

# The Square Root Algorithm (without zeroes)

(Written in the style of a computer program. For Eighth grade.)

**Note:** As you follow the algorithm below you will need to carefully keep track of the following variables:

R, X, Y, *Difference*, *Sum*, *Product*

1. Enclose the number in a “house” as you would enclose a long division problem. Starting at the decimal point, and working out in both directions, draw short vertical lines that separate the number into pairs of two digits. Make sure that there are at least as many digit-pairs after the decimal place as the number of decimal places that are needed in the answer. Add ending zeroes, if needed. (e.g., In order to calculate  $\sqrt{45}$  to three decimal, we would need to add three pairs of ending zeroes and do  $\sqrt{45.00\ 00\ 00}$ .)
2. Let R be equal to the left-most digit-pair (which may be a single digit) that is inside the “house”. Circle it. Draw a small box, large enough to hold one digit, well to the left of R.
3. Let X be a single digit (somewhere from 0 to 9), such that it is as large as possible, but  $X^2$  is still less than or equal to R. Write X both in the box, and immediately below the box.
4. Underneath the digit that is below the box, write down the *Sum* of X plus X. Write the result of squaring X below R, and below that, write the *Difference* of R minus the square of X.
5. If there are no more digit pairs to bring down, then goto step 11.
6. Bring down the next digit-pair, combining it with, and writing it next to, the *Difference* (that was just found). This now forms the new value for R. Circle it.
7. Draw a small box to the right of the *Sum*. If the digit-pair just brought down is the first one after the decimal place, then write a decimal point above this box.
8. We must now choose a special single digit (somewhere between 0 and 9) that will be written both in the box and directly below the box. This special digit below the box will be called Y, and the new value for X will be the result of taking the *Sum* (found to the left of the box), and attaching to the end of it, the special digit in the box. (This means that Y will be equal to the last digit of the new value for X.) This special digit is chosen such that the result of X times Y is as large as possible, but still less than or equal to R. Write the correct choice for this special digit both in the box and below the box.
9. Underneath R, write the *Product* of X times Y, and then subtract it from R, writing this new *Difference* underneath it all.
10. Underneath X and Y, write the *Sum* of X plus Y. Goto step 5.
11. The answer to the square root problem is found by reading the digits in the boxes from top to bottom, with the decimal point possibly in the middle. If the *Difference* is zero, then the answer is exact; otherwise it is an approximation.

**Example:** Calculate  $\sqrt{780.0849}$

**Solution:** The work is shown below. The values for R are circled. The Y values (2, 7, 9, 3) are the single digits immediately below the boxes. The X values (2, 47, 549, 5583) are the numbers ending with the digit in the box. Each step number corresponds to the step number in the above algorithm.

Step1: The number is divided into 4 digit-pairs. Step2: R=7. Step3: X=2. Step4: *Sum*=4, *Difference*=3. Step6: R=380. Step8: Trying different “special” digits, we see that  $48 \cdot 8$  is bigger than R, and  $47 \cdot 7$  is less than R. The correct special digit is therefore 7, which we write both in the

$$\begin{array}{r}
 \boxed{2} \quad \boxed{7}80.\mid 08\mid 49 \\
 \underline{2} \quad \underline{-4} \\
 4\ \boxed{7} \quad \boxed{3}\ 80 \\
 \underline{7} \quad \underline{-329} \\
 54\ \boxed{9} \quad \boxed{5}\ 108 \\
 \underline{9} \quad \underline{-4941} \\
 558\ \boxed{3} \quad \boxed{1}\ 6749 \\
 \underline{3} \quad \underline{-16749} \\
 5586 \quad \boxed{0}
 \end{array}$$

box and below the box. Step9: The product of  $47 \cdot 7$  (329) is written below R. The *Difference* is 51. Step10: The *Sum* of  $47+7$  (54) is written below. We go back up to step5.

Step6: R=5108. Step7: We write a decimal point above the box. Step8: The special digit is 9, making X=549 and Y=9.

Step9: *Difference*=167. Step10: *Sum*=558. Step6:

R=16749. Step8: X=5583, Y=3. Step9: *Difference*=0.

Step10: *Sum*=5586, goto step5. Step5: goto step11.

Step11: **Our final answer is 27.93** (exactly).