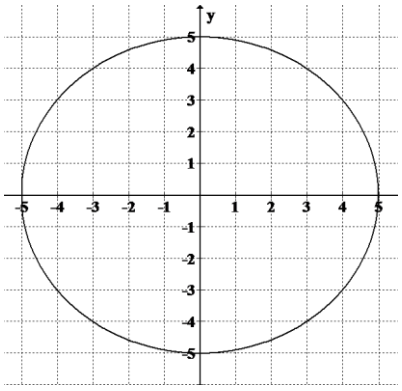
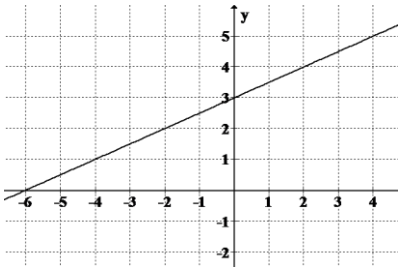
Cartesian Geometry – Part I ANSWERS

Problem Set #1

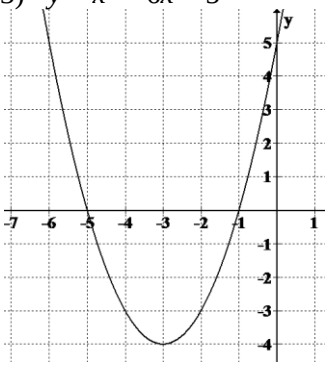
1) $x^2 + y^2 = 25$



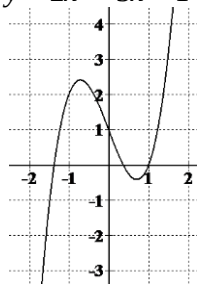
2) $y = \frac{1}{2}x + 3$



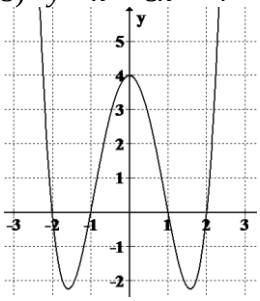
3) $y = x^2 + 6x + 5$



4) $y = 2x^3 - 3x + 1$

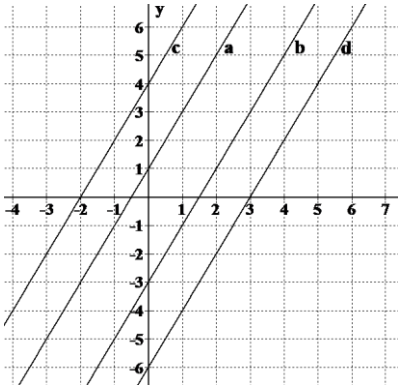


5) $y = x^4 - 5x^2 + 4$



Problem Set #2

1)



2) Where the line hits the *y*-axis.

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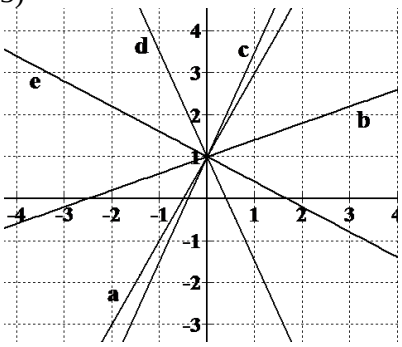
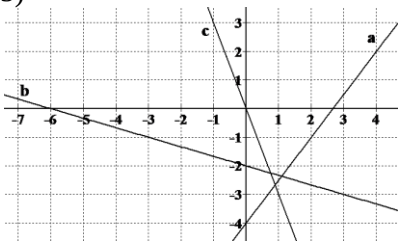
3)



4) How the line tilts (the slope).

5)



Problem Set #3

1)

a) 4

b) $-\frac{5}{2}$

c) $\frac{2}{3}$

d) $\frac{5}{2}$

e) $\frac{2}{5}$

f) $\frac{2}{5}$

2) Lines with negative slopes point up and to the left (and down and to the right) whereas lines with positive slopes point up and to the right (and down and to the left).

3) 1

4) Its slope is less than 1.

5) Its slope is greater than 1.

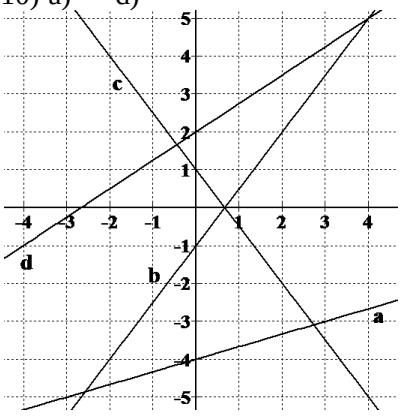
6) The slopes are equal.

7) The slopes are opposite reciprocals.

8) 0

9) Undefined or infinite slope.

10) a) – d)



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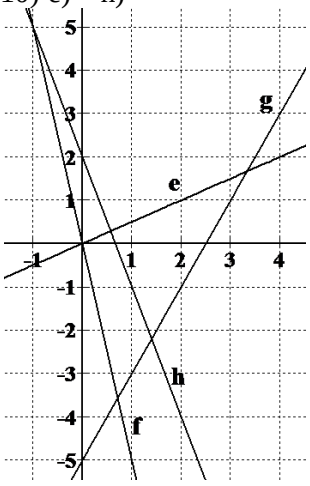
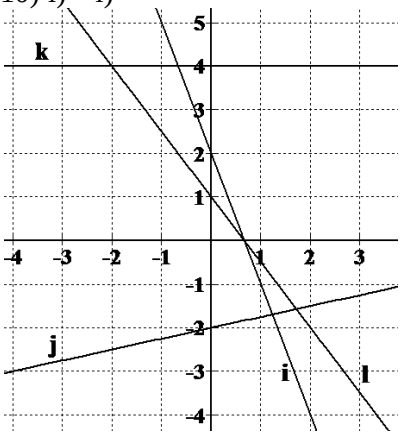
10) e) – h)



10) i) – l)



11) 10c

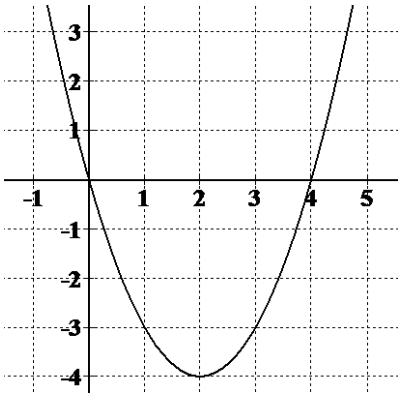
12)

a) (0,0), (-1,5), (1, -3) for example.

b) x can be any real number.

c) $y \geq -4$

d)



e) Yes. It shows that the graph spans all of the x values and does not go below -4 on the y-axis.

13)

a) First equation:

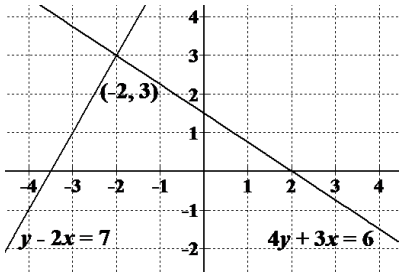
$(0, \frac{3}{2})$, (2, 0), (6, -3)

Second equation:

(0, 7), (1, 9), (-2, 3)

b) (-2, 3)

c)



d) (-2, 3)

Cartesian Geometry – Part I ANSWERS

Problem Set #4

1)

a) $y = \frac{2}{3}x - 9$ or

$-2x + 3y = -27$

b) $y = \frac{1}{2}x$ or $-x + 2y = 0$

c) $y = -\frac{5}{2}x + 6$ or

$5x + 2y = 12$

d) $y = -3x - 7$ or $3x + y = -7$

e) $y = 9$

f) $x = -8$

2)

a) $y = -3x + 1$

b) $y = \frac{2}{3}x + 5$

c) $y = x + 2$

d) $y = \frac{1}{3}x + 3$

e) $y = \frac{4}{3}x - 3$

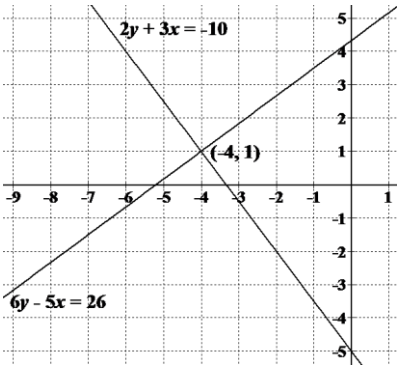
f) $y = \frac{3}{4}x + \frac{1}{2}$

g) $y = \frac{4}{5}x + \frac{1}{5}$

3) $-\frac{1}{2}x + y = 3$ or $-x + 2y = 6$ or

$2y - x - 6 = 0$ for example.

4)



Problem Set #5

1)

a) $y = \frac{1}{3}x + 7$ or $-x + 3y = 21$

b) $y = -\frac{2}{3}x + 2$ or

$2x + 3y = 6$

c) $y = x$

d) $y = -6$

e) $y = -\frac{5}{6}x - \frac{17}{3}$ or

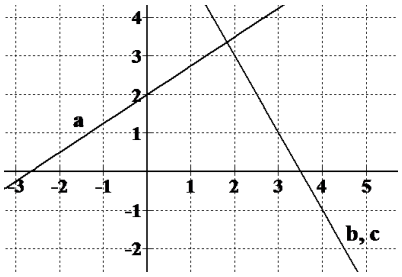
$5x + 6y = -34$

f) $y = -\frac{4}{3}x + \frac{11}{3}$ or

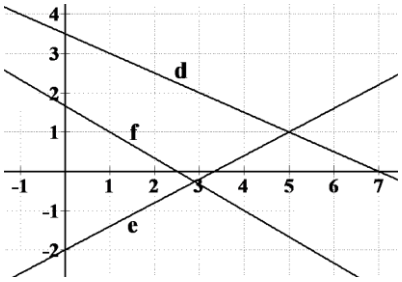
$4x + 3y = 11$

Cartesian Geometry – Part I ANSWERS

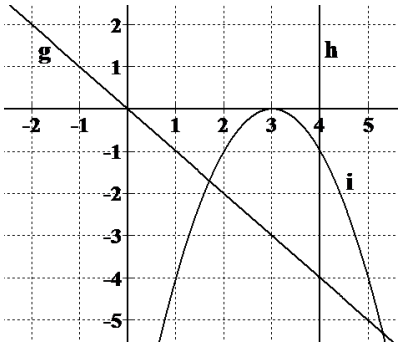
2) a) – c)



2) d) – f)



2) g) – i)



3)

a) $\frac{2}{3}$

b) -4 or (0, -4)

c) 6 or (6, 0)

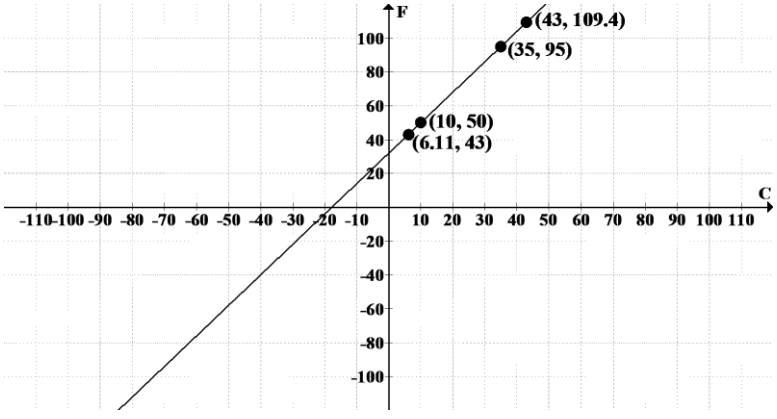
d) $y = -6$ where $x = -3$

e) (0, -4), (6, 0), (-3, -6) for example.

f) (-12, -12)

g) (9, 2)

4)



Cartesian Geometry – Part I ANSWERS

4) cont'd

a) *y*-intercept. This shows the value of F when C = 0.

b) Slope. An increase in 5°C is the same as an increase of 9°F.

c) 35° C

d) 50° F

e) $6\frac{1}{9}°$

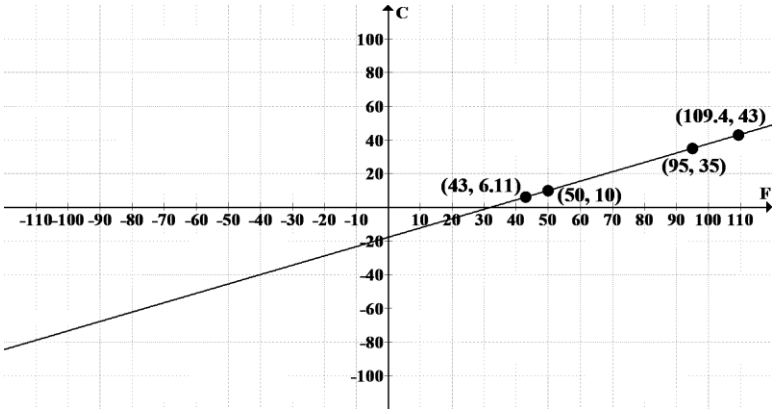
f) $109\frac{2}{5}°$

---

5)

a) *y*-intercept which is $\frac{5}{9}\cdot 32$.

b) Slope.



c) 35° C

d) 50° F

e) $6\frac{1}{9}°$

f) $109\frac{2}{5}°$

6)

a) -40°

b) -40°C is equal to -40°F.

7)

a) $y = \frac{2}{3}x + 6$

b) $Y = -5x + 3$

c) $y = \frac{1}{3}x - 3$

d) $y = \frac{2}{5}x + 2$

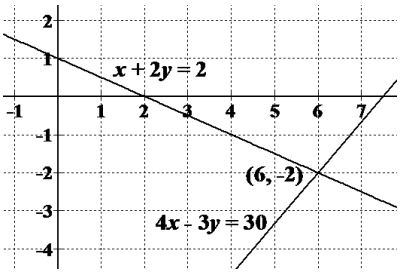
e) $y = \frac{2}{3}x - \frac{2}{3}$

f) $y = -2x + 4$

g) $y = -\frac{4}{3}x + \frac{8}{3}$

8) $(2, -\frac{7}{2})$

9) (6, -2)



Cartesian Geometry – Part I ANSWERS

Problem Set #6

1)

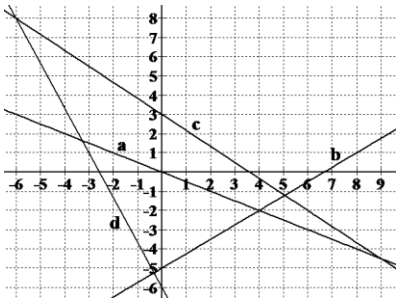
a) $y = -x + 5$

b) $y = -\frac{2}{3}x$

c) $y = \frac{1}{2}x + 2$

d) $y = \frac{5}{4}x - \frac{3}{2}$

2)



3) (-6, 8) See graph in problem 2.

4) $(\frac{96}{19}, -\frac{23}{19})$ or $(5\frac{1}{19}, -1\frac{4}{19})$

See graph in problem 2.

5)

a) $y = \frac{2}{3}x + 5$

b) $y = -\frac{1}{2}x - 4$

c) $y = \frac{1}{2}x + \frac{13}{2}$

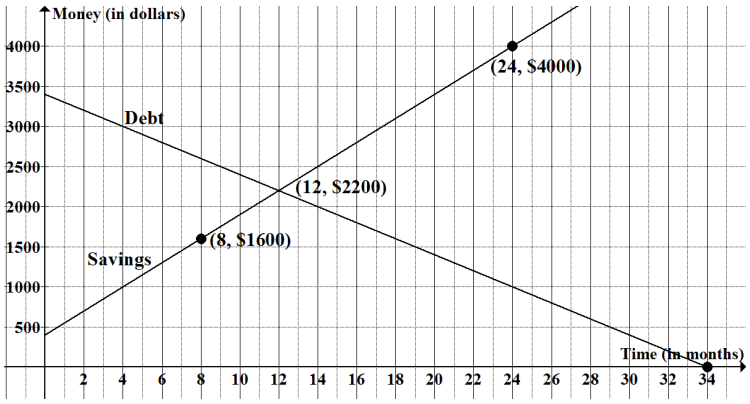
d) $y = -\frac{2}{5}x + 8$

e) $y = \frac{5}{2}x - 21$

6) a) $y = -100x + 3400$

b) $y = 150x + 400$

c)



d) $4000

e) 8 months

f) After 34 months.

g) At (12, 2200). Jason's net worth is $0 at this point.

Trigonometry – Part II ANSWERS

Problem Set #1

1)
   a) $\frac{\sqrt{2}}{2} \approx 0.707$
   b) $\frac{\sqrt{2}}{2} \approx 0.707$
   c) 1
   d) $\frac{1}{2}$
   e) $\frac{\sqrt{3}}{2} \approx 0.866$
   f) $\sqrt{3} \approx 1.732$
   g) $\frac{\sqrt{3}}{2} \approx 0.866$
   h) $\frac{1}{2}$
   i) $\frac{\sqrt{3}}{3} \approx 0.577$
   j) 1
   k) 0
   l) 0
   m) 0
   n) 1
   o) Undefined.

2) Answers may vary.
   a) sin(10°) ≈ 0.174
      cos(10°) ≈ 0.985
      tan(10°) ≈ 0.176
   b) sin(55°) ≈ 0.819
      cos(55°) ≈ 0.574
      tan(55°) ≈ 1.43
   c) sin(35°) ≈ 0.574
      cos(35°) ≈ 0.819
      tan(35°) ≈ 0.700
   d) sin(75°) ≈ 0.966
      cos(75°) ≈ 0.259
      tan(75°) ≈ 3.73
   e) sin(15°) ≈ 0.259
      cos(15°) ≈ 0.966
      tan(15°) ≈ 0.268

Problem Set #2

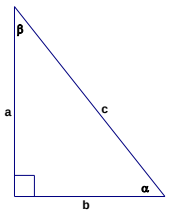
1)
   a) ≈ 0.1736
   b) ≈ 0.985
   c) ≈ 5.6713
   d) ≈ 0.9063
   e) ≈ 0.616
   f) ≈ 0.1405

2)
   a) cos(angle) = $^4/_5$
      sin(angle) = $^3/_5$
      tan(angle) = $^3/_4$
   b) cos(angle) = $\frac{40}{50} = \frac{4}{5}$
      sin(angle) = $\frac{30}{50} = \frac{3}{5}$
      tan(angle) = $\frac{30}{40} = \frac{3}{4}$
   c) cos(angle) = $\frac{8}{10} = \frac{4}{5}$
      sin(angle) = $\frac{6}{10} = \frac{3}{5}$
      tan(angle) = $\frac{6}{8} = \frac{3}{4}$
   d) cos(angle) = $\frac{5}{13}$
      sin(angle) = $\frac{12}{13}$
      tan(angle) = $\frac{12}{5} = 2\frac{2}{5}$
   e) cos(angle) = $\frac{10}{26} = \frac{5}{13}$
      sin(angle) = $\frac{24}{26} = \frac{12}{13}$
      tan(angle) = $\frac{24}{10} = \frac{12}{5} = 2\frac{2}{5}$
   f) cos(angle) = $\frac{45}{53}$
      sin(angle) = $\frac{28}{53}$
      tan(angle) = $\frac{28}{45}$
   g) cos(angle) = $\frac{28}{53}$
      sin(angle) = $\frac{45}{53}$
      tan(angle) = $\frac{45}{28} = 1\frac{17}{28}$

3) Answers may vary.

Trigonometry – Part II ANSWERS

4) Use the triangle below for a) – d) when appropriate. Answers may vary.



a) Using Ptolemy's Difference Formula:

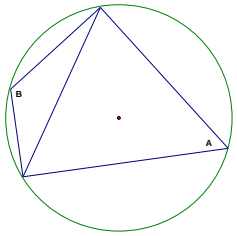
$\sin(180 - \alpha) =$

$\sin(180)\cos(\alpha) -$

$\sin(\alpha)\cos(180)$

$\sin(180 - \alpha) = \sin(\alpha)$

If we think of the angle as inscribed in a circle, then the sine is the ratio of the chord to the diameter. An inscribed angle on the other side of this chord will be supplementary to the original angle and will have the same sine.



sinA = sinB

b) Using Ptolemy's Difference Formula:

$\sin(90 - \alpha) =$

$\sin(90)\cos(\alpha) -$

$\sin(\alpha)\cos(90)$

$\sin(90 - \alpha) = \cos(\alpha)$

This also says that if two angles add to 90°, then the sine of one angle equals the cosine of the other. In the above drawing, we know that $\beta + \alpha = 90°$, and therefore $\beta = 90° - \alpha$. In this way we can conclude that $\sin(\alpha) = \cos(\beta) = \frac{a}{c}$

c) $\sin^2(\propto) = \frac{a^2}{c^2}$

$\cos^2(\propto) = \frac{b^2}{c^2}$

$\sin^2(\propto) + \cos^2(\propto) =$

$\frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$

d) $\frac{\sin(\propto)}{\cos(\propto)} = \frac{\frac{\text{opp}}{\text{hyp}}}{\frac{\text{adj}}{\text{hyp}}} = \frac{\text{opp}}{\text{adj}} =$

$\tan(\propto)$

4 e) Recall that the sine of an inscribed angle is equal to the ratio of the chord this angle subtends and the diameter of the circle. If *d* is the diameter of the circle,

then $\sin(A) = \frac{a}{d}$

$d = \frac{a}{\sin(A)}$

$\sin(B) = \frac{b}{d}$

$d = \frac{b}{\sin(B)}$

$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$

$\frac{a}{b} = \frac{\sin(A)}{\sin(B)}$

5)

a) $x \approx 59.36$

b) $x \approx 37.1$

c) $x \approx 3.53$

Trigonometry – Part II ANSWERS

6)

7) See Problem Set #1, Pr #1

Problem Set #3

1)

a) ≈ 0.766
b) ≈ 4.33
c) ≈ 0.839
d) ≈ 0.996
e) ≈ 0.087
f) ≈ 0.087
g) ≈ 0.087
h) ≈ 11.43
i) ≈ 0.996
j) ≈ 0.996
k) $= \frac{\sqrt{3}}{2} \approx 0.866$

2)

a) $\sin^2(27°) + \cos^2(27°) = 1.$
$\cos(27°) = \sqrt{1 - 0.454^2}$
≈ 0.891
$\tan(27°) = \frac{\sin(27°)}{\cos(27°)} \approx 0.506$

b) $\sin(86°) \approx 0.998$
$\tan(86°) \approx 14.3$

c) $\tan(13°) \approx 0.231$. Thus $\frac{\text{opp}}{\text{adj}} = \frac{0.231}{1}$. Pythagorean Theorem: hyp ≈ 1.03.
$\sin(13°) \approx \frac{0.231}{1.0263} \approx 0.225$
$\cos(13°) \approx 0.974$

3) $\tan(\propto) = \frac{1}{\tan(\beta)}$

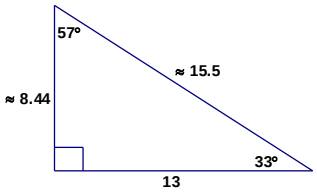
4)

a) 45°, 135°
b) 60°
c) 45°
d) 90°
e) 30°
f) 60°

5)

a) $x \approx 6.58$
b) $x \approx 12.31$
c) $x \approx 6.14$
d) $x \approx 8.4$
e) $x \approx 4.1$

6)



7) See Problem Set #1; Pr #1

Problem Set #4

1)

a) ≈ 0.342
b) ≈ 0.9397
c) ≈ 2.7475
d) ≈ 0.9397
e) ≈ 0.342
f) ≈ 0.364
g) ≈ 1.11
h) ≈ 0.8192
i) ≈ 0.9657
j) $\frac{\sqrt{2}}{2} \approx 0.7071$
k) $\frac{1}{2}$
l) 0

Trigonometry – Part II ANSWERS

2)

a) $\cos(58°) \approx 0.530$
$\tan(58°) \approx 1.60$

b) $\sin(14°) \approx 0.243$
$\tan(14°) \approx 0.250$

c) $\sin(49°) \approx 0.755$
$\cos(49°) \approx 0.656$

3)

a) $\sin\theta = \sqrt{1 - \cos^2\theta}$

b) $\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

4)

a) Draw a right triangle with a leg of length 1 and a hypotenuse of length 4. This shows that $\sin^{-1}(0.25) \approx 14.5°$, 165.5°

b) $\approx 26°$

c) $\approx 40.5°, 139.5°$

d) $\approx 49.5°$

e) $\approx 26.5°$

f) $\approx 63.5°$

5)

a) $x \approx 5.71$

b) $x \approx 17.1$

c) $x \approx 25.3$

d) $x \approx 6.4$

e) $x \approx 2.73$

f) $x \approx 16.3$

g) $x \approx 10.7$

h) $x \approx 9.4$, $y \approx 12.7$

6) ≈ 602 m

7) See Problem Set #1 – Pr #1

Problem Set #5

1)

a) $\sqrt{3}$
b) $\frac{\sqrt{3}}{2}$
c) ≈ 0.47
d) $\frac{\sqrt{3}}{2}$
e) $\frac{\sqrt{2}}{2}$
f) $\frac{\sqrt{3}}{3}$
g) $\frac{1}{2}$
h) 1
i) ≈ 0.42
j) ≈ 0.82
k) 0
l) ≈ 0.91
m) 0
n) Undefined.
o) 0
p) ≈ 0.91
q) 1
r) $\frac{\sqrt{2}}{2}$
s) ≈ 1.43
t)
u) ≈ 0.57
v) 1

2)

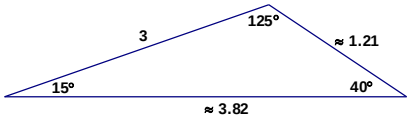
a) 30°, 150°
b) 45°
c) $\approx 14.5°$, $\approx 165.5°$
d) 90°
e) 45°, 135°
f) $\approx 36.9°$
g) 60°
h) 45°
i) $\approx 26.6°$
j) 60°, 120°
k) 0°
l) $\approx 75.5°$
m) 90°
n) 0°, 180°
o) 30°
p) 60°
q) $\approx 84.3°$
r) 30°
s) $\approx 14°$
t) $\approx 76°$
u) 0°
v) Und.

3)

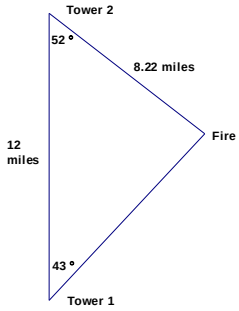
a) $x \approx 73.2$
b) $\theta \approx 25.8°$
c) $x \approx 21.1$
d) $\theta \approx 43.6°$
e) $x \approx 8.11$
f) $\theta \approx 59°$
g) $x \approx 14.6$
h) $x \approx 5.2$
i) $x \approx 4.54$
j) $\theta \approx 57.1°$

Trigonometry – Part II ANSWERS

4)



5) ≈ 4.16 miles
6) ≈ 327 m
7) ≈ 58.3°
8)



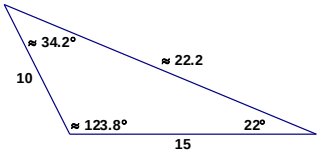
The closer tower is ≈ 8.22 miles from the fire.

Problem Set #6

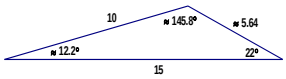
1)

a) $x \approx 30.6$
b) $\theta \approx 32.9°$
c) $x \approx 18.2$
d) $x \approx 2.13$
e) $x \approx 26.3$
f) $\theta \approx 18.3°$
g) $\theta \approx 43.7°$
h) $x \approx 4.24$
i) $\theta \approx 71.8°$
j) $x \approx 12.5$

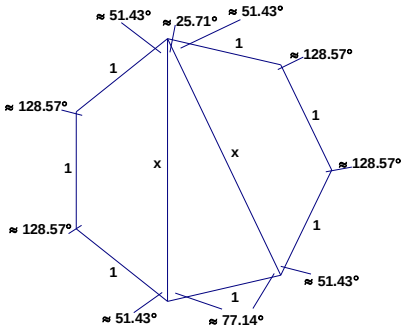
2)



OR



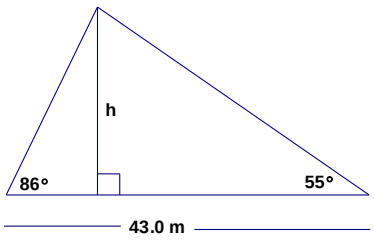
3) ≈ 43.3 ft.
4) ≈ 16.32 ft.
5) ≈ 49.27°
6) $x \approx 2.247$.



7)

a) ≈ 51.84°
b) ≈ 719 ft.
c) ≈ 58.3°. See problem #7 from the previous problem set.

8) Setup the problem like:



NOTE: Not in correct proportion.
Use the Law of Sines to find one of the other sides of the big triangle and you should easily be able to derive h ≈ 55.8 m.