac{\sqrt{3}}{2}} = \frac{1}{\frac{2-\sqrt{3}}{2}}$
$= \frac{2}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = 4 + 2\sqrt{3}$
8) 1
9) $\frac{1}{2}$
10) 3
11) $\frac{3}{2} = 1\frac{1}{2}$
12) Infinite.
13) Φ
14) $\Phi + 1 = \Phi^2$
15) $\frac{\pi}{2}$
16)

$S_1 = \frac{\pi}{2} + \frac{\pi}{2}\cdot\frac{1}{\Phi} + \frac{\pi}{2}\left(\frac{1}{\Phi}\right)^2 + \frac{\pi}{2}\left(\frac{1}{\Phi}\right)^3 + \dots$

$= \sum_{i=0}^{\infty} \frac{\pi}{2}\left(\frac{1}{\Phi}\right)^i \rightarrow$

$\frac{\pi}{2}\left(\sum_{i=0}^{\infty} \left(\frac{1}{\Phi}\right)^i\right) \rightarrow \frac{\pi}{2}\left(\frac{1}{1-\frac{1}{\Phi}}\right)$

$\rightarrow \frac{\pi}{2}\left(\frac{1}{\frac{\Phi-1}{\Phi}}\right) \rightarrow \frac{\pi}{2}\left(\frac{\Phi}{\Phi-1}\right)$ and given that $\Phi - 1 = \frac{1}{\Phi}$ we get a final answer of $\frac{\pi\Phi^2}{2}$

17) $X_n = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n-1}}$

$X_{n+1} = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n}}$

$X_{n+2} = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+1}}$

$X_{n+1} + X_{n+2} = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n}} + \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+1}}$

$X_{n+1} + X_{n+2} = \frac{\pi}{2}\left(\frac{1}{\Phi^{n}} + \frac{1}{\Phi^{n+1}}\right)$

$X_{n+1} + X_{n+2} = \frac{\pi}{2}\cdot\frac{\Phi+1}{\Phi^{n+1}}$

$X_{n+1} + X_{n+2} = \frac{\pi}{2}\cdot\frac{\Phi^2}{\Phi^{n+1}}$

$X_{n+1} + X_{n+2} = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n-1}}$

$X_{n+1} + X_{n+2} = X_n$

18)

$S_{n+2} = X_{n+2} + X_{n+3} + X_{n+4}\dots$

$= \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+1}} + \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+2}} + \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+3}} + \dots$

$= \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+1}}\left(1 + \frac{1}{\Phi} + \frac{1}{\Phi^2} + \frac{1}{\Phi^3} + \cdots\right)$

$= \frac{\pi}{2}\cdot\frac{1}{\Phi^{n+1}}\left(\Phi^2\right) = \frac{\pi}{2}\cdot\frac{1}{\Phi^{n-1}} = X_n$

19) The area of the rectangle is Φ so that would also be the sum of the squares. If you want to sum up the squares then assume the first square has an area of 1. Then the sum of the squares is

$1^2 + \frac{1}{\Phi^2} + \frac{1}{\Phi^4} + \frac{1}{\Phi^6} + \dots =$

$\sum_{i=0}^{\infty}\left(\frac{1}{\Phi^2}\right)^i = \frac{1}{1-\frac{1}{\Phi^2}} = \frac{1}{\frac{\Phi^2-1}{\Phi^2}} =$

$\frac{\Phi^2}{\Phi^2-1} = \frac{\Phi^2}{(\Phi+1)-1} = \Phi.$

Math & Music ANSWERS

Problem Set #1

The Fifth has a ratio of **3:2**
The Fourth has a ratio of **4:3**
The Third has a ratio of **5:4**
The Sixth has a ratio of **5:3**
The Octave has a ratio of **2:1**

The ratios of these intervals are the same regardless of what note you start on.

Problem Set #2

1) E 660 Hz
2) D $586\frac{2}{3}$ Hz
3) A 440 Hz
4) F 704 Hz
5) F 2816 Hz
6) F 88 Hz
7) B 990 Hz
8) G $782\frac{2}{9}$ Hz
9) A 880 Hz
10) A fifth and a fourth above is an octave.

Problem Set #3

1)

#	Name	Length	Frequency
1.	C	60 cm	264 Hz
2.	D	$53\frac{1}{3}$ cm	297 Hz
3.	E	48 cm	330 Hz
4.	F	45 cm	352 Hz
5.	G	40 cm	396 Hz
6.	A	36 cm	440 Hz
7.	B	32 cm	495 Hz
8.	C	30 cm	528 Hz
9.	D	$26\frac{2}{3}$ cm	594 Hz
10.	E	24 cm	660 Hz
11.	F	$22\frac{1}{2}$ cm	704 Hz
12.	G	20 cm	792 Hz
13.	A	18 cm	880 Hz
14.	B	16 cm	990 Hz
15.	C	15 cm	1056 Hz
16.	E	12 cm	1320 Hz
17.	G	10 cm	1584 Hz
18.	B	8 cm	1980 Hz
19.	C	$7\frac{1}{2}$ cm	2112 Hz
20.	D	$6\frac{2}{3}$ cm	2376 Hz
21.	E	6 cm	2640 Hz

2)

#	Ratio
#2:#1	9:8
#3:#2	10:9
#4:#3	16:15
#5:#4	9:8
#6:#5	10:9
#7:#6	9:8
#8:#7	16:15
#9:#8	9:8
#10:#9	10:9
#11:#10	16:15
#12:#11	9:8
#13:#12	10:9
#14:#13	9:8
#15:#14	16:15
#16:#15	5:4
#17:#16	6:5
#18:#17	5:4
#19:#18	16:15
#20:#19	9:8
#21:#20	10:9

Some of the whole steps are 9:8, and some are 10:9. The half steps are 16:15. But two half steps produce 256:225, which is greater than either 9:8 or 10:9.

3) If we started instead from the D (297Hz) instead of the C, then it turns out that only two of the resulting notes (G and B) in the scale will have the exact same frequencies as those that were produced by the C scale. This D scale would produce two completely new notes (F# and C#), and most troublesome, two of the notes should be the same, but have slightly different frequencies - E334.13 and A445.5.

4) 9:8
5) 9:8
6) 9:8
7)
   a) G 396 Hz
   b) 3:2 Fifth
8)
   a) F 352 Hz
   b) 4:3 Fourth
9)
   a) $F^{\#}$ 373.4 Hz
   b) $\sqrt{2}$: 1 Diminished fifth or augmented fourth.

Problem Set #4

1) $\frac{9}{8} \cdot \frac{9}{8} \cdot L = \frac{4}{3} \rightarrow \frac{3^2}{2^3} \cdot \frac{3^2}{2^3} \cdot L = \frac{2^2}{3}$

   $\rightarrow L = \frac{2^8}{3^5} \rightarrow L = \frac{256}{243}$

2) 256: 243
3) $\left(\frac{256}{243}\right)^2 \neq \frac{9}{8}$. Doesn't work!
4) 2187:2048
5) 2187:2048 is ≈ 1.36% greater than 256:243.
6) $\left(\frac{9}{8}\right)\left(\frac{9}{8}\right)\left(\frac{256}{243}\right)\left(\frac{9}{8}\right)\left(\frac{256}{243}\right)\left(\frac{9}{8}\right)\left(\frac{9}{8}\right) = 2$

7) C 264; D 297; E $334\frac{1}{8}$;

   F 352; G 396; A 445.5;

   B 501.2; C 528.

8) $\left(\frac{9}{8}\right)^6 \approx 2.027$ so this doesn't work. It's off by ≈ 1.36%.
9) This should be a point of classroom discussion, and is also addressed in the rest of this problem set.
10) $440 \cdot \frac{5}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = 434.57$. The ratio of these two notes is 80:81. The ending notes frequency is 1.25% greater.

11) $469\frac{221}{256}$ Hz

    which is an $A^{\#}$ or B^{b}.

12) ≈ 3568Hz
13) 3520 Hz
14) $\frac{\left(\frac{3}{2}\right)^{12}}{2^7} = \frac{3^{12}}{2^{19}} = \frac{531441}{524288}$
15) 12 fifths is greater by ≈ 1.36%. See problem #5.

Problem Set #5

1) Answers may vary.
2) Answers may vary.
3) From *Exponential Growth* – Problem Set #1 - Pr #10.

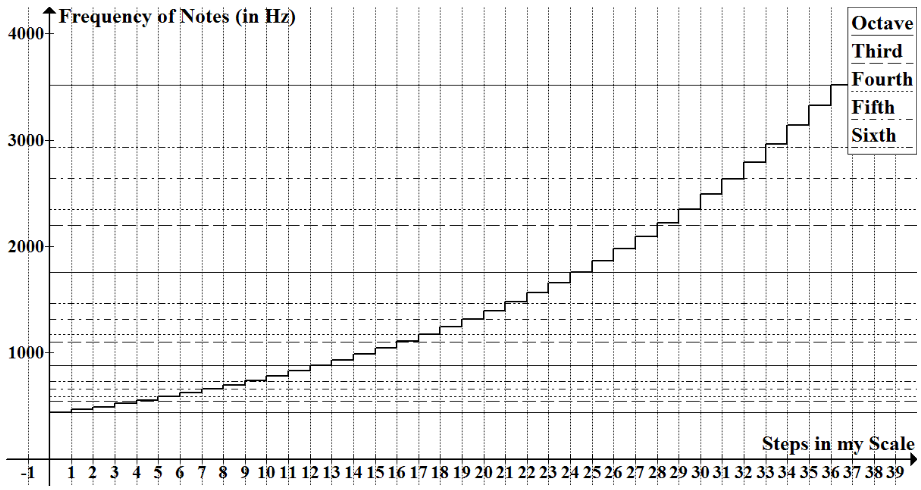
APR ≈ 10.40895137%

After 1 year	$485.80
2 years	$536.37
3 years	$592.20
4 years	$653.84
5 years	$721.90
6 years	$797.04
7 years	$880.00

4)

A	440 Hz
	485.80 Hz
	536.37 Hz
	592.20 Hz
	653.84 Hz
	721.90 Hz
	797.04 Hz
A	880 Hz

5) Answers may vary. The best scales (the scales which have notes that come closest to the important intervals) appear to have 12-steps, 17-steps or 19-steps. Below, the graph for the standard 12-step equitempered scale is shown.



NOTE: The solid lines represent octaves while the different types of dotted lines represent thirds, fourths, fifths and sixths.

Problem Set #6

1) 12

2)

Note	Hz
A	440
$A^{\#}$ or B^{b}	466.16
B	493.88
C	523.25
$C^{\#}$ or D^{b}	554.37
D	587.33
$D^{\#}$ or E^{b}	622.25
E	659.26
F	698.46
$F^{\#}$ or G^{b}	739.99
G	783.99
$G^{\#}$ or A^{b}	830.61
A	880

3) $\approx$1.122: 1
4) $\approx$ 1.0595:1
5) 2:1
6) $\approx$ 1.498:1
7) $\approx$ 1.335:1
8) $\approx$ 1.260:1
9) $\approx$ 1.682:1
10) 3520 Hz
11) 3520 Hz

12) Because a half-step increases the frequency by $\sqrt[12]{2}$, and a fifth is equal to 7 half-steps, we can say that a fifth increases the frequency by $2^{\frac{7}{12}}$. Therefore we can say $2^7 = \left(2^{\frac{7}{12}}\right)^{12}$

13) $98 \cdot 2^{(\frac{x}{12})} = 293.67$, which leads to x = 19 half-steps.

14) 20 half-steps.
15) 70 half-steps.
16) F
17) $A^{\#}$ or B^{b}
18) $C^{\#}$ or D^{b}
19) B