Proofs ANSWERS

Problem Set #1

1) $^5/_4$
2) 6
3) 6
4) $\frac{y+3}{3}$
5) $^x/_y$ <u>or</u> $^5/_3$
6) The exterior angles of any polygon add to 360°.
7) 1. Givens as stated.
   2. AC ≅ CB (def. of midpoint)
   3. ∠DAC ≅ ∠ECB (Corresponding ∠ th.)
   4. ΔDAC ≅ ΔECB (SAS ≅ th.)
   5. ∠DCA ≅ ∠EBC (def. ≅ figures)
   6. ∴CD || BE (Corresponding ∠ th. Converse)
8) 1. Given parallelogram ABCD.
   2. Draw BD (post. 1)
   3. AD || BC ; AB || CD (def. of parallelogram)
   4. ∠ABD ≅ ∠BDC (alternate interior ∠ th.)
   5. ∠CBD ≅ ∠ADB (alternate interior ∠ th.)
   6. ΔCBD ≅ ΔADB (ASA ≅ th.)
   7. ∴AB ≅ CD; BC ≅ AD (def. ≅ figures)
9) 1. Given //gram ABCD.
   2. Draw BD (post. 1)
   3. AD || BC ; AB || CD (def. of parallelogram)
   4. ∠ABD ≅ ∠BDC (alternate interior ∠ th.)
   5. ∠CBD ≅ ∠ADB (alternate interior ∠ th.)
   6. ΔCBD ≅ ΔADB (ASA ≅ th.)
   7. ∴∠A ≅ ∠C (def. ≅ figures)
   8. Similarly, AC can be drawn to show that ∠B ≅ ∠D.
10) 1. Givens as stated.
   2. AB ≅ CD (parallelogram side th.)
   3. ∠A ≅ ∠C (parallelogram ∠ th.)
   4. ΔABE ≅ ΔCDF (SAS ≅ th.)
   5. ∴BE ≅ FD (def. ≅ figures)
11) 1. Givens as stated.
   2. ∠2 + ∠1 = 180° (Supplementary ∠ th.)
   3. ∠5 + ∠6 = 180° (Supplementary ∠ th.)
   4. ∠2 + ∠1 = ∠5 + ∠6 (C.N.1 or Transitive)
   5. ∠2 ≅ ∠5 (parallelogram ∠ th.)
   6. ∠1 = ∠6 (C.N.3 or subtraction)
   7. BE = FD (parallelogram side th.)
   8. ΔBEA ≅ ΔDFC (SAS ≅ th.)
   9. ∴∠A ≅ ∠C (def. ≅ figures)
12) Diagonals bisect each other.
   1. Given parallelogram ABCD.
   2. AD || BC (def. of parallelogram)
   3. ∠ADB ≅ ∠CBD (Alternate Int. ∠ th.)
   4. AD ≅ BC (parallelogram side th.)
   5. ∠DEA ≅ ∠BEC (Vertical ∠ th.)

Proofs ANSWERS

6. ΔDEA ≅ ΔBEC (AAS ≅ th.)
7. DE ≅ EB; AE ≅ EC (def. ≅ figures)
8. ∴ The diagonals of the parallelogram ABCD bisect each other. (def. of bisect)

13)
1. Given rectangle ABCD
2. ∠ A, ∠ B, ∠ C, ∠ D are all right (90°). (Def of rectangle)
3. ∠A + ∠B = 180° and ∠A + ∠D = 180° (CN2)
4. ∴ **BC ∥ AD and AB ∥ CD** (Elements I-28, <u>or</u> converse of Same-Side-Interior Angle Theorem)
5. Draw AC (Post 1)
6. ∠BCA ≅ ∠DAC (Elements I-29a, <u>or</u> Alt. Interior Angle Th.)
7. AC ≅ AC (Reflexive Property)
8. ΔABC ≅ ΔACD (HL Δ Congruency Th.)
9. ∴ **AB ≅ CD and BC ≅ AD** (Def. of ≅ figs)

14)
1. Givens as shown.
2. ∠BDE ≅ ∠BAC (Corresponding ∠ th.)
3. Δ BDE ~ ΔBAC (AA ~ th.)
4. BD:BE = BA:BC (def ~ figures)
5. BD:BA = BE:BC (Th. V-16)
6. ∴BD:DA = BE:EC (Th. V-17)

Problem Set #2

1) $\frac{2}{3}$
2) $\frac{a-7}{3}$
3) $\frac{6}{20} = \frac{3}{10}$
4) $\frac{y-3}{3}$
5) $\frac{x}{5}$ or $\frac{4}{y}$ or $\frac{z}{w}$
6) AB = 21; CE = 12; CB = 28
7) BD = 27; CE = 12; BE = 36
8) AD = 1; CE = 1½; CB = 7½
9) BD = 16; AD = 8; CB = 30
10) Answers may vary.

11)
1. Givens as stated.
2. ∠A ≅ ∠E (alternate interior ∠ th.)
3. ∠B ≅ ∠D (alternate interior ∠ th.)
4. ∴ΔABC ≅ ΔEDC (ASA ≅ th.)

12)
1. Givens as stated.
2. AC ≅ CE (def. of midpoint)
3. BC ≅ CD (def. of midpoint)
4. ∠ACB ≅ ∠ ECD (Vertical ∠ th.)
5. ∴ΔABC ≅ ΔEDC (SAS ≅ th.)

13) The diagonals are congruent.
1. Given rectangle ABCD.
2. ∠ADC, ∠BCD are right (def. of rectangle)
3. ∠ADC ≅ ∠BCD (Post. 4)
4. AD ≅ BC, AB ≅ DC (rectangle side th.)
5. ΔADC ≅ ΔBCD (SAS ≅ th.)
6. ∴AC ≅ BD (def. ≅ figures)

Proofs ANSWERS

14) 1. Givens as shown.
2. ∠CXA ≅ ∠FXD (vertical ∠ th.)
3. ΔAXC ≅ ΔDXF (SAS ≅ Th.)
4. ∠A ≅ ∠D (def. ≅ figures)
5. ∠BXA ≅ ∠EXD (vertical ∠ th.)
6. ΔBXA ≅ ΔEXD (ASA ≅ th.)
7. ∴BX ≅ XE (def. ≅ figures)

15) a) The diagonals are perpendicular.
1. Given rhombus ABCD.
2. DE ≅ EB (Parallelogram diagonal th.)
3. DC ≅ BC (def. of rhombus)
4. ΔDEC ≅ ΔBEC (SSS ≅ th.)
5. ∠DEC ≅ ∠BEC (def. ≅ figures)
6. ∴EC ⊥ DB (def. perpendicular)

b) rhombus = $32\sqrt{3}$; rectangle = $64\sqrt{3}$

c) rhombus:rectangle = 1:2

16) 1. Givens as stated.
2. ΔABC ≅ ΔACD (SSS ≅ th.)
3. ∠7 ≅ ∠8 (def. ≅ figures)
4. ΔBCE ≅ ΔDCE (SAS ≅ th.)
5. ∴EB ≅ ED (def. ≅ figures)

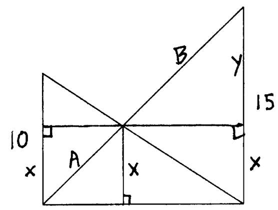
17) 1. Givens as stated.
2. ∠1 ≅ ∠4 (alternate interior ∠ th.)
3. ΔABD ≅ ΔCDB (SAS ≅ th.)
4. ∠2 ≅ ∠3 (def. ≅ figures)
5. ∴BC || AD (alternate interior ∠ converse)

18) 1. Givens as stated.
2. ∠ADC ≅ ∠CBA (parallelogram angle th.)
3. ∠5 ≅ ∠4 (C.N.3 or subtraction)
4. BC ≅ AD (parallelogram side th.)
5. BC || AD (def. parallelogram)
6. ∠7 ≅ ∠2 (alternate interior ∠ converse)
7. ∴ΔADY ≅ ΔCBX (ASA ≅ th.)

19) 1. Given ΔABC; AD bisects ∠BAC
2. ∠1 ≅ ∠2 (def. bisect)
3. Draw line ***ℓ*** through B parallel to AD (Th. I-31)
4. Extend AC to intersect with line ***ℓ*** at point Q (Post 2)
5. ∠2 ≅ ∠Q (corresponding ∠ th.)
6. ∠1 ≅ ∠ABQ (alternate interior ∠ th.)
7. ∠Q ≅ ∠ABQ (C.N.1)
8. QA ≅ AB (isosceles triangle converse)
9. DC:BD = AC:QA (Δ proportionality th.)
10. ∴DC:BD = AC:AB (substitution)

Proofs ANSWERS

20)



1. Given the diagram above, by similar triangles, we know A:B = 10:15, and also A:B = X:Y. Therefore X:Y=10:15 → X = ⅔Y
2. X+Y = 15
3. ⅔Y + Y = 15
4. Y = 9; X = 6.

21)

a) 250,000 stadia.
b) About 0.86% error.

Problem Set #3

1)
1. Givens as stated.
2. ∠BCF ≅ ∠DCG (vertical ∠ th.)
3. ΔBCF ≅ ΔDCG (AAS ≅ th.)
4. BF ≅ GD (def. ≅ figures)
5. ∠BFC + ∠BFA = 180°; ∠CGD + ∠EGD = 180° (Suppl. ∠ Th.)
6. ∠BFC ≅ ∠CGD (post 4)
7. ∠BFA = 90°; ∠DGE = 90° (C.N.3 or subtraction)
8. ∠BFA = ∠DGE (C.N.1 <u>or</u> post. 4 <u>or</u> transitive)
9. ΔBFA ≅ ΔDGE (SAS ≅ th.)
10. ∴∠A ≅ ∠E (def. ≅ figures)

2)
1. Given ΔABC; D is midpoint of AC; E is midpoint of BC.
2. CD ≅ DA; CE ≅ EB (def. of midpoint)
3. CA=CD+DA; CB=CE+EB (seg. add. post)
4. CA=CD+CD; CB=CE+CE (substitution)
5. CA=CD+CD → CA=2CD → CA:CD = 2:1; CB=CE+CE → CB=2CE → CB:CE = 2:1 (algebra)
6. CA:CD = CB:CE (transitive or CN1)
7. CA:CB = CD:CE (Th. V-16)
8. ΔACB ~ ΔDCE (SAS ~ th.)
9. ∠CED ≅ ∠B (def. ~ figures)
10. ∴**DE || AB** (corresponding ∠ converse)
11. CA:AB = CD:DE (def. ~ figures)
12. CA:CD = AB:DE (Th. V-16)
13. ∴**AB:DE = 2:1** <u>or</u> **DE=½AB** (transitive or C.N.1)

3) BD=6; AB=9; CE=4

4) AD=10⅔; AB=26⅔; BE=15

5) AB=10; BE=3.3; CE=7.7

6) BD=6; AB=9¾; CB=26

Proofs ANSWERS

7) a) CD=14.4 b) AC=25
c) 23⅓

8)
1. Givens as stated.
2. ∠BAC ≅ ∠BCA (isosceles Δ Th.)
3. ∠2 ≅ ∠4 (isosceles Δ Th.)
4. ∴∠1 ≅ ∠3 (C.N.3 or subtraction)

9)
1. Givens as stated.
2. ∠BAC ≅ ∠BCA (isosceles Δ Th.)
3. ∠2 ≅ ∠1; ∠3 ≅ ∠4 (def. bisect)
4. ∠BAC = ∠1+∠2; ∠BCA = ∠3+∠4 (angle addition post.)
5. ∠BAC = 2(∠2); ∠BCA = 2(∠4) (substitution)
6. 2(∠2) = 2(∠4) (transitive <u>or</u> C.N.1)
7. ∠2 = ∠4 (algebra)
8. ∴AD ≅ CD (isosceles Δ converse)

10) Yes, it does work! This is theorem VI-31 from *The Elements.*

11) 7

12) $\sqrt{5c}$

13) $\frac{4w}{3y}$

14) $\frac{7}{w+7}$

15) $\frac{3-x}{x}$

16) a) $y = 4\frac{4}{5}$; $x = 5\frac{5}{7}$

b) $y = \frac{70}{3}$; $x = 14$

17) $PQ = \frac{20}{3}$ (because BQ:QE = BC:YE)
$CY = 5\sqrt{13}$
<u>Explanation</u>: Draw ZY, where Z is the midpoint of BC. The center of the hexagon is W. Triangle ZWC is a 30-60-90 triangle. $ZW=5\sqrt{3}$; $ZY=10\sqrt{3}$.
Using triangle ZYC, we get $CY=5\sqrt{13}$

18)
1. Given.
2. Th. I-23
3. Th. III-21
4. Th. I-32b
5. C.N.3 [note: there's no AA ~ th. in Euclid]
6. def. of equiangular [similar]
7. Th. VI-4
8. Th. VI-16
9. C.N.2
10. from drawing
11. Th. III-21
12. Th. I-32b
13. C.N.3
14. def. of equiangular [similar]
15. Th. VI-4
16. Th. VI-16
17. C.N.2 [from steps 8 & 16]
18. Th. V-1
19. from drawing

Problem Set #4

1) a) 31°

b) $\frac{180° - x}{2} = 90° - ½x$

c) 180° – 2y

2) a) 90° b) 90°

Proofs ANSWERS

3)
1. Givens as stated.
2. BD ≅ AE (rectangle diagonal th.)
3. BD ≅ AC (parallelogram side th.)
4. AE ≅ AC (transitive or C.N.1)
5. ∴ΔACE is isosceles (def. isosceles)

4)
a) BF=CF=15; XF = 9; BX=DX=12; EX=16; ED=EB=20
b) All the triangles are similar to each other!
c) All angles are 90°, 36.9°, or 53.1°, except ∠AED ≈ 106.2°, and ∠DFC ≈ 73.8°

5) ∠A=∠B=∠5=108°; ∠F=∠6=90°; ∠7=162°; ∠8=∠9= 9° ; ∠1=∠2=∠3=∠4=45°; ∠10=∠12=36°; ∠11=∠13=72°

6)
1. Givens as stated.
2. AB≅BC≅AC (def. equilateral)
3. ∠A≅∠B≅∠C (Isosceles triangle th.)
4. ∠DCB≅∠FBA≅∠EAC (Subtraction or C.N.3)
5. ΔDCB≅ΔFBA≅ΔEAC (ASA ≅ th.)
6. DB≅AF≅EC; DC≅FB≅EA (def. ≅ figures)
7. DE≅DF≅EF (Subtraction or C.N.3)
8. ∴ΔDEF is equilateral (def. equilateral)

7) Because ∠1 ≅ ∠2 and ∠7 ≅ ∠8, we know that ΔABC ≅ ΔADC, which means that BC≅CD. Now we can say that ΔBCE ≅ ΔDCE. Therefore ∠5≅∠6 and AC bisects ∠BED.

8)
1. Given ΔABC; ∠ACB is a right angle; D is the midpoint of AB.
2. Draw CD. (Post 1)
3. Draw a line through D parallel to AC meeting BC at E, and a line through D parallel to BC meeting AC at F. (Th. I-31)
4. ∠BCA ≅ ∠BED = 90°; ∠BCA ≅ ∠DFA = 90° (corresponding ∠ th.)
5. ΔBED ~ ΔBCA; ΔDFA ~ ΔBCA (AA ~ th.)
6. BD=DA → BD:DA=1:1 (def. of midpoint)
7. BE:EC=1:1 → BE=EC AF:FC=1:1 → AF=FC (Δ proportionality th.)
8. ΔCFD ≅ ΔAFD; ΔBED ≅ ΔCED (SAS ≅ th.)
9. BD≅CD; AD≅CD (def. ≅ figures)
10. **∴ΔBDC, ΔCDA are isosceles** (def. isosceles)
11. EDFC is a parallelogram (def. ▱)
12. ED ≅ CF; EC ≅ DF (▱ side th.)
13. area ΔCDA = ½CA·DF = CF·DF; area ΔBDC = ½BC·ED = EC·ED (def. area of triangle)
14. area ΔBDC = EC·ED = DF·CF (substitution)
15. **∴area ΔBDC = area ΔCDA** (transitive or C.N.1)

Proofs ANSWERS

9) XY = $\frac{\sqrt{3}}{8}$; CY = $\frac{\sqrt{7}}{12}$

10) ∠6 = 72°; ∠2 = 36°

11) 1. Givens as stated.
2. ∠5 ≅ ∠6 (isosceles triangle th.)
3. ∠6 = ∠3 + ∠B; ∠5 = ∠1 + ∠A (Δ exterior angle th.)
4. ∠3 + ∠B = ∠1 + ∠A (C.N.1)
5. ∠A = ∠B (C.N.3)
6. AC ≅ CB (isos. Δ conv.)
7. ∴ΔABC is isosceles (def. isosceles)

12) 1. Givens as stated.
2. ∠BDA, ∠BCA are rt. angles (Th. of Thales)
3. ∠BDA = ∠BCA (Post 4)
4. ∠DAB = ½arcBD; ∠DAC = ½arcDC (Inscribed Angle Th.)
5. ∠DAB ≅ ∠DAC (transitive or C.N.1)
6. ΔDBA ~ ΔCEA (AA Th.)
7. AD:BD = AC:CE (def. similar figures)
8. AB:BE = AC:CE (Th. V-16)
9. AB:AC=BE:CE (∠Angle Bisector Th.)
10. AB:BE = (AB+AC):(BE+CE) (Th. V-12)
11. AD:BD = (AB+AC):(BE+CE) (Transitive)
12. ∴**AD**:**BD = (AB+AC):BC** (segment add post.)

13) a) X = $\frac{24}{11} = 2\frac{2}{11}$; Y = $\frac{77}{8} = 9\frac{5}{8}$

b) X = $\frac{32}{11} = 2\frac{10}{11}$; Y = $\frac{12}{11} = 1\frac{1}{11}$

14) a) b = 4½ b) a = $\frac{20}{3}$ = 6⅔

c) c = $\frac{16}{5} = 3\frac{1}{5}$

15) 1. Givens as stated.
2. EB ≅ EC (isosceles triangle converse)
3. AE ≅ ED (isosceles triangle converse)
4. ∴ΔABE ≅ ΔDCE (SSS ≅ th.)

16) 1. Givens as stated.
2. ∠5 ≅ ∠2; ∠6 ≅ ∠3 (corresponding ∠ th.)
3. ∠5 ≅ ∠6 (transitive or C.N.1)
4. BE≅CE; AE≅ED; EF≅EG (isosceles Δ converse)
5. ∠F + ∠2 = 180° → ∠F = 180° – ∠2; ∠G + ∠3 = 180° → ∠G = 180° – ∠3 (supplementary ∠th.)
6. ∠G = 180° – ∠2 (substitution)
7. ∠G ≅ ∠F (transitive or C.N.1)
8. ΔAFE ≅ ΔDGE (AAS ≅ th.)
9. AF ≅ GD (def. ≅ figures)
10. FB ≅ GC (subtraction or C.N.3)
11. ∠9 ≅ ∠2; ∠12 ≅ ∠3 (vertical ∠ th.)
12. ∠9 ≅ ∠12 (transitive or C.N.1)
13. ΔAFB ≅ ΔGDC (SAS ≅ th.)
14. ∴∠7 ≅ ∠8 (def. ≅ figures)

Proofs ANSWERS

17) Draw a line through A that is parallel to BC intersecting DC at a point labeled E. ABCE is a parallelogram. AE is congruent to BC, and therefore also congruent to AD, which makes ∠AED is congruent to ∠D. And since ∠AED is congruent to ∠C, ∠C must be congruent to ∠D.

18)
1. Given DF,CF,BH,AH are ∠ bisectors of ABCD.
2. ∠DAE = ∠BAE; ∠CDE = ∠ADE (def. bisect)
3. ∠DAB = ∠DAE + ∠BAE; ∠ADC = ∠CDE + ∠ADE (segment add. post.)
4. ∠DAB=∠DAE+∠DAE → ∠DAB=2∠DAE
∠ADC=∠ADE+∠ADE → ∠ADC=2∠ADE (substitution)
5. DC ∥ AB (def. parallelogram)
6. ∠DAB+∠ADC = 180° (same-side int. ∠ th.)
7. 2∠DAE + 2∠ADE = 180° (substitution)
8. ∠DAE + ∠ADE = 90° (division by 2)
9. ∠DEA + ∠DAE + ∠ADE = 180° (Δ interior angle th.)
10. ∠DEA + 90° = 180° → ∠DEA = 90° (substitution & algebra)
11. ∠HEF = 90° (vertical angle th.)
12. Similarly, it can be shown that ΔDFC, ΔCGB, ΔAHB are right triangles, and the other angles of EFGH are right angles.
13. ∴EFGH is a rectangle. (def. rectangle)

19) Here's one possible proof:
1. Given trapezoid ABCD, with AB ∥ CD and AD not parallel to BC, and EF is a median of sides AD and BC.
2. Draw line DF. Label the point where it intersects with AB (both extended) as X. (post. 1 and 2)
3. ∠CFD ≅ ∠XFB (vertical angle th.)
4. ∠CDF ≅ ∠FXB (alternate interior ∠ th.)
5. CF ≅ FB (def. median)
6. ΔCDF ≅ ΔFXB (AAS ≅ th.)
7. DF ≅ FX (def. ≅ figures)
8. DE ≅ EA (def. median)
9. DX = DF + FX; DA = DE + EA (segment add. post.)
10. DX=DF+DF → DX=2DF → DX:DF=2:1
DA=DE+DE → DA=2DE → DA:DE=2:1 (substitution)
11. ΔDEF ~ ΔDAX (SAS ~ th.)
12. ∠DEF ≅ ∠A (def similar figures)
13. ∴EF ∥ AB (corresponding angle converse)

Here's another very different approach:
1. Given trapezoid ABCD, with AB ∥ CD and AD not parallel to BC, and EF is a median of sides AD and BC.
2. Extend AD & BC to meet at point Q. (post. 2)

3. AE=ED=x; CF=FB=y (def. median) also, let DQ=z, and CQ=w
4. AD = DE+EA = 2x; CB = CF+FB = 2y (segment addition post.)
5. z:2x = w:2y (Δ proportionality th.)
6. z:x = w:y (multiplication or Th. V-4)
7. z:(x+z) = w:(y+w) (Th. V-18)
8. z:w = (x+z):(y+w) (Th. V-16)
9. ΔQDC ~ ΔQEF (SAS ~ th.)
10. ∠QDC ≅ ∠QEF (def. similar figures)
11. ∠QDC ≅ ∠QAB (Corresponding ∠ Th.)
12. ∠QEF ≅ ∠QAB (C.N.1)
13. ∴ EF || AB (Corr. ∠ Converse)

Problem Set #5

I. The Pentagon & the Golden Triangle

1) There are only two differently shaped triangles.

2) 1. Given regular pentagon ABCDE inscribed in a circle.
   2. All five sides of the pentagon are equal. (def. of regular)
   3. All five arcs are equal. (equal chord th.)
   4. Each of the three angles at each vertex of the pentagon are equal, including ∠QDC ≅ ∠CAD (inscribed ∠ th.)
   5. ∴ ΔDCQ ~ ΔACD (AA ~ th.)

3) All angles are either 36°, 72° or 108°.

4) 1. Given regular pentagon ABCDE inscribed in a circle.
   2. ΔDCQ ~ ΔACD (as shown above)
   3. DC:DQ = AD:AC = 1:1 → DQ=DC (def. ~ figures)
   4. ∠CAD ≅ ∠ADB (as shown above)
   5. DQ = AQ (isosceles triangle th.)
   6. DC = AQ (transitive or C.N.1)
   7. DC:QC = AC:DC (def. ~ figures)
   8. $DC^2 = AC \cdot QC$ (algebra)
   9. ∴ $AQ^2 = AC \cdot QC$ (substitution)

5) AQ=1; AC≈1.618; QC≈0.618; PQ≈0.382

6) D:S ≈ 1.618:1; S:D ≈ 0.618:1

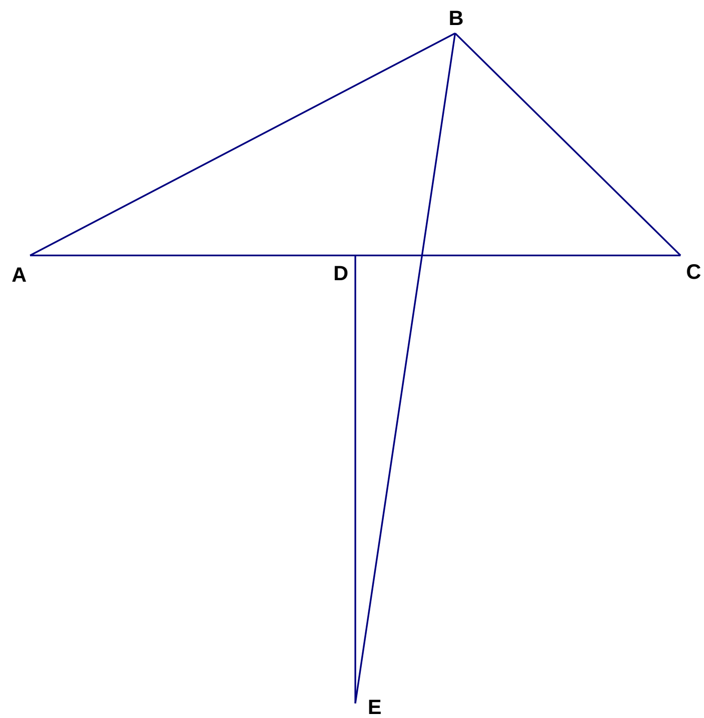
7)
   a) 0.618 from J.
   b) W:L ≈ 1.618:1
   c) L:W ≈ 0.618:1
   d) L:S ≈ 1.618:1
   e) W:S ≈ 2.618:1

Proofs ANSWERS

8)
1. Given
2. Th. II-11
3. Post. 3
4. Th. I-2
5. Post. 1
6. Th. IV-5
7. C.N.1
8. Th. III-37
9. Th. III-32
10. Th. III-20
11. C.N.1
12. C.N.2
13. from drawing
    [angle addition post.]
14. Th. I-32a
15. C.N.1
16. Def. of circle
17. Th. I-5
18. C.N.1
19. Th. I-6
20. C.N.1 [steps 4 & 19]
21. Th. I-6
22. C.N.1 or substitution
    [steps 13 & 21]
23. C.N.1 [steps 17 & 22]

II. All Triangles are Isosceles!?

The easiest way to see the error is to construct this triangle.

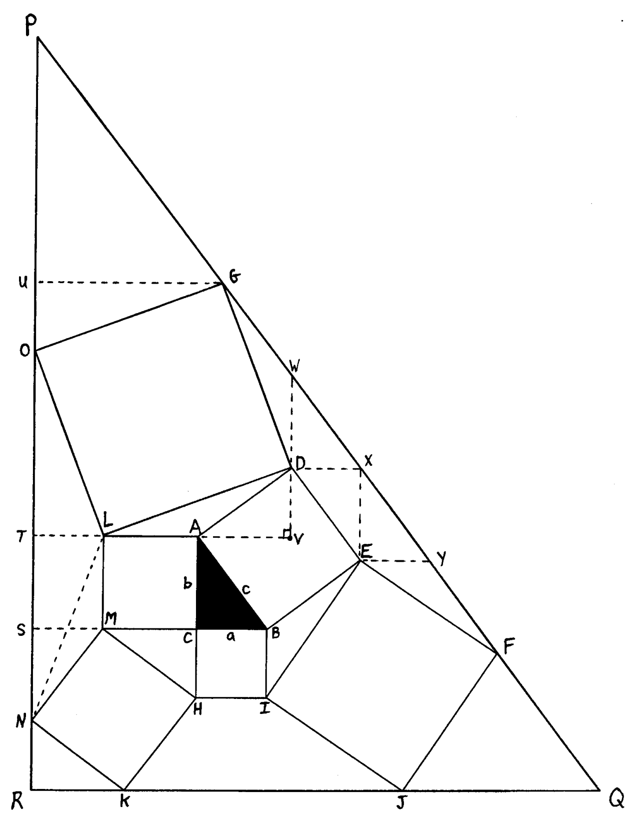


Point E will usually lie outside of ΔABC.

III. A Pythagorean Curiosity

1) Squares AE and NH are congruent. Triangles ABC, MHC and KNR are all congruent, and similar to ΔPQR. All the squares are similar to one another.

2) *To prove LM is parallel to ON:*
First, we draw line LN. Considering the four angles that meet at point A (and ignoring the dotted line), we can say that $m\angle LAD = 180° - m\angle CAB$. Likewise, $m\angle LMN = 180° - m\angle CMH$. Since $\angle CMH \cong \angle CAB$, we now know $\angle LAD \cong \angle LMN$. And because squares AE and MK are congruent, we can say that $\Delta LAD \cong \Delta LMN$ (SAS). It then follows that $OL \cong LN$ and $\angle NOL \cong \angle ONL$, which we will call θ. Letting $\alpha = \angle MLN$ $(=\angle ALD)$, and considering the angles surrounding point L, we can say that $\angle OLN = 180° - 2\alpha$. Considering ΔOLN, we get $\angle OLN = 180° - 2\theta$, which allows us to say that $\alpha = \theta$. It follows that the alternate interior angles $\angle MLN$ and $\angle ONL$ are equal and LM is parallel to ON.



Proofs ANSWERS

3) a) All of these triangles have an area equal to the area of ΔABC, which is ½ab. The reasoning is as follows. Since ΔMHC is congruent to ΔABC, their areas are equal. To find the area of ΔADL extend line LA and drop a perpendicular line down from D to point V. This creates ΔADV, which is congruent to ΔABC. Therefore ΔADL also has an area of ½ab. Similarly, it can be shown that ΔIBE also has an area equal to ½ab, as well.

b) Points T and S come from extending LA and CM to the left. It can be shown that TS = SN = b, that OT = 2b, and that TL = a. Therefore, trapezoid LMNO has an area of $^5/_2$ab, which is 5 times the area of ΔABC. Similarly, it can be shown that trapezoid HIJK has the same area. In order to show that trapezoid DEFG has again that same area, we extend horizontal and vertical lines from points D and E, thereby dividing the trapezoid into five triangles. These five triangles have equal area because ΔDWG ≅ ΔLAD, ΔIBE ≅ ΔEYF, and the remaining three triangles (ΔWXD, ΔEDX, ΔXYE) are all congruent to ΔABC.

c) Since NO, KJ, and FG are four times longer, respectively, than b, a, and c, the areas of the squares drawn off NO, KJ, and FG are $16a^2$, $16b^2$ and $16c^2$, respectively.

d) Surprisingly, the squares NH, EJ and LG do not follow the typical Pythagorean relationship. In fact, if ΔABC is isosceles, then square EJ is congruent to square LG. Since square NH is congruent to square AE, its area is c^2. By using ΔLVD to calculate the length of LD, we can determine that the area of square LG is $4b^2 + a^2$. Similarly, the area of square EJ turns out to be $4a^2 + b^2$.

e) By drawing a horizontal line from point G, we create ΔGUO, which is congruent to ΔOTL. We therefore know that UG = 2b. Since ΔPUG is similar to ΔABC, we can say that $PU = 2b^2/a$, and that the area of ΔOGP is $\frac{1}{2}(2b^2/a + a)(2b)$, which works out to $(2b^3 + a^2b)/a$. Similarly, the area of ΔJFQ is $(2a^3 + b^2a)/b$.

4) a) These squares follow the normal Pythagorean relationship: $ON^2 + FG^2 = KJ^2$

b) This one is a pleasant surprise. Using our answer from 3d, above, we can say that $EJ^2 + LG^2 = 5NH^2$.

5) a) Using our work from 3e, we get $RQ = 2b + 4a + 2a^2/b$, which works out to $\frac{2}{b}(a+b)^2$. $RP = \frac{2}{a}(a+b)^2$.

b) Of course, all of these ratios must be equal. It can be expressed as $\frac{2}{ab}(a+b)^2:1$ or as $2(a+b)^2:ab$.

c) The ratio of the areas is the square of the ratio of the lengths. The answer is $\frac{4}{a^2b^2}(a+b)^4:1$ or $4(a+b)^4:a^2b^2$