Possibility & Probability – Part II ANSWERS

Problem Set #1

1)
   a) $6! = 720$
   b) $\frac{8!}{2!} = 20{,}160$

2) $5 \cdot 12 \cdot 5 = 300$
3) 45,697,600
4) 2,730
5) 3,628,800
6)
   a) 720
   b) 120
   c) 60
   d) 5
   e) 1
   f) 10
   g) 5
   h) 1

7)
   a) $\frac{5}{9} \cdot \frac{4}{8} = \frac{5}{18}$
   b) $\frac{4}{9} \cdot \frac{3}{8} = \frac{1}{6}$
   c) $({}_5C_1)({}_4C_1) \div {}_9C_2 = \frac{5}{9}$

   or more simply

   $1 - (\frac{5}{18} + \frac{1}{6}) = \frac{5}{9}$

8)
   a) $\frac{4}{52} = \frac{1}{13}$
   b) $\frac{4}{52} = \frac{1}{13}$
   c) $\frac{13}{52} = \frac{1}{4}$
   d) $\frac{8}{52} = \frac{2}{13}$
   e) $\frac{16}{52} = \frac{4}{13}$

9)
   a) $\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} \approx 0.603\%$
   b) $\frac{16}{1326} \approx 1.21\%$

10) ${}_{10}P_3 = 720$

11) 120

12) $({}_6C_5)\left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^1 + ({}_6C_6)\left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^0$

$= \frac{19}{4096} \approx 0.46\%$

Another option is as follows:

Total number of possible outcomes = $4^6 = 4096$.

Number of ways to get all 6 correct = 1.
Number of ways to get the first one wrong and then the last 5 correct = 3.
If only one is wrong, the number of possible positions for the wrong problem = 6
Number of ways to get one wrong and 5 correct = $3 \cdot 6 = 18$

Total number of ways to get at least 5 correct = 1+18 = 19

Probability of getting at least five correct = $\frac{19}{4096}$

Possibility & Probability – Part II ANSWERS

Problem Set #2

1)
   a) $5! = 120$
   b) $\frac{6!}{3!} = 120$

2) 6

3)
   a) $5! = 120$
   b) $5^5 = 3125$

4)
   a) ${}_{52}C_5 = 2{,}598{,}960$
   b) ${}_{52}C_{13} \approx 6.35 \cdot 10^{11}$

5)
   a) ${}_8C_3 = 56$
   b) ${}_8C_5 = 56$

6)
   a) $\frac{4}{12} \cdot \frac{3}{11} = \frac{1}{11}$
   b) $\frac{5}{12} \cdot \frac{4}{11} = \frac{5}{33}$

7) ${}_8P_3 = 336$

8)
   a) $\frac{4}{36} = \frac{1}{9}$
   b) $\frac{1}{36}$
   c) $\frac{11}{36}$

9)
   a) $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$
   b) $\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$
   c) $\frac{1}{221} + \frac{1}{17} = \frac{14}{221}$

10)
   a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
   b) $x^4 + 8x^3 + 24x^2 + 32x + 16$
   c) $x^4 - 40x^3 + 600x^2 - 4000x + 10000$

11)
   a) $\frac{1}{2^4} = \frac{1}{16}$
   b) $\frac{4}{16} = \frac{1}{4}$
   c) $\frac{6}{16} = \frac{3}{8}$
   d) $\frac{6}{16} + \frac{4}{16} = \frac{10}{16} = \frac{5}{8}$

12) $10 + 8 - 4 = 14$

Problem Set #3

1)
   a) $\frac{6!}{2!2!} = 180$
   b) $\frac{5!}{3!} = 20$
   c) $\frac{10!}{4!3!2!} = 12{,}600$

2) $4^7 = 16{,}384$

3) $4! = 24$

4) ${}_{60}C_{59} = {}_{60}C_1 = 60$

5)
   a) $({}_4C_2)({}_6C_2) = 90$
   b) ${}_{10}C_4 - {}_6C_4 = 195$
   c) ${}_6C_4 + {}_4C_4 = 16$

6) $8 \cdot 7 = 56$

7) $\frac{11!}{8!3!} = 165$

8) $1 \div {}_{20}P_3 = \frac{1}{6840}$

Possibility & Probability – Part II ANSWERS

9)

a) $\frac{4}{36} = \frac{1}{9}$

b) $\frac{2}{36} = \frac{1}{18}$

c) $\frac{10}{36} = \frac{5}{18}$

d) $\frac{11}{36}$

10)

a) ${}_2C_2 \div {}_{12}C_2 = \frac{1}{66}$ or $\frac{2}{12} \cdot \frac{1}{11} = \frac{1}{66}$

b) $({}_2C_1)({}_4C_1) \div ({}_{12}C_2) = \frac{4}{33}$

or $\frac{2}{12} \cdot \frac{4}{11} + \frac{4}{12} \cdot \frac{2}{11} = \frac{4}{33}$

c) $({}_{10}C_2) \div ({}_{12}C_2) = \frac{15}{22}$ or $\frac{10}{12} \cdot \frac{9}{11} = \frac{15}{22}$

11)

a) $\frac{1}{32}$

b) $\frac{5}{32}$

c) $\frac{10}{32} = \frac{5}{16}$

d) $\frac{16}{32} = \frac{1}{2}$

12) $\frac{28}{52} = \frac{7}{13}$

13) Put the ten coins in a row and separate the different groups by three lines (thus creating four groups). For example if 0 is a coin, then one possible distribution would be:

000|00|0|0000

Thus the number of ways to arrange 10 zeros and three lines is $\frac{13!}{10!3!} = 286$.

14)

a) ${}_{26}C_{13} \div {}_{52}C_{13} = \frac{19}{1160054}$ $\approx \frac{1}{61055} \approx 0.00164\%$

b) ${}_{40}C_{13} \div {}_{52}C_{13} \approx \frac{1}{53}$ $\approx 1.89\%$

c) $\approx 1 - 1.89\% = 98.1\%$

d) $4^{13} \div {}_{52}C_{13} \approx \frac{1}{9462}$ $\approx 0.0106\%$

e) $({}_{13}C_{11})({}_{39}C_2) \div {}_{52}C_{13}$ $\approx \frac{1}{10986334}$

$\approx 0.0000091\%$

Problem Set #4

1)

a) 5040
b) 3360

2)

a) 336
b) 512

3) 495

4)

a) 720
b) 72
c) 144

5)

a) 4845
b) 116,280

6) ${}_{12}C_3 = 220$

7) 20,180,160

Possibility & Probability – Part II ANSWERS

8)
- a) $({}_6C_2)({}_{10}C_3) = 1800$
- b) ${}_{16}C_5 - {}_{10}C_5 = 4116$

9)
- a) $\frac{6}{216} = \frac{1}{36}$
- b) $\frac{10}{216} = \frac{5}{108}$
- c) $\frac{15}{216} = \frac{5}{72}$

10)
- a) $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
- b) $x^7 - 14x^6 + 84x^5 - 280x^4 + 560x^3 - 672x^2 + 448x - 128$

11)
- a) $\frac{35}{128}$
- b) $\frac{29}{128}$
- c) $\frac{29}{128}$

12) Either you pick the 9 best players or the 9 worst players so $\frac{2}{48620} = \frac{1}{24310}$

13)
- a) $10^3 = 1000$
- b) ${}_{10}P_3 = 720$
- c) ${}_{10}C_3 = 120$

Challenge Problem Set

1)
- a) $({}_7C_4)\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^3 \approx 5.77\%$
- b) $({}_7C_4)\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^3 + ({}_7C_5)\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^2 + ({}_7C_6)\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^1 + ({}_7C_7)\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^0 \approx 7.03\%$

2)
- a) $\frac{{}_{48}C_{13}}{{}_{52}C_{13}} \approx 30.4\%$
- b) $\frac{({}_4C_1)({}_{48}C_{12})}{{}_{52}C_{13}} \approx 43.9\%$
- c) $\frac{({}_4C_2)({}_{48}C_{11})}{{}_{52}C_{13}} \approx 21.35\%$
- d) $\frac{({}_4C_3)({}_{48}C_{10})}{{}_{52}C_{13}} \approx 4.12\%$
- e) $\frac{({}_4C_4)({}_{48}C_9)}{{}_{52}C_{13}} \approx 0.264\%$
- f) $\approx 1 - 30.4\% \approx 69.6\%$
- g) $\approx 1 - (30.4\% + 43.9\%) \approx 25.7\%$

3)
- a) $2^{10} = 1024$
- b) ${}_{10}C_3 + {}_{10}C_4 = 330$

4)
- a) $\frac{({}_4C_3)({}_{48}C_2)}{{}_{52}C_5} \approx 0.174\%$
- b) $\frac{({}_{13}C_1)({}_4C_3)({}_{12}C_2)({}_4C_1)({}_4C_1)}{{}_{52}C_5} \approx 2.11\%$
- c) $\frac{{}_{26}C_5}{{}_{52}C_5} \approx 2.53\%$
- d) $\frac{({}_{13}C_1)({}_4C_3)({}_{12}C_1)({}_4C_2)}{{}_{52}C_5} \approx 0.144\%$

Possibility & Probability – Part II ANSWERS

(Challenge Problem Set – cont'd)

5) Ignoring leap years…

a) It's easier to calculate the probability that no two or more people in the group have the same birthday:

$$\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}\cdot\frac{361}{365}\cdot\frac{360}{365}\cdot\frac{359}{365}\cdot\frac{358}{365}\cdot\frac{357}{365}\cdot\frac{356}{365}$$

≈ 88.31%

Thus the probability that at least two people will have the same birthday in a group of 10 is

≈ 100% – 88.31% = 11.69%

b) 23

6)

a) ${}_{39}C_{13} \div {}_{52}C_{13} \approx 1.28\%$

b) ≈ 100% – 1.28% = 98.72%

c) $({}_{13}C_4)({}_{39}C_9) \div {}_{52}C_{13} \approx$ 23.9%

d) ${}_{16}C_{13} \div {}_{52}C_{13} = \frac{1}{1133952785}$

≈ 0.0000000882

e) $({}_{13}C_1)({}_{4}C_4)({}_{48}C_9) \div {}_{52}C_{13}$ ≈ 3.43%

7)

a) $({}_{1}C_1)\,({}_{51}C_4) \div {}_{52}C_5 \approx 9.62\%$

b) $({}_{13}C_2)({}_{13}C_2)({}_{26}C_1) \div {}_{52}C_5$

≈ 6.09%

c) $[({}_{13}C_3)({}_{39}C_2) + ({}_{13}C_4)({}_{39}C_1) + ({}_{13}C_5)({}_{39}C_0)] \div {}_{52}C_5$ ≈ 9.28%

d) The easiest approach to this problem is to find the probability of getting all four suits and then subtract that from one. A 5-card hand with all four suits means one suit is repeated.
$1 - [({}_{4}C_1)({}_{13}C_2)({}_{13}C_1)({}_{13}C_1)({}_{13}C_1) \div {}_{52}C_5] \approx 73.6\%$

Possibility & Probability – Part II ANSWERS

(Challenge Problem Set – cont'd)

8)

a) $({}_{12}C_4)({}_8C_4)({}_4C_4) \div 3! = 5775$. We have to divide by 3! because the order in which the groups were chosen doesn't matter.

b) Here are three ways to do this problem:

i) We first choose two people to be in Betty and Sue's group: ${}_{10}C_2 = 45$. Then we choose the other two groups: $({}_8C_4)({}_4C_4) = 70$. Therefore the number of ways to have Betty and Sue in the same group is $45 \cdot 70 \div 2 = 1575$. (We divide by two because the order of the last two groups doesn't matter.)
The probability that Betty and Sue are in the same group is: $1575 \div 5775 = 3/11 \approx 27.3\%$

ii) This problem is the same as asking: if Betty is in the first position in a line, what is the probability that Sue is in one of the next three positions?
Answer: 3/11.

iii) Again thinking of it as Betty in the first position in a line, we can now ask:
What is the probability that Sue isn't in second position?
Answer: 10/11
What is the probability that Sue isn't in third position?
Answer: 9/10
What is the probability that Sue isn't in second position?
Answer: 8/9
Therefore, the probability that Sue isn't in any of these positions is: $\frac{10}{11} \cdot \frac{9}{10} \cdot \frac{8}{9} = \frac{8}{11}$
So the probability that she is in the first three positions is $1 - \frac{8}{11} = \frac{3}{11}$

c) $({}_{12}C_4)({}_8C_4)({}_4C_4) = 34,650$. This is the same as #8a except the order in which the groups were chosen does matter, so we don't divide by 3!.

Trigonometry – Part III ANSWERS

Problem Set #1

1)

a) sin(170°) = sin(10°) ≈ 0.1736

b) cos(80°) = sin(10°) ≈ 0.1736

c) $\cos^2(10°) + \sin^2(10°) = 1$ yields cos(10°) ≈ 0.985

d) $\tan(10°) = \frac{\sin(10°)}{\cos(10°)}$ ≈ 0.1762

e) cos(170°) = -cos(10°) ≈ -0.985

f) tan(170°) = -tan(10°) ≈ -0.1762

2)

a) $\frac{1}{2}$

b) 0

c) ≈0.423

d) $\frac{\sqrt{3}}{2}$

e) $\frac{\sqrt{2}}{2}$

f) $\frac{\sqrt{3}}{3}$

g) $\sqrt{3}$

h) 1

i) ≈ 0.466

j) $\frac{1}{2}$

k) 0

l) $\frac{\sqrt{2}}{2}$

m) 0

n) Undefined

o) $\frac{\sqrt{3}}{2}$

p) ≈ 0.906

q) 1

r) ≈ 0.906

s) 1

t) $\frac{\sqrt{3}}{2}$

u) $-\frac{1}{2}$

v) $-\sqrt{3}$

3)

a) 30°, 150°

b) 45°

c) ≈ 53.1°, ≈ 127°

d) ≈ 26.6°

e) 45° , 135°

f) ≈ 75.5°

g) 90°

h) 0° , 180°

i) 150°

j) 120°

4)

a) θ ≈ 38.9°

b) x ≈ 112

c) x ≈ 14.7

d) x ≈ 12.4

e) $x = \sqrt{85}$ ≈ 9.22

f) x ≈ 3.1

g) θ ≈ 23°

h) θ = 48°

i) x ≈ 5.88

j) x ≈ 15

Problem Set #2

1)

a) Using ΔAXB, $\cos(A) = \frac{AX}{c}$ thus $c\cdot\cos(A) = AX$.

b) $CX = a\cdot\cos(C)$
$BY = c\cdot\cos(B)$
$CY = b\cdot\cos(C)$
$AZ = b\cdot\cos(A)$
$BZ = a\cdot\cos(B)$

c) #1 = $b\cdot c\cdot\cos(A)$
#2 = $b\cdot a\cdot\cos(C)$
#3 = $a\cdot b\cdot\cos(C)$
#4 = $a\cdot c\cdot\cos(B)$
#5 = $c\cdot a\cdot\cos(B)$
#6 = $c\cdot b\cdot\cos(A)$

Equal pairs: #1 & #6; #2 & #3; #4 & #5.

Trigonometry – Part III ANSWERS

1d) Explanation:

<u>Step 1</u>: #6=#1; #5=#4; #3=#2
<u>Step 2</u>: #5 + #6 = c^2;
#1 + #2 = b^2
<u>Step 3</u>: Add #3 to both sides.
<u>Step 4</u>: #4 + #3 = a^2
<u>Step 5</u>: #3 = $a \cdot b \cdot \cos(C)$.
Subtract 2(#3) from both sides and then you have the Law of Cosines!

2)

a) $h^2 = b^2 - x^2$
b) $h^2 = c^2 - (a - x)^2$
c) Set the two above equations equal to each other:

$b^2 - x^2 = c^2 - (a - x)^2$
$b^2 - x^2 = c^2 - (a^2 - 2ax + x^2)$
$b^2 - x^2 = c^2 - a^2 + 2ax - x^2$
$c^2 = a^2 + b^2 - 2ax$

Because $\cos(C) = \frac{x}{b}$,

$x = b \cdot \cos(C)$
$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$
The Law of Cosines!

3) For the *Proof using Squares*, imagine that angle A is the obtuse angle. Then two of the altitudes will fall "outside" of the squares, which results in rectangles #2 and #5 being larger than the squares. Then, instead of having c^2 = #5 + #6, we get c^2 = #5 – #6, and instead of b^2 = #2 + #1, we get b^2 = #2–#1. The rest of the proof follows as before.

For the *Proof without Squares*, imagine that angle B is the obtuse angle. Then the altitude h will fall "outside" the triangle, and x would be larger than a. Then, instead of side c being divided between two pieces, x and a–x, the two pieces would be a and x–a. The rest of the proof follows as before.

4) Answers may vary.

5)

a) $x \approx 8.48$
b) $\theta \approx 78.5°$

6)

a) $x \approx 105$
b) $x \approx 3.38$
c) $\theta \approx 34.2°$
d) $\theta \approx 26.9°$
e) $x \approx 7.41$
f) $\theta \approx 51.3°$
g) $x = \sqrt{39} \approx 6.24$
h) $x \approx 6.65$

Trigonometry – Part III ANSWERS

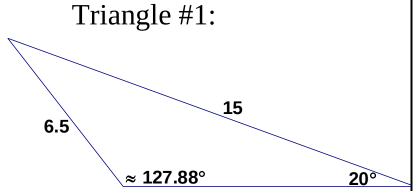
Problem Set #3

1) The *Law of Sines* is used when a problem is dealing with two angles and two sides.
The *Law of Cosines* is used when a problem is dealing with all three sides and one angle.
The *Law of Tangents*, which we will learn about next, is an exception to the above rules.

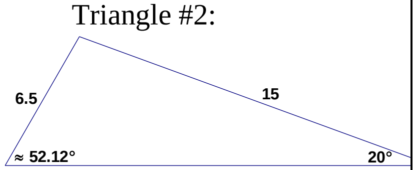
2)
- a) $\theta \approx 57.8°$
- b) $x \approx 37.5$
- c) $\theta \approx 34.03°$
- d) $\theta \approx 52.1°$ <u>or</u> $\theta \approx 127.9°$

3)
- a) Using the *Law of Sines*, sin(θ) yields two possible answers.
- b) If only two sides and an angle not between them are known (SSA).
- c)

Triangle #1:



Triangle #2:



- d) See 2d.

4)
- a) $-\frac{\sqrt{2}}{2}$
- b) $\frac{\sqrt{2}}{2}$
- c) -1
- d) $-\frac{\sqrt{3}}{2}$
- e) $\frac{1}{2}$
- f) $-\frac{\sqrt{3}}{3}$
- g) $\approx 63.43°$
- h) $120°$
- i) ≈ 0.78
- j) ≈ 0.94
- k) ≈ 0.34
- l) ≈ 0.94
- m) -0.34
- n) $\approx 30°$
- o) $120°$
- p) $\approx 17.5°$, $162.5°$

5)
- a) $x \approx 0.573$
- b) $x \approx 7.7$
- c) $x \approx 384.8$
- d) Not a possible triangle.
- e) $\theta \approx 79.4°, \propto \approx 45.6°$ <u>or</u> $\theta \approx 100.6°, \propto \approx 24.4°$
- f) $\theta \approx 47.4°$
  $\propto \approx 70.6°$
- g) $x \approx 3.78$
- h) $x \approx 13.88$ <u>or</u> $x \approx 3.24$
- i) $\theta \approx 38.21°$
  $\propto \approx 21.79°$

Trigonometry – Part III ANSWERS

Problem Set #4

1) ΔPCQ is similar to ΔBCR. Therefore ∠CPQ = ∠CRB and PQ is parallel to RT.

2)

- ACQ = 180° – ∠A – ∠B
- ∠PQC = $\frac{1}{2}$∠ACQ

  ∠PQC = 90° – $\frac{1}{2}$∠A – $\frac{1}{2}$∠B
- ∠PCQ = ∠A + ∠B
- ∝ = $\frac{1}{2}$(∠A + ∠B)
- ∠CQA = ∝ = $\frac{1}{2}$(∠A + ∠B)
- ∠TQB = ∝ = $\frac{1}{2}$(∠A + ∠B)
- ∠AQP = ∠ CQA + ∠PQC

  ∠AQP = $\frac{1}{2}$(∠A + ∠B) + (90° – $\frac{1}{2}$∠A – $\frac{1}{2}$∠B)

  ∠AQP = 90°
- ∠ATR = 90°
- θ + ∝ = ∠A

  θ = ∠A – $\frac{1}{2}$(∠A + ∠B)

  θ = $\frac{1}{2}$(∠A – ∠B)

3) ΔPCQ and ΔBCR; ΔAQP , ΔATR and ΔTQB.

4) tan(θ) = $\frac{\text{TB}}{x}$ therefore

TB = x·tan(θ)

5) RT = x·tan(∝)

6) $x = \frac{\text{TB}}{\tan(\theta)}$ and $x = \frac{\text{RT}}{\tan(\propto)}$

therefore $\frac{\text{TB}}{\tan(\theta)} = \frac{\text{RT}}{\tan(\propto)}$

$\frac{\tan(\theta)}{\tan(\propto)} = \frac{\text{TB}}{\text{RT}}$

$\frac{\tan\,[^1/_2(\text{A}-\text{B})]}{\tan\,[^1/_2(\text{A}+\text{B})]} = \frac{\text{a}-\text{b}}{\text{a}+\text{b}}$

(TB : RT = (a – b) : (a + b) through similar triangles)

The Law of Tangents!

7)

a) The average of A and B.
b) The difference between the average and either A or B.

8) θ + ∝ = 60° .

$\frac{\tan\,[^1/_2(\theta-\propto)]}{\tan\,[^1/_2 60]} = \frac{5-3}{5+3}$

$\tan[^1/_2\,(\theta-\propto)] = \frac{1}{4}\tan(30)$

$\tan[^1/_2\,(\theta-\propto)] \approx 0.1443$

$\frac{1}{2}(\theta-\propto) \approx 8.21°$

The values of the desired angles are therefore 8.21° above and below the average, 30°. So we get:

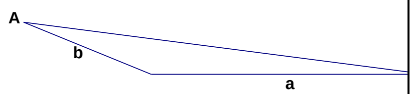
θ ≈ 30° + 8.21° ≈ 38.21°
α ≈ 30° – 8.21° ≈ 21.79°.

And, as we expected,
θ + ∝ = 60°

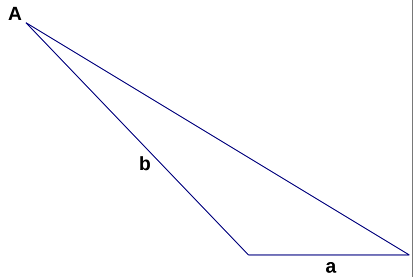
Trigonometry – Part III ANSWERS

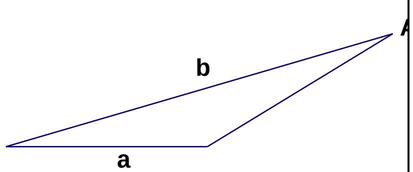
9)

a) Not ambiguous. The known side across from the known angle is longer than the other known side.



b) Two possible triangles.





c) Not a possible triangle.

10)

a) $x \approx 11.04$
b) $x \approx 156.65$
c) $\theta \approx 125.1°$
d) $\theta \approx 75.964°$
e) $x \approx 62.26$
f) $\theta \approx 66.05°$, $\propto \approx 59.95°$
<u>or</u> $\theta \approx 113.95°$, $\propto \approx 12.05°$
g) $\theta \approx 55.16°$, $\propto \approx 56.84°$
h) $x \approx 31.61$
i) $\propto \approx 67.46°$
j) $x \approx 14.154$

Problem Set #5

1)

a) $\frac{\sqrt{3}}{2}$
b) $\frac{2\sqrt{3}}{3}$
c) $\frac{\sqrt{2}}{2}$
d) $\sqrt{2}$
e) $\sqrt{3}$
f) $\frac{\sqrt{3}}{3}$
g) ≈ 0.94
h) ≈ 1.064
i) ≈ 0.28
j) ≈ 3.63
k) ≈ 3.08
l) ≈ 0.325

2)

a) $\theta \approx 56.25°$
b) $\theta \approx 34.85°$
c) $\propto \approx 67.15°$ <u>or</u>
$\propto \approx 112.85°$
d) $x \approx 11.87$
e) $\theta \approx 147.94°$
f) $\theta \approx 63.47°$, $x \approx 8.87$ <u>or</u>
$\theta \approx 116.53°$, $x \approx 4.4$
g) Impossible Triangle.
h) $\theta \approx 34.5°$, $x \approx 8.71$
i) $\theta \approx 141.24°$
j) Impossible triangle.
$(5 + 12 < 18)$.

3) $x \approx 2.76$ km

4) See the next page.

5) a) $\tan^2(\theta) + 1 = \sec^2(\theta)$
b) $1 + \cot^2(\theta) = \csc^2(\theta)$

6) a) $\cos(17°) \approx 0.9563$
$\tan(17°) \approx 0.3057$
$\sec(17°) \approx 1.0457$
$\csc(17°) \approx 3.42$
$\cot(17°) \approx 3.271$

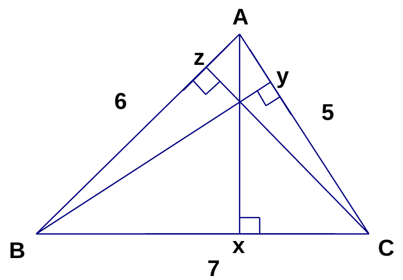
Trigonometry – Part III ANSWERS

6)

b) $\sin(53°) \approx 0.7986$
$\tan(53°) \approx 1.33$
$\sec(53°) \approx 1.662$
$\csc(53°) \approx 1.252$
$\cot(53°) \approx 0.754$

c) $\sin(39°) \approx 0.629$
$\cos(39°) \approx 0.777$
$\sec(39°) \approx 1.287$
$\csc(39°) \approx 1.59$
$\cot(39°) \approx 1.235$

7)



Area ≈ 14.7
Ax ≈ 4.2
Cz ≈ 4.9
By ≈ 5.88

8) ≈ 63.72 ft.

4)

	0°	30°	45°	60°	90°	120°	135°	150°	180°
cos(θ)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
sin(θ)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan(θ)	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Un.	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
sec(θ)	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Un.	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1
csc(θ)	Un.	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Un.
cot(θ)	Un.	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	Un.

Trigonometry – Part III ANSWERS

Problem Set #6

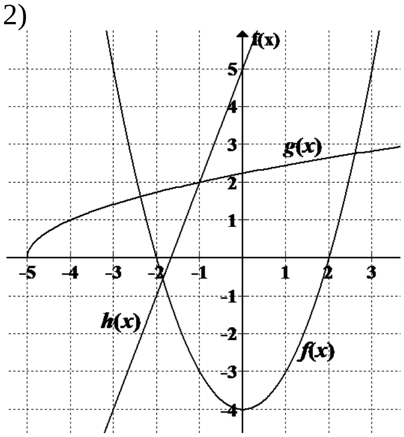
1) Answers may vary.
2)
   a) $\frac{\sqrt{3}}{3}$
   b) 2
   c) ≈ -0.9397
   d) ≈ 0.6428
   e) ≈ 1.556
   f) ≈ 0.6428
   g) ≈ -5.759
   h) $-\sqrt{3}$
3)
   a) $\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$
   b) $\frac{\sqrt{2-\sqrt{2}}}{2}$
   c) $\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$
4)
   a) $x \approx 2.684$
   b) $x \approx 4.45$
   c) θ ≈ 35.3°
   d) θ ≈ 101.41° or θ ≈ 18.6°
   e) $x \approx 9.554$
      ∝ ≈ 23.46°
      $\beta \approx 84.54°$
   f) $x \approx 17.92$
   g) θ ≈ 56.44°
   h) θ ≈ 37.6°
   i) $x \approx 5.56$
   j) $x \approx 108.13$ or $x \approx 33.3$
5) $\frac{\sqrt{3}}{2}$
6) $\frac{\sqrt{2}}{2}$
7) ≈ 1.43
8) $\frac{1}{2}$
9) ≈ -0.985
10) −1
11) $\frac{2\sqrt{3}}{3}$
12) $\sqrt{2}$
13) $-\frac{\sqrt{3}}{3}$
14) ≈ 0.174
15) ≈ 5.76
16) ≈ 0.643
17) ≈ 1.556
18) 60° and 120°
19) 30°
20) 60°
21) 30°
22) 60°
23) Undefined.
24) ≈ 11.54° and 168.46°
25) 30° and 150°
26) ≈ 108.4°
27) ≈ 161.6°
28) 135°
29) ≈ 102.64°, ≈ 45.21°, ≈ 32.15°
30) ≈ 33.7 miles.
31) Perimeter ≈ 3.11
    Area is ≈ 95.5% of circle
32) The balloon has risen ≈ 756 ft during that minute and is travelling at a speed of 756 ft/min which is the same as 8.6 mph.

Cartesian Geometry – Part III ANSWERS

Problem Set #1

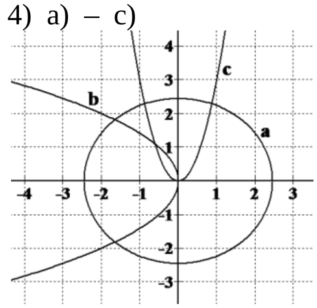
1)
- a) 21
- b) 5
- c) 7
- d) 5
- e) $\sqrt{2}$
- f) $\sqrt{x^2 - 2}$
- g) 21
- h) 8
- i) -3
- j) $9x^2 + 30x + 21$
- k) $3x^2 - 7$
- l) $3x + 8$

2)

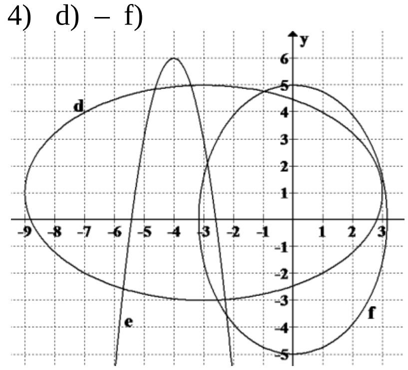


3) Answers may vary.

4) a) – c)

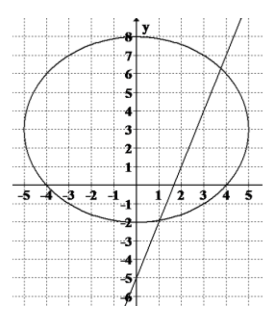


4) d) – f)



5) Exact Points of Intersection are: $\left(\frac{24+\sqrt{186}}{10}, \frac{22+3\sqrt{186}}{10}\right)$, $\left(\frac{24-\sqrt{186}}{10}, \frac{22-3\sqrt{186}}{10}\right)$

Decimal approximations are:
(≈ 3.764, ≈ 6.292),
(≈ 1.0362, ≈ -1.8915)



Cartesian Geometry – Part III ANSWERS

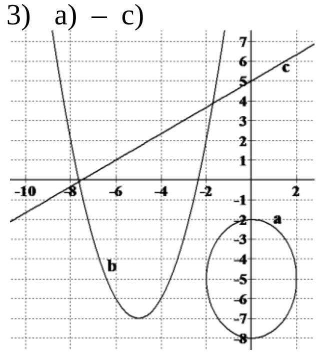
Problem Set #2

1) $\sqrt{58} \approx 7.62$

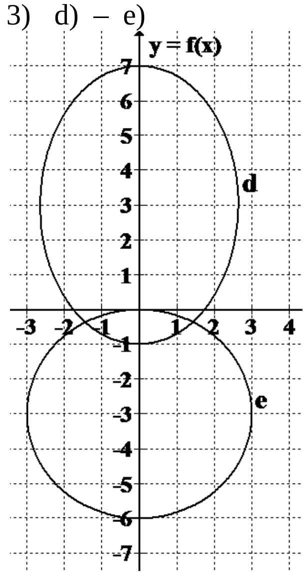
2) ℝ is the set of all real numbers.
ℤ is the set of integers.

- $f(x) = \sin(x)$
  Domain:
  All real numbers (ℝ)
  Range: $-1 \leq f(x) \leq 1$
- $f(x) = \cos(x)$
  Domain: ℝ
  Range: $-1 \leq f(x) \leq 1$
- $f(x) = \tan(x)$
  Domain:
  ℝ except $x \neq (2n + 1)\cdot 90°$ where n can be any integer ($n \in \mathbb{Z}$)
  Range: ℝ
- $f(x) = \sec(x)$
  Domain: ℝ
  except $x \neq (2n + 1)\cdot 90°$
  $n \in \mathbb{Z}$
  Range:
  $f(x) \geq 1$ or $f(x) \leq -1$
- $f(x) = \csc(x)$
  Domain: ℝ
  except $x \neq n\cdot 180°$
  $n \in \mathbb{Z}$
  Range:
  $f(x) \geq 1$ or $f(x) \leq -1$
- $f(x) = \cot(x)$
  Domain: ℝ
  except $x \neq n\cdot 180°$
  $n \in \mathbb{Z}$
  Range: ℝ

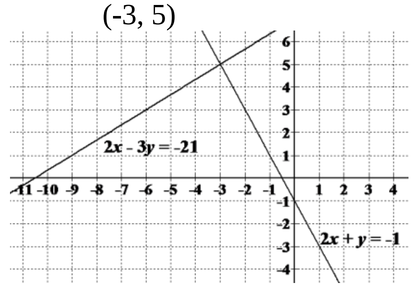
3) a) – c)



3) d) – e)

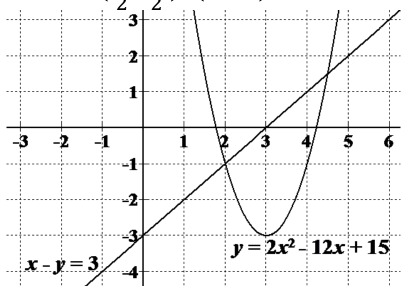


4)
a) Point of intersection:
(-3, 5)



Cartesian Geometry – Part III ANSWERS

4) b) Points of intersection:
$(\frac{9}{2}, \frac{3}{2})$, (2, -1)

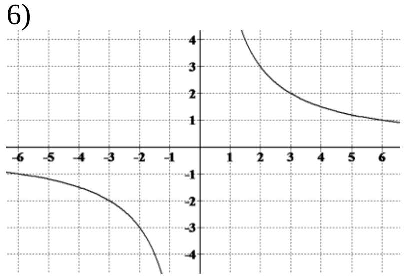


5)

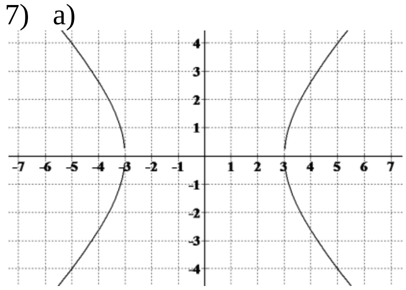
a) Domain: $\mathbb{R}$ except $x \neq \pm 3$
Range: $\mathbb{R}$ except $f(x) \neq 0$

b) Domain: $x \leq 5$
Range: $f(x) \geq 4$

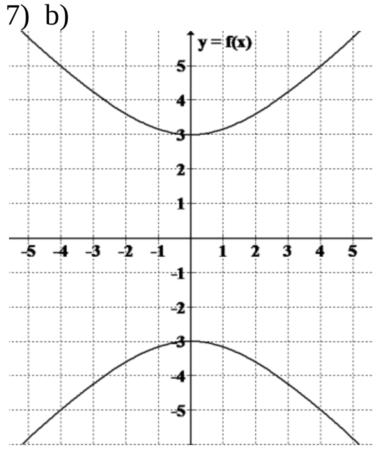
6)



7) a)

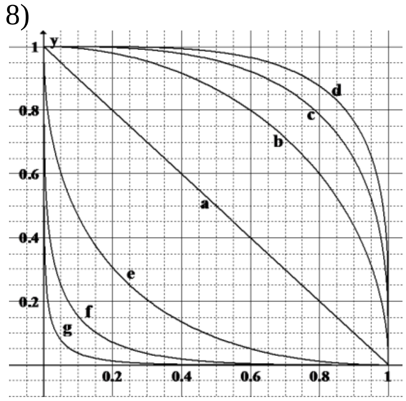


7) b)



7) c) $x^2 + y^2 = -9$ has only imaginary solutions.

8)

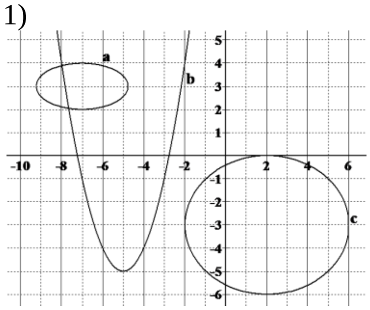


9) As n gets larger, or gets closer to zero, the graph bows further away from the line $x + y = 1$.

Cartesian Geometry – Part III ANSWERS

Problem Set #3

1)



2)

a) Domain: $\mathbb{R}$
Range: $f(x) \leq 5$

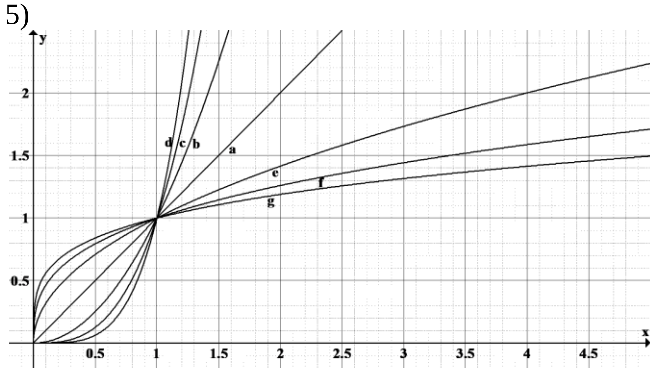
b) Domain: $x \geq -3$
Range: $f(x) \leq 8$

3) $4\sqrt{5} \approx 8.944$

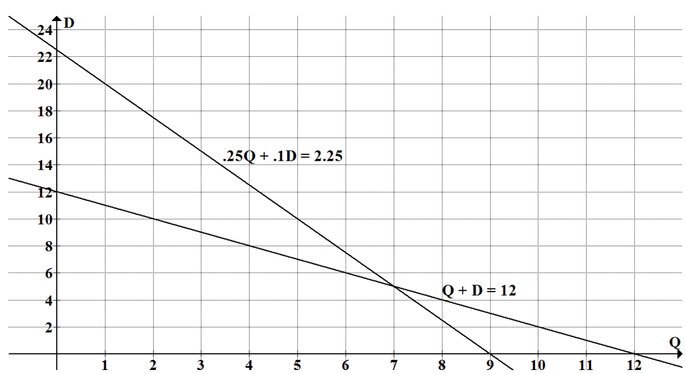
4)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

5)



6) Q + D = 12 & .25Q + .1D = 2.25; See below graph.
Solution: 7 Quarters and 5 Dimes



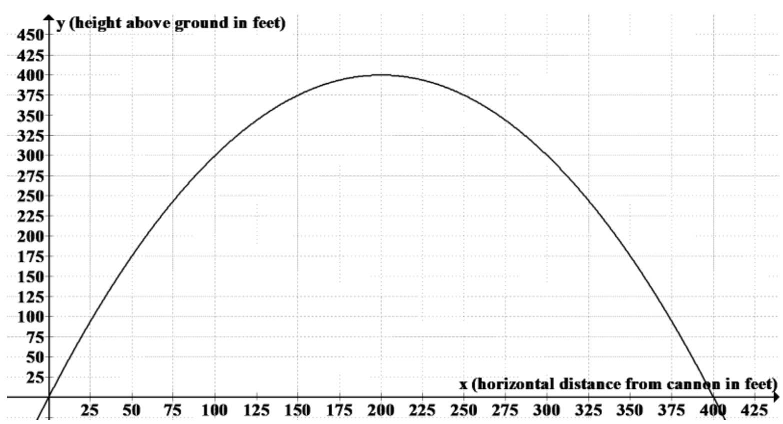
Cartesian Geometry – Part III ANSWERS

7)

a) 2π
b) $x^2 + y^2 = 1$
c) (-1, 0)
d) (0, 1)
e) $(\frac{1}{2}, \frac{\sqrt{3}}{2})$
f) $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
g) $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
h) $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
i) $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
j) $(\frac{\sqrt{3}}{2}, \frac{1}{2})$
k) $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
l) (0, -1)
m) $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
n) $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

8)

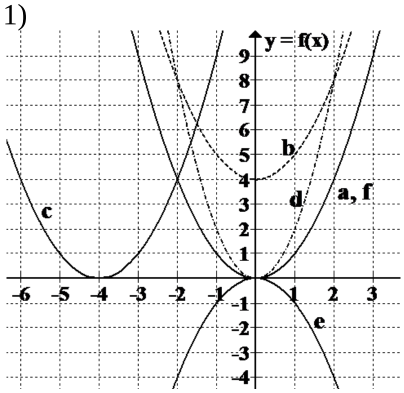
a) See below graph
b) ≈ 76°
c) 400 ft.
d) (100, 300)
<u>or</u> (300, 300)
e) $(200 - 100\sqrt{2}, 200)$
<u>or</u> $(200 + 100\sqrt{2}, 200)$
which is also
(≈ 58.6, 200)
<u>or</u> (≈ 341.42, 200)
f) 400 ft.
g) ≈ 447.2 ft.



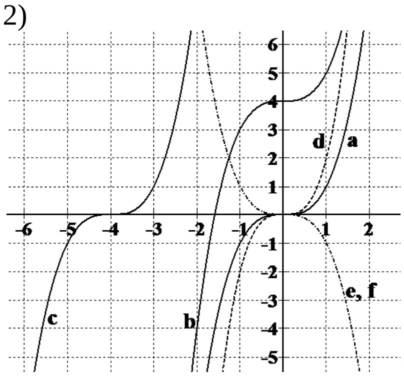
Cartesian Geometry – Part III ANSWERS

Problem Set #4

1)



2)



3)

a) As drawn.
b) Moves up 4.
c) Moves left 4.
d) All the *y*-values are doubled, which stretches it in the *y*-direction, and makes it look thinner.
e) Reflects across the *x*-axis.
f) Reflects across the *y*-axis.

4)

a) $\frac{\pi}{3}$
b) 60°
c) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

5) 45°, $\frac{\sqrt{2}}{2}$

6) 30°, $\frac{1}{2}$

7) 90°, 0

8) 45°, 1

9) 180°, 0

10) 135°, $-\frac{\sqrt{2}}{2}$

11) Answers may vary. We will use: $y = 833\frac{1}{3}x + 6500$ where *y* is dollars and *x* is years after 1982.

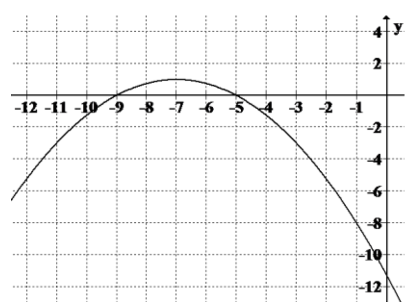
12)

a) $11,500
b) $21,500
c) -$3,500
d) $34,000

13) Answers may vary.

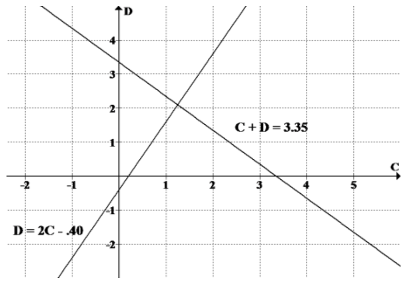
14) That is where the data begins.

15) Roots: -9, -5



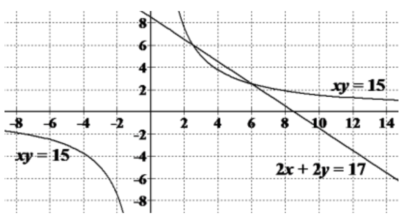
Cartesian Geometry – Part III ANSWERS

16 a)   C+D=3.35; D=2C – 0.40



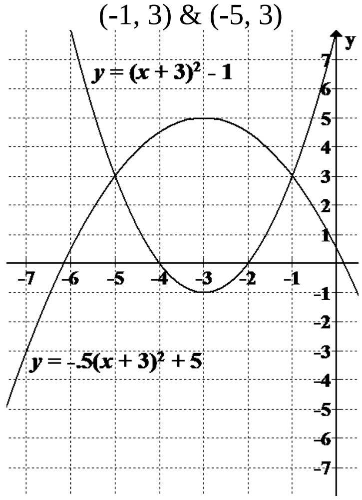
donut = \$2.10; coffee = \$1.25.

16) b) $xy = 15$; $2x + 2y = 17$

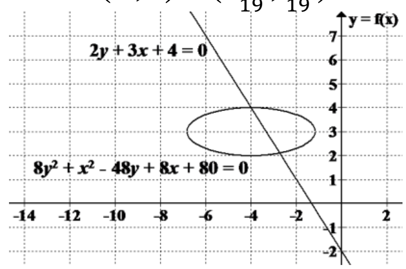


The rectangle is $2\frac{1}{2}$ x 6

17) a)   Points of intersection:
(-1, 3) & (-5, 3)



17) b)   Points of Intersection:
(-4, 4) & ($-\frac{52}{19}$, $\frac{40}{19}$)



Problem Set #5

1)   Answers may vary.

2)

- a)   30° ; $\frac{\sqrt{3}}{2}$
- b)   90° ; 1
- c)   90° ; 0
- d)   60° ; $\sqrt{3}$
- e)   150° ; $\frac{1}{2}$
- f)   135° ; -1 ;
- g)   210° ; $-\frac{2\sqrt{3}}{3}$;
- h)   300° ; $-\frac{2\sqrt{3}}{3}$;

3)

- a) $\frac{\pi}{4}$; $\frac{\sqrt{2}}{2}$
- b) $\frac{2\pi}{3}$; $\frac{\sqrt{3}}{2}$
- c)   0 ; 1
- d) $\frac{3\pi}{4}$; -1
- e) $\frac{3\pi}{2}$; -1
- f) $\frac{4\pi}{3}$; $\frac{\sqrt{3}}{3}$

Cartesian Geometry – Part III ANSWERS

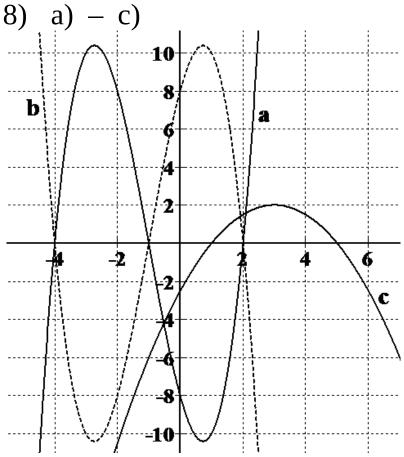
4) D : R = 180 : π

5)
- a) 45°
- b) 330°
- c) 144°
- d) 720°
- e) 270°
- f) ≈114.6°

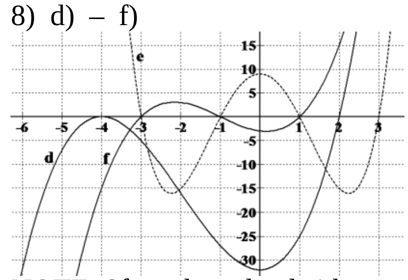
6)
- a) $\frac{\pi}{2}$
- b) $\frac{5\pi}{6}$
- c) $\frac{6\pi}{5}$
- d) 20π

7) $f(x) = 5x^3 + 5x^2 - 30x$.

8) a) – c)

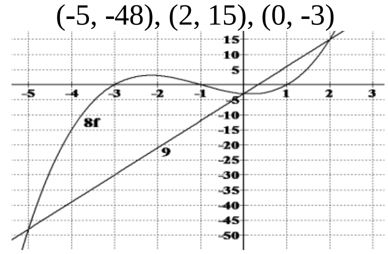


8) d) – f)

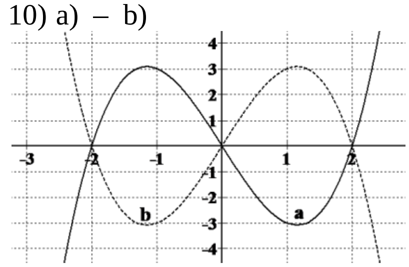


NOTE: 8f can be solved either by factoring or by polynomial long division.

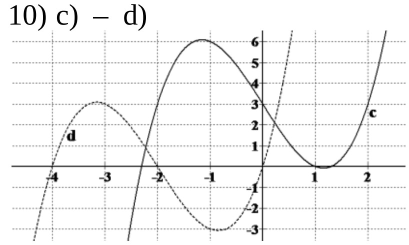
9) Points of intersection: (-5, -48), (2, 15), (0, -3)



10) a) – b)



10) c) – d)



11) No. At 50 feet along the ground from Frank, the ball is on its way down and is only 50 feet above the ground.

Cartesian Geometry – Part III ANSWERS

Problem Set #6

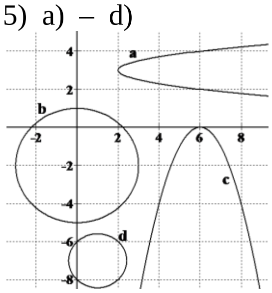
1)
- a) Domain: $\mathbb{R}$ except $x \neq 0$.
  Range: $\mathbb{R}$ except $f(x) \neq 0$
- b) Domain: $\mathbb{R}$.
  Range: $f(x) \geq -7$
- c) Domain: $x \geq 5$.
  Range: $f(x) \geq 10$.

2)
- a) 60°
- b) 315°
- c) 120°
- d) 247.5°
- e) ≈ 57.3°
- f) ≈ 212°

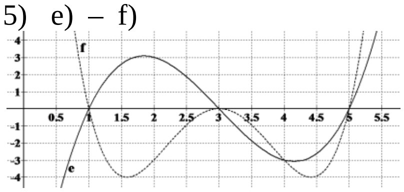
3)
- a) $\frac{\pi}{2}$ c) π
- b) $\frac{11\pi}{6}$ d) $\frac{11\pi}{9}$

4)
- a) $-\frac{1}{2}$ d) -2
- b) $-\frac{\sqrt{2}}{2}$ e) $-\sqrt{2}$
- c) $-\frac{\sqrt{3}}{3}$ f) $-\sqrt{3}$

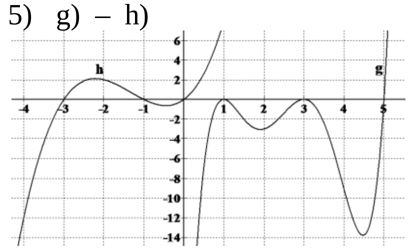
5) a) – d)



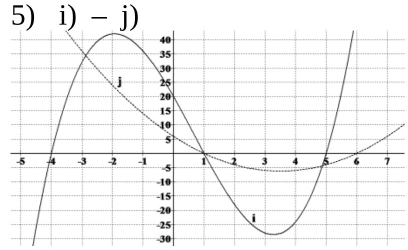
5) e) – f)



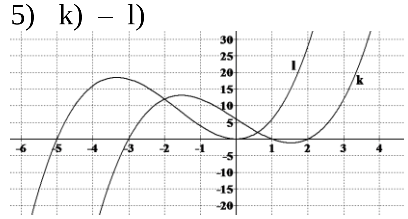
5) g) – h)



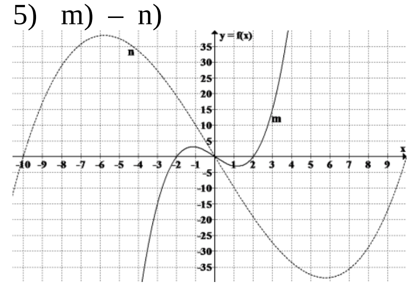
5) i) – j)



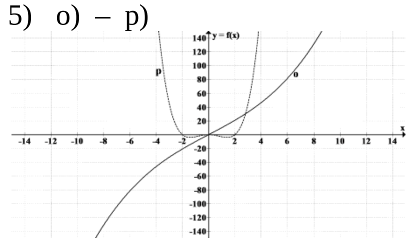
5) k) – l)



5) m) – n)



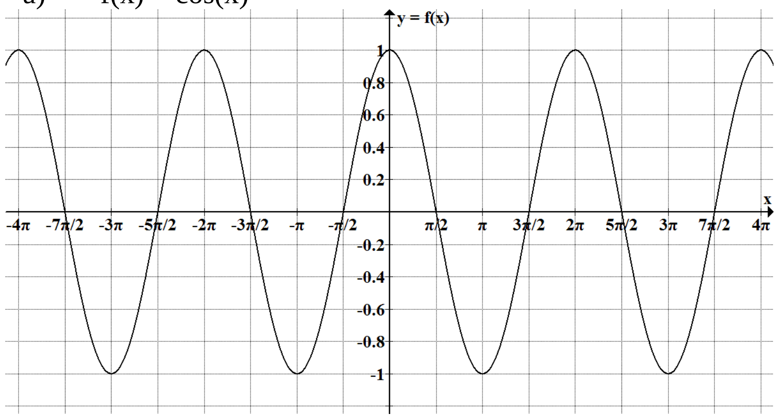
5) o) – p)



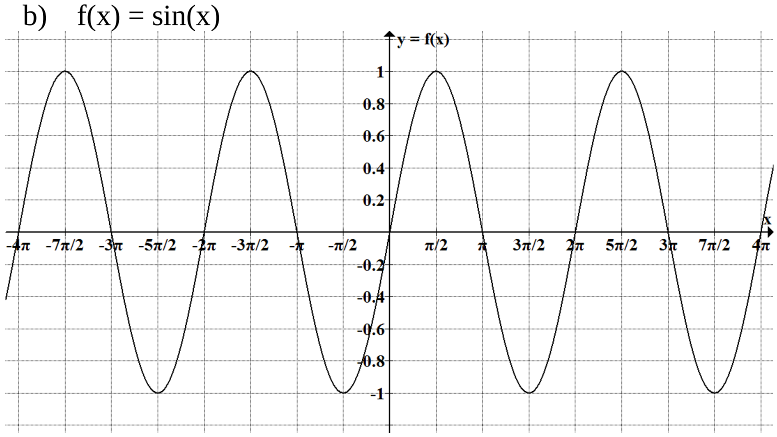
Cartesian Geometry – Part III ANSWERS

6)

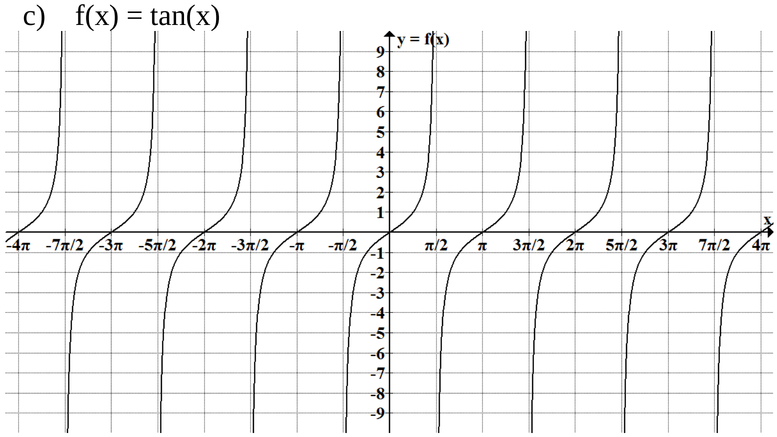
a) f(x) = cos(x)



b) f(x) = sin(x)



c) f(x) = tan(x)



Logarithms – Part III ANSWERS

Problem Set #1

1) 2
2) -1
3) 4
4) 3
5) 0
6) -2
7) 0
8) 0
9) x
10) 0
11) 1
12) 5
13) z
14) 10
15) 3
16) -3
17) ⅓
18) -⅓
19) Undefined.
20) No solution.
21) $\log_b x + \log_b y$
22) $\log_b x - \log_b y$
23) $x\log_b a$
24) x
25) x
26) $\log_b a = \frac{\log_x a}{\log_x b}$
27) 5
28) 1
29) 10
30) 12
31) 14
32) 8
33) $x = 4$
34) $x = 4$
35) $x = 6$
36) $x = 0$
37) $x = -1$
38) $x = \frac{2}{3}$
39) $x = \frac{4}{11}$
40) $x = 3$
41) $x = -10$
42) $x \approx 0.64864$
43) $x \approx 2.8614$
44) $x \approx 2.8614$

Problem Set #2

1) 2
2) -3
3) 0
4) -1
5) 1
6) 9
7) 5
8) 53
9) $\log_b a = \frac{\log_x a}{\log_x b}$
10)
    a) ≈ 2.2146
    b) ≈ 0.5283
    c) $x \approx 2.2146$
11) Take log base-8 of both sides, then use the property from Problem Set #1, Pr #23.
12) $x = 3^{15} = 14{,}348{,}907$
13) $x = 13$
14) $x \approx 1.9459$
15) $x = 64$
16) $x = -\frac{2}{7}$
17) $x = -1$
18) $x = \frac{21}{2} = 10\frac{1}{2}$
19) $x = 21$
20) $x \approx 10.8965$
21)
    a) $5509.91
    b) $5517.93
    c) $5518.65
    d) $5518.6756464
    Note that calculators with smaller memory will give less accurate results. Compare this with the next problem.
    e) $5518.6756467
    Note that calculators with smaller memory will give less accurate results. Compare with the previous problem.
    f) Answers may vary.

22)
- a) First: $r \approx 0.0039624$
  Second: $r \approx 0.0039546$
- b) The first formula is for annual growth rate, and the second formula is for continuous growth rate.
- c) In the year 2313
- d) We assume that the rate of growth will remain constant indefinitely.

Problem Set #3

1) $x \approx 1.8957$
2) $x = 2$
3) $x \approx 2.2732$
4) $x = -\frac{4}{13}$
5) $x \approx -0.2782$
6) $x \approx 0.0587$
7) $x \approx 0.5986$
8) $x = -1, 4$
9) $(e^x - 4)(e^x + 1) = 0$
   Thus $e^x = 4 \rightarrow x \approx 1.3863$
   ($e^x = -1$ has no real solution).
10) $x = 3$
11) $x \approx 33.85$
12)
    - a) ≈ 13.94%
    - b) ≈ 13.863%
13)
    - a) Continuous ≈ 0.0672
      Annual ≈ 0.0695
    - b) ≈ 18 million
    - c) ≈ 265 million (Note that in 2010, Brazil's entire population was about 193 million).
    - d) Answers may vary.
14)
    - a) $9042.78
    - b) ≈ 9.3 years

Problem Set #4

1) $x = -\frac{8}{23}$
2) $x \approx 0.667942$
3) $x \approx 1.391$
4) $x = 11$
5) $x \approx 7.3891$
6) $x \approx -0.83582$
7) $x = 85\frac{1}{3}$
8) $x \approx 2.463$
9) $x = 6, 1$
10) $x = 2$
11) ≈ 6.952%
12) ≈ 930,600,000 years
13) ≈ 13.6%
14)
    - a) ≈ 23.12 years
    - b) ≈ 36.65 years
    - c) ≈ 140.1 years
15) ≈ 19.4 years
16) ≈ $557.71

Problem Set #5

1) $x = 2$
2) $x = 2$
3) $x = 8^{512}$
4) $x = -3$
5) $x = 27$
6) $x = 3\sqrt[3]{3} \approx 4.33$
7) $x = \frac{1}{27}$
8) $x \approx 1.443$
9) $x \approx 0.614$
10) $x \approx -0.614$
11) $x = 432$
12) $x = \frac{5}{2} = 2\frac{1}{2}$
13) $x = \frac{13}{5} = 2\frac{3}{5}$

Logarithms – Part III ANSWERS

14) No real solution.
15) $x \approx 2.65$
16) No real solution.
17) $x \approx -8{,}886{,}108$
18) $x \approx 779$
19) $x = 3, 1$
20) $x \approx 6.614$
21) ≈ 15 years
22)
   a) ≈ 94 million people
   b) ≈ 1.28 billion people
   c) ≈ 7.34%
   d) ≈ 4.65%
   e) During the year 2335

Complex Numbers – Part II

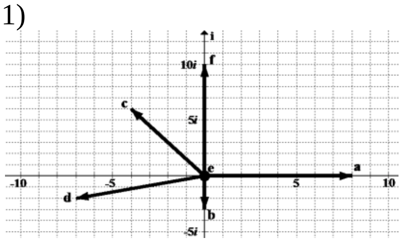
Problem Set #1

1) $11 + 7i$
2) 29
3) $21 + 20i$
4) 50
5) -1
6) $-i$
7) 1
8) -1
9) -9
10) $-1000i$
11) $52 + 16i$
12) $x^2 - 8x + 16$
13) $15 - 8i$
14) $-9 + 46i$
15) $-3i$
16) $\frac{12}{17} - \frac{3}{17}i$
17) $-8 + 6i$
18) $(x + \sqrt{3})(x - \sqrt{3})$
19) $(x + 5i)(x - 5i)$
20) $(x + 3)(x - 3)(x + 3i)(x - 3i)$
21) $x = -8, -5$
22) $x = -6 \pm 2i$
23) $x = \frac{3}{2} \pm \frac{\sqrt{15}}{2}i$
24) $x = \frac{3 \pm \sqrt{33}}{2}$
25) $\frac{1}{2}$
26) $-\frac{\sqrt{3}}{2}$
27) 0
28) $\sqrt{2}$
29) 2
30) $\frac{\sqrt{3}}{3}$
31) 0
32) Undefined.
33) $-\frac{2\sqrt{3}}{3}$
34) $-\sqrt{3}$
35) $-\frac{1}{2}$
36) $-\frac{1}{2}$

Complex Numbers – Part II ANSWERS

Problem Set #2

1)



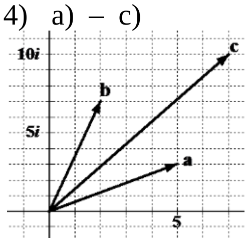
2)

a) 5
b) $\sqrt{5}$
c) $\sqrt{130}$
d) 10
e) 7
f) 10

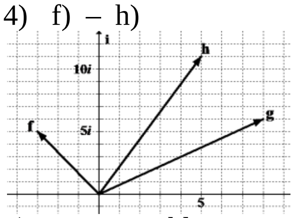
3)

a) 9
b) $\frac{11}{36} + \frac{\sqrt{5}i}{3}$
c) $8i$
d) -1

4) a) – c)

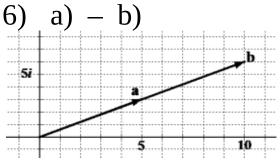


4) f) – h)

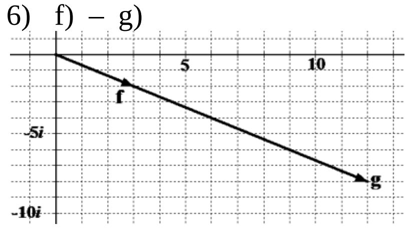


5) One possible answer: The resulting vector is found by placing the two given vectors "tip to tail".

6) a) – b)

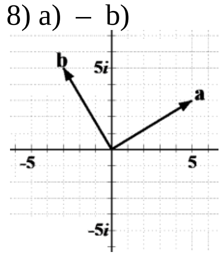


6) f) – g)

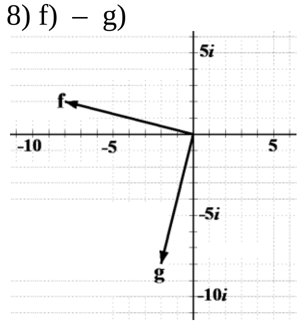


7) The resulting vector maintains the same angle (direction), and its magnitude becomes *n* times greater (where *n* is the real number).

8) a) – b)

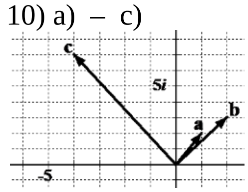


8) f) – g)

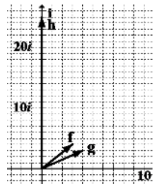


9) Multiplying by *i* simply rotates the vector counterclockwise, by 90°.

10) a) – c)



10) f) – h)



11) The resulting vector has a magnitude equal to the product of the magnitudes of the given vectors, and an angle (formed with the right portion of the real axis) equal to the sum of the two given vectors' angles.

Problem Set #3

1)
   a) $6 + 8i$
   b) $r = 10$
   c) $\theta = \sin^{-1}(\frac{4}{5}) = \cos^{-1}(\frac{3}{5})$
   $= \tan^{-1}(\frac{4}{3}) \approx 53.1°$
   ≈ 0.927 radians.

2)
   a) $5\sqrt{2} + 5\sqrt{2}i$
   b) $5\sqrt{2} + 5\sqrt{2}i$
   c) $1 + \sqrt{3}i$
   d) $7i$
   e) -6
   f) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
   g) $-10i$
   h) $\frac{5}{2} - \frac{5\sqrt{3}}{2}i$

3)
   a) $\text{cis}(135°) = \text{cis}(\frac{3\pi}{4})$
   b) $\text{cis}(30°) = \text{cis}(\frac{\pi}{6})$
   c) $\sqrt{2}\text{cis}(45°) = \sqrt{2}\text{cis}(\frac{\pi}{4})$
   d) $\text{cis}(225°) = \text{cis}(\frac{5\pi}{4})$
   e) $4\text{cis}(330°) = 4\text{cis}(\frac{11\pi}{6})$
   f) $7\text{cis}(90°) = 7\text{cis}(\frac{\pi}{2})$

4) $i(a + bi) = -b + ai$
   $i(r\cdot\text{cis}\ \theta) = r\cdot\text{cis}\ (\theta + \frac{\pi}{2})$

5) $(a + bi)(c + di)$
   $= (ac - bd) + (ad - bc)i$
   $(r_1\cdot\text{cis}\ \theta_1)(r_2\cdot\text{cis}\ \theta_2)$
   $= r_2r_1\cdot\text{cis}(\theta_1+\theta_2)$.
   This means that when we multiply two complex numbers in polar form, we multiply the lengths of the vectors and add the angles.

Problem Set #4

1)
   a) $4\sqrt{3} + 4i$
   b) $\frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$
   c) $-5i$
   d) $-\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{6}i$
   e) $\frac{1}{4} - \frac{\sqrt{3}}{4}i$
   f) i

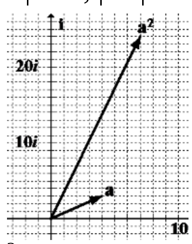
2)
   a) $\text{cis}(120°) = \text{cis}(\frac{2\pi}{3})$
   b) $\approx \sqrt{34}\ \text{cis}\ (59.0°)$
   $\approx \sqrt{34}\ \text{cis}\ (1.03)$
   c) $20\ \text{cis}(315°) = 20\ \text{cis}(\frac{7\pi}{4})$
   d) $\approx 13\text{cis}\ (247.5°)$
   $\approx 13\ \text{cis}(4.32)$

3)
   a) $r\cdot\text{cis}\ \theta = r\cdot\cos\theta + (r\cdot\sin\theta)i$
   b) $a + bi =$
   $\sqrt{a^2 + b^2}\ \text{cis}\ [\tan^{-1}(b/a)]$
   $0 \le \theta \le 2\pi$ so π may need to be added to θ depending on which quadrant the complex number is in and what kind of calculator you are using.

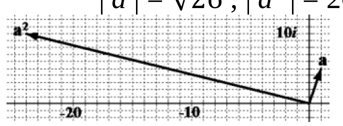
4)
- a) $-3\sqrt{3} - 3i$
- b) $-6\sqrt{2} + 6\sqrt{2}i$
- c) $0.851 - 2.88i$
- d) $31.5 - 24.6i$

5)
- a) $7\sqrt{2}\,\text{cis}\,(135°)$
  $= 7\sqrt{2}\,\text{cis}\,(\frac{7\pi}{4})$
- b) $\approx \sqrt{65}\,\text{cis}\,(240.3°)$
  $\approx \sqrt{65}\,\text{cis}\,(4.19)$

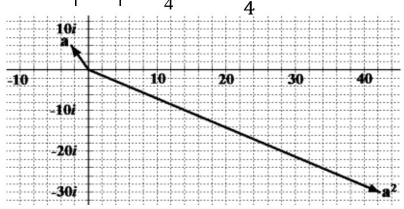
6) a) $a^2 = 7 + 24i$
$|\,a\,| = 5$, $|\,a^2\,| = 25$



b) $a^2 = -24 + 10i$
$|\,a\,| = \sqrt{26}$, $|\,a^2\,| = 26$



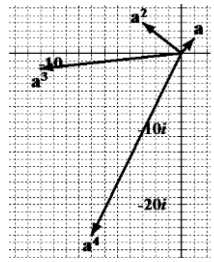
c) $a^2 = -\frac{119}{4} - 30i$
$|\,a\,| = \frac{13}{2} = 6\frac{1}{2}$,
$|\,a^2\,| = \frac{169}{4} = 42\frac{1}{4}$



7) $(r\cdot\text{cis}\,\theta)^2 = r^2\,\text{cis}(2\theta)$. When we square a complex number in polar form, we simply square the length and double the angle.

8) $a^2 = -3 + 4i$, $a^3 = -11 - 2i$, $a^4 = -7 - 24i$

$|\,a\,| = \sqrt{5}$, $|\,a^2\,| = 5$,
$|\,a^3\,| = 5\sqrt{5}$, $|\,a^4\,| = 25$



9) $(r\cdot\text{cis}\,\theta)^n = r^n\,\text{cis}(n\cdot\theta)$. When we take a complex number in polar form to an exponent, we simply take the length to that exponent and then multiply the angle by the exponent.

10)
- a) -16
- b) -16
- c) $500\sqrt{2} + 500\sqrt{2}i$
- d) $500\sqrt{2} + 500\sqrt{2}i$

11) Polar form is easier.

Problem Set #5

1)
- a) $3 + 3\sqrt{3}i$
- b) $\frac{1}{2} - \frac{1}{2}i$
- c) $-5i$
- d) $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$
- e) $\approx 0.684 + 1.88i$
- f) $\approx -7.66 + 6.43i$
- g) $-6.88 + 2.45i$
- h) $-12.7 + 2.80i$

Note that $13\cdot\text{cis}(6.5) =$
$13\cdot\text{cis}(6.5 - 2\pi)$
$\approx 13\cdot\text{cis}(0.217)$

2) a) $\approx 3\sqrt{5}\ \text{cis}(116.6°)$
$\approx 3\sqrt{5}\ \text{cis}(2.03)$

b) $10\,\text{cis}\,(30°) = 10\,\text{cis}\left(\frac{\pi}{6}\right)$

c) $\approx \sqrt{58}\,\text{cis}\,(203.2°)$
$\approx \sqrt{58}\,\text{cis}\,(3.55)$

d) $\text{cis}\,(120°) = \text{cis}\left(\frac{2\pi}{3}\right)$

e) $4\text{cis}\,(225°) = 4\text{cis}\left(\frac{5\pi}{4}\right)$

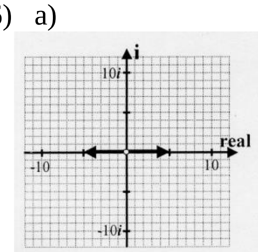
f) $\sqrt{7}\,\text{cis}\,(90°) = \sqrt{7}\ \text{cis}\left(\frac{\pi}{2}\right)$

3) A spiral.

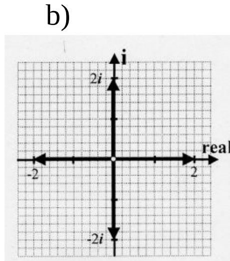
4) If the magnitude > 1,
the spiral expands.
If the magnitude = 1,
the graph stays on the unit circle.
If the magnitude < 1,
the spiral contracts.

5) …fall on the unit circle.

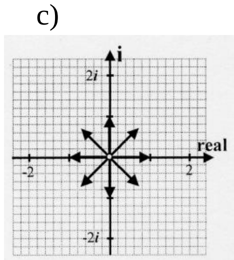
6) a)



b)



c)



7) …fall on the unit circle.

8) $\text{cis}(0) = 1$

$\text{cis}\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\text{cis}\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Problem Set #6

1)

a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \text{cis}(\frac{\pi}{4})$

$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \text{cis}(\frac{5\pi}{4})$

b) $3\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 3\text{cis}(\frac{\pi}{4})$

$-3\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 3\text{cis}(\frac{5\pi}{4})$

c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \text{cis}(\frac{\pi}{4})$

$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \text{cis}(\frac{5\pi}{4})$

$-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \text{cis}(\frac{3\pi}{4})$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \text{cis}(\frac{7\pi}{4})$

d) $2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\text{cis}(\frac{\pi}{4})$

$-2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\text{cis}(\frac{7\pi}{4})$

$2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\text{cis}(\frac{3\pi}{4})$

$-2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\text{cis}(\frac{5\pi}{4})$

e) $-5 = 5\text{cis}(180°)$

$5\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 5\text{cis}(\frac{\pi}{3})$

$5\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 5\text{cis}(\frac{5\pi}{3})$

Complex Numbers – Part II ANSWERS

f) $2i = 2\text{cis}(\frac{\pi}{2})$

$\sqrt{3} - i = \frac{1}{2}\text{cis}(\frac{11\pi}{6})$

$-\sqrt{3} - i = \frac{1}{2}\text{cis}(\frac{7\pi}{6})$

g) $\sqrt{2} + \sqrt{2}i = 2\text{cis}(\frac{\pi}{4})$

$\approx -1.93 + 0.581i = 2\text{cis}\left(\frac{11\pi}{12}\right)$

$\approx 0.518 - 1.93i = 2\text{cis}\left(\frac{19\pi}{12}\right)$

2) Answers may vary. Continue working through this section.

3)

a) 2^{21}

b) $9^{7\log_9 8}$

c) $f(x) = 10^{x\log 5}$

d) $f(x) = e^{x\ln 5}$

e) 3^{2i}

f) $e^{i\ln 13}$

4)

a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

b) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

c) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$

d) i

e) $e^{i\pi} = -1$

5)

a) We know: $e^{i\ln(a)} = a^i$

And: $e^{i\ln(a)} = \text{cis}[\ln(a)]$

Therefore: $a^i = \text{cis}[\ln(a)]$

b) $\left(e^{ix}\right)^i = [\text{cis}(x)]^i$

$e^{-x} = [\text{cis}(x)]^i$

c) From b: $[\text{cis}(x)]^i = e^{-x}$

Multiply from a:

$r^i[\text{cis}(x)]^i = e^{-x}\text{cis}[\ln(r)]$

$[r \cdot \text{cis}(x)]^i = e^{-x}\text{cis}[\ln(r)]$

6)

a) $\text{cis}(\ln 13) \approx \text{cis}(2.56)$
$\approx -0.838 + 0.545\,i$

b) $\text{cis}(\ln 7) \approx \text{cis}(1.95)$
$\approx -0.366 + 0.93\,i$

c) $e^{-\frac{\pi}{4}} \approx 0.456$

d) $e^{-\frac{2\pi}{3}} \approx 0.123$

e) $\text{cis}(\frac{\pi}{2}) = i$ therefore

$i^i = \left[\text{cis}\left(\frac{\pi}{2}\right)\right]^i$

$= e^{-\frac{\pi}{2}} \approx 0.208$

f) $e^{-\frac{\pi}{6}}\text{cis}(\ln 8)$
$\approx 0.592\,\text{cis}(2.08)$
$\approx -0.288 + 0.517\,i$

g) $\left[6\,\text{cis}\left(\frac{2\pi}{3}\right)\right]^i$
$= e^{-\frac{2\pi}{3}}\text{cis}(\ln 6)$
$\approx 0.123\text{cis}(1.79)$
$\approx -0.027 + 0.120\,i$