11th Grade Assignment – Week #9

Important Notes for this Week:

- As I mentioned in the lecture, this week is unusual. Some of you have seen logarithms before, while others have not. Either way, the intention is to dive into logarithms for just this week. If logarithms are new for you, it could be a lot of work. We will then return to logarithms later in the year. The hope is that the work this week will make the logarithm work later in year go more smoothly and deeply.
- The work this week is taken from the *Logarithms* unit from my 10th grade workbook (found also later in this document). For those of you who did that last year, this will be a valuable review.
- Note that the **Power and Base Tables** are included in the (below) *Logarithms* unit.

Group Assignment:

For Tuesday

- Try to work through **Problem Set #3** in the (below) *Logarithms* unit. I will give the answers to these problems – the **Laws of Logarithms!** – in tomorrow's lecture.
- If you have extra time, pick and choose key problems to work on together from the other problem sets.

For Thursday

- Together, look through the Proof of the Change of Base Formula, which is found on the next page (top of the right column). Help each other understand any confusing steps.
- Continue to work together through the *Logarithms* problem sets. Especially help each other out on the problems that are causing confusion.

Individual Work

- *Trigonometry test.* Take the trigonometry test found on the next two pages.
- *Logarithms.* Your task this week is to work through, as much as you can, the problem sets in the *Logarithms* unit. It is important that you carefully pick and choose which problems to focus on. It is likely that you won't get through it all. So choose carefully what to work on.

Trig II Test

Trig mental math (1 point each)

Each of the below problems should be done in your head, without the use of a calculator. Only write down the final answer. All ten problems should be completed within 2 minutes.

For #1-7, give exact answers.

1) $\sin(30°)$

2) $\cos(30°)$

3) $\tan(30°)$

4) $\sin(45°)$

5) $\cos(90°)$

6) $\tan(45°)$

7) $\sin(90°)$

For #8-10, give decimal approximations.

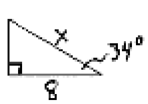
8) $\cos(25°)$

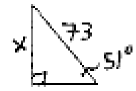
9) $\sin(10°)$

10) $\tan(70°)$

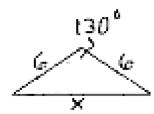
For the rest of the problems, you may either use a calculator in order to give the answer as a decimal approximation, or you can write your answer in terms of a trig function, such as 7 sin(42°).

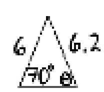
11) Find the indicated variable.
(4 points each)

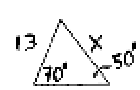
a) 

b) 

c) 

d) 

e) 

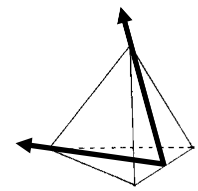
f) 

12) A train climbs up a mountain on a track that has a constant inclination of 2.3°. How much elevation has the train gained after 10 miles? (4 points)

13) Explain what this identity means, and why it is true.
(4 points)

$\sin(180° - \theta) = \sin(\theta)$

14) Find the dihedral angle of a regular tetrahedron. (A tetrahedron is a solid that has four equilateral triangular faces. The dihedral angle is the angle at which two faces come together, as shown here.) (4 points)



Notes for Logarithms, Part II

(from the Teacher's edition)

Power and Base Tables

For the most part, the students should not use calculators for this unit. It will then be necessary to use *Power and Base Tables* for many of the problems.

Summary of Topics

The following topics are covered during this unit:

- Fractional and negative exponents. Hopefully, this is a review from ninth grade.
- Basic logarithm calculations, such as $\log_8 512$.
- Changing between log and exponent form. For example, $\log_2 8$ can be written as $2^3 = 8$.
- Laws of logarithms. The students are led to discover these laws for themselves, and then they need to know how to use them. The laws of logarithms are shown together in Problem Set #4.
- Change of base formula. See details below.
- Solving equations using logarithms. The students should come to realize that when an equation has a variable in the exponent, we should usually simplify it as much as possible, and then change it into log form. For example, when solving this: $\frac{1}{8}\,10^{4x-7} - 70 = 55$, we first simplify it to

  $$10^{4x-7} = 1000,$$

  which then gets changed into log form as:

  $$\log_{10} 1000 = 4x - 7,$$

  leading to an answer of $5/2$.
- Using logarithms for difficult calculations. See details below.

Change of base formula $\quad \log_a x = \frac{\log_b x}{\log_b a}$

- Using the Change of Base Formula

The change of base formula allows us to change the base to something that is more convenient than what is given. For example,
if we are to solve $\log_8 16$, then we can change the problem into base 2 and instead solve $\frac{\log_2 16}{\log_2 8}$, which leads to an answer of $4/3$.

Proof of the Change of Base Formula

Let $\log_a x = c$, which is also $a^c = x$

Take the $\log_b$ of both sides:

$$\log_b (a^c) = \log_b x$$

$$c \bullet \log_b a = \log_b x$$

$$c = \frac{\log_b x}{\log_b a}$$

subbing back into the original equation gives:

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$$

Table of common logarithms

The Table of Common Logarithms is found below (from our *High School Source Book*). It allows you to look up values for common logarithms (which is a base 10 logarithm) between $\log_{10} 1$ and $\log_{10} 10$ (yielding answers between 0 and 1). The table can also be used in reverse to find values of base 10 exponential functions between 10^0 and 10^1.
We can then use the table to help find other values, such as $\log 619$ and $10^{4.85}$. (Note that the students will need to be told that if no base is specified, then the default is base 10.) For more details, see the worked-out examples on Problem Set #5.

Using logarithms for difficult calculations

The original purpose of logarithms was to save time spent on tedious calculations for mathematicians, astronomers, and engineers. With the aid of a common log table, roots and exponents become division and multiplication problems, and division and multiplication become basic addition and subtraction. For more details, see the worked-out examples on Problem Set #7.

Logarithms – Part II

Problem Set #1

Review

Calculate each. Use the *Power and Base Tables*, if needed. No Calculators!

1) $(\frac{2}{3})^{-1}$
2) $(\frac{2}{3})^{0}$
3) $(\frac{2}{3})^{3}$
4) $(\frac{2}{3})^{-3}$
5) 400^{2}
6) 400^{-2}
7) $400^{\frac{1}{2}}$
8) $400^{-\frac{1}{2}}$
9) $1{,}000{,}000^{\frac{1}{2}}$
10) $1{,}000{,}000^{\frac{1}{3}}$
11) $1{,}000{,}000^{-\frac{1}{3}}$
12) $1{,}000{,}000^{-\frac{2}{3}}$
13) $64^{-\frac{1}{2}}$
14) $64^{5/2}$
15) $64^{-4/3}$
16) $64^{-5/6}$
17) $64^{-2/3}$
18) $\log_4 16$
19) $\log_3 27$
20) $\log_3 81$
21) $\log_{49} 7$
22) $\log_7 49$
23) $\log_{16} 4$
24) $\log_4 64$
25) $\log_{64} 4$
26) $8^{\frac{1}{3}}$
27) 8^{3}
28) $8^{-\frac{1}{3}}$
29) 8^{-3}
30) $\log_5 25$
31) $\log_{10} 10000$
32) $\log_3 \frac{1}{3}$
33) $\log_2 \frac{1}{4}$
34) $\log_8 (1/8)$
35) $\log_8 64$
36) $\log_2 1024$
37) $\log_8 512$
38) $\log_8 (\frac{1}{512})$
39) $\log_8 2$
40) $\log_8 (\frac{1}{2})$
41) $\log_8 0$
42) $\log_9 81$
43) $\log_9 (\frac{1}{81})$
44) $\log_9 3$
45) $\log_9 (\frac{1}{3})$
46) $\log_9 27$
47) $\log_9 (-3)$

48) Change to exponent form:
Example: $\log_2 8 = 3$
Solution: $2^3 = 8$
a) $\log_4 64 = 3$
b) $\log_{10} 0.1 = -1$
c) $\log_{16} (\frac{1}{4}) = -\frac{1}{2}$

49) Change to log form:
a) $6^2 = 36$
b) $6^{-2} = 1/36$
c) $16^{\frac{3}{4}} = 8$

Power and Base Tables

2^{nd} Power

N	N^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

3^{rd} Power

N	N^3
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

4^{th} Power

N	N^4
1	1
2	16
3	81
4	256
5	625
6	1296
7	2401
8	4096
9	6561
10	10000

5^{th} Power

N	N^5
1	1
2	32
3	243
4	1024
5	3125
6	7776
7	16807
8	32768
9	59049
10	100000

Base 2

N	2^N
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Base 3

N	3^N
1	3
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
9	19683
10	59049

Base 4

N	4^N
1	4
2	16
3	64
4	256
5	1024
6	4096
7	16384
8	65536

Base 5

N	5^N
1	5
2	25
3	125
4	625
5	3125
6	15625
7	78125

Base 6

N	6^N
1	6
2	36
3	216
4	1296
5	7776
6	46656

Base 7

N	7^N
1	7
2	49
3	343
4	2401
5	16807
6	117649

Base 8

N	8^N
1	8
2	64
3	512
4	4096
5	32768
6	262144

Base 9

N	9^N
1	9
2	81
3	729
4	6561
5	59049
6	531441

— Logarithms – Part II —

Problem Set #2

Calculate each.

1) 9^2
2) $9^{1/2}$
3) 9^{-2}
4) $9^{-1/2}$
5) $8{,}000{,}000^{1/3}$
6) $8{,}000{,}000^{-1/3}$
7) $8{,}000{,}000^{2/3}$
8) $8{,}000{,}000^{-2/3}$
9) $1{,}000{,}000{,}000{,}000^{1/4}$
10) $\log_3 9$
11) $\log_2 16$
12) $\log_4 (1/4)$
13) $\log_4 1$
14) $\log_4 2$
15) $\log_4 (\frac{1}{16})$
16) $\log_4 (-1/2)$
17) $\log_{25} (\frac{1}{5})$
18) $\log_6 \sqrt{6}$
19) Change to exponent form:
    Example: $\log_2 8 = 3$
    Solution: $2^3 = 8$
    a) $\log_{10} 100000 = 5$
    b) $\log_4 (\frac{1}{64}) = -3$
    c) $\log_3 4x = 5$
20) Change to log form:
    a) $7^3 = 343$
    b) $8^{-3} = \frac{1}{512}$
    c) $9^{4x+7} = 285$
21) $\log_{100} 1000000$
22) $\log_{100} 10$
23) $\log_{100} 1000$
24) $\log_{100} 0.1$
25) $\log_{100} 0.01$
26) $\log_{100} 0.001$
27) $\log_9 729$
28) $\log_9 (\frac{1}{729})$
29) $\log_3 (\frac{1}{729})$
30) $\log_9 (\frac{1}{3})$
31) $\log_3 (\frac{1}{9})$
32) 36^2
33) $36^{1/2}$
34) 36^{-2}
35) $36^{-1/2}$
36) $\log_8 16$
37) $\log_8 4$
38) $\log_2 0$
39) $\log_8 1$
40) $\log_{37} (\frac{1}{37})$
41) $\log_{81} 3$
42) $\log_8 (\frac{1}{256})$
43) $\log_5 (-25)$
44) $\log_{25} (\frac{1}{125})$
45) $\log_{27} 81$
46) $\log_{81} (\frac{1}{27})$

Solve for X. It may help to rewrite the equation in exponential or log form.

47) $3^X = 81$
48) $x^4 = 16$
49) $10^X = \frac{1}{1000}$
50) $\log_x 8 = 3$
51) $2^{3X-1} = 32$
52) $\log_4 x = -2$
53) $\log_3 9x = 5$
54) $2 + 3 \log_8(1-2x) = 0$
55) $\frac{1}{8} 10^{4x-7} - 70 = 55$

— Logarithms – Part II —

Problem Set #3

Deriving the Laws of Logarithms!

Calculate each. Use the *Power and Base Tables*, as needed.

1) a) $\log_2 16$
   b) $\log_2 64$
   c) $\log_2 (64 \cdot 16)$

2) a) $\log_{10} 1000$
   b) $\log_{10} 100,000$
   c) $\log_{10} (100,000 \cdot 1000)$

3) $\log_3 (9 \cdot 27)$

4) Derive a Law of Logarithms!
   $\log_b (M \cdot N) =$

5) a) $\log_{10} 100,000$
   b) $\log_{10} 1000$
   c) $\log_{10} (100,000 \div 1000)$

6) a) $\log_3 2187$
   b) $\log_3 243$
   c) $\log_3 (2187 \div 243)$

7) $\log_2 (512 \div 32)$

8) Derive a Law of Logarithms!
   $\log_b (M \div N) =$

9) a) $\log_2 8$
   b) $\log_2 (8^3)$

10) a) $\log_{10} 1000$
    b) $\log_{10} (1000^5)$

11) $\log_3 (9^7)$

12) Derive a Law of Logarithms
    $\log_b (N^k) =$

13) a) $\log_2 8$
    b) $\log_2 (^1/_8)$

14) a) $\log_{10} 100,000$
    b) $\log_{10} \left(\frac{1}{100000}\right)$

15) How are $\log_b (^1/_N)$ and $\log_b N$ related to one another? Write a Law of Logarithms that expresses this.

16) a) $\log_3 81$
    b) $\log_{81} 3$

17) a) $\log_{10} 100$
    b) $\log_{100} 10$

18) How are $\log_a b$ and $\log_b a$ related to one another? Write a Law of Logarithms that expresses this.

19) $\log_3 (3^7)$

20) $\log_{10} (10^6)$

21) Derive a Law of Logarithms
    $\log_b (b^k) =$

22) $5^{\log_5 625}$

23) $10^{\log_{10} 1000}$

24) Derive a Law of Logarithms
    $b^{\log_b N} =$

25) What can the following Logarithm Law be used for?

$$\log_a x = \frac{\log_b x}{\log_b a}$$

— Logarithms – Part II —

Problem Set #4

1) Review. Calculate each.

a) $9^{5/2}$

b) 9^2

c) $9^{3/2}$

d) 9^1

e) $9^{½}$

f) 9^0

g) $9^{-½}$

h) 9^{-1}

i) $9^{-3/2}$

j) 9^{-2}

k) $9^{-5/2}$

l) $\log_{20} 400$

m) $\log_{20} 8000$

n) $\log_{25} 625$

o) $\log_{25} (\frac{1}{625})$

p) $\log_5 (\frac{1}{625})$

q) $\log_5 (\frac{1}{25})$

r) $\log_{25} (\frac{1}{5})$

s) $\log_5 (-25)$

t) $\log_7 (\frac{1}{7})$

u) $\log_{27} 243$

v) $\log_{27} (\frac{1}{243})$

The Laws of Logarithms

- $\log_b (M{\cdot}N) = \log_b M + \log_b N$
- $\log_b (^M/_N) = \log_b M - \log_b N$
- $\log_b N^k = k \cdot \log_b N$
- $\log_b (^1/_N) = -\log_b N$
- $\log_a b = \frac{1}{\log_b a}$
- $\log_b (b^k) = k$
- $b^{\log_b N} = N$
- Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

2) For each of the above laws, explain what it means or how it can be useful.

3) Use one of the Laws of Logarithms in order to evaluate each logarithm. Do not use a calculator, but you may need to use the *Power and Base Tables*.

a) $\log_2 (16{\cdot}32)$

b) $\log_4 \left(\frac{16384}{256}\right)$

c) $\log_5 (125^4)$

d) $\log_{125} 5$

e) $\log_3 \left(\frac{1}{27}\right)$

f) $\log_5 (5^8)$

g) $8^{\log_8 64}$

4) The following problems were on Problem Set #2 (Do you recall how you did them?) Now, use the *change of base formula*. (Think about what the common base should be.)

a) $\log_{27} 81$

b) $\log_8 4$

c) $\log_{16} (\frac{1}{8})$

5) First estimate the answer to one decimal place, then use your calculator (and the *change of base formula*) to give an answer rounded to three significant figures.

a) $\log_2 15$

b) $\log_4 300$

c) $\log_3 2$

d) $\log_3 0.4$

e) $3^{5.23}$

f) $4^{-2.91}$

Problem Set #5

1) Use the Common Log Table to calculate each problem (without a calculator). Remember that the log table can only be used to find the log (base 10) of a number between 1 and 10, and it can be used to find 10^x (antilog x) where x is between 0 and 1.

Example: log 619

Solution:

$\log 619 = \log(6.19 \cdot 10^2)$
$= \log 6.19 + \log 10^2$
$\approx 0.7917 + 2$
≈ 2.7917

Example: $10^{4.85}$

Solution:

$10^{4.85} = 10^{(4+0.85)}$
$= 10^4\ 10^{0.85}$
$\approx 10^4 \cdot 7.08$
$\approx 70{,}800$

a) log 8920
b) log 870,000
c) log 0.0056
d) $10^{2.75}$
e) $10^{7.1}$
f) $10^{-3.26}$

2) Expand each expression as much as possible.

Example: $\log_2\left(\frac{8x^5}{y \cdot z}\right)$

Solution:

$\log_2\left(\frac{8x^5}{y \cdot z}\right) = \log_2(8x^5) - \log_2(y \cdot z)$
$= \log_2 8 + \log_2 x^5 - (\log_2 y + \log_2 z)$
$= 3 + 5\log_2 x - \log_2 y - \log_2 z$

a) $\log_2(16x^2)$
b) $\log_5\left(\frac{125x}{y}\right)$
c) $\log_5\left(\frac{625xy}{z^6}\right)$
d) $\log_{10}\left(\frac{x}{10y^3}\right)$

3) Condense each expression. (i.e., rewrite as one logarithm.)

Example: $4 + \log_2 x - 3\log_2 y$

Solution: $4 + \log_2 x - 3\log_2 y$
$= \log_2 16 + \log_2 x - \log_2(y^3)$
$= \log_2(16x) - \log_2(y^3)$
$= \log_2\left(\frac{16x}{y^3}\right)$

a) $\log_3 x + \log_3 a$
b) $\log_7 d - \log_7 8$
c) $6 + 5\log_2 x$
d) $\log_3 x - 2\log_3 y - 5\log_3 z$

4) Solve for X (possibly in terms of other variables). Use a calculator only when necessary.

a) $7^x = 34$
b) $100^x = 20$
c) $8^x = \frac{1}{2}$
d) $z^x = w$
e) $\log_3 x = 5$
f) $\log_3 40 = x$
g) $\log_x 40 = 5$
h) $\log_{20} x = \frac{1}{3}$
i) $10^{2X+4} = 0.001$
j) $\log_5 x = -3$
k) $\frac{2}{3} 6^{4x+3} - 43 = 57$

— Logarithms – Part II —

Problem Set #6

1) Review. Calculate each.

a) $125{,}000^{1/3}$

b) $125{,}000^{-1/3}$

c) $125{,}000^{2/3}$

d) $125{,}000^{-2/3}$

e) $32^{2/5}$

f) $32^{-4/5}$

g) $\log_{16}\left(\frac{1}{16}\right)$

h) $\log_{16} 256$

i) $\log_{16} 1$

j) $\log_{16} 2$

k) $\log_{16}\left(\frac{1}{256}\right)$

l) $\log_{16}(-\frac{1}{4})$

m) $\log_8 32$

n) $\log_8 2$

o) $\log_2 0$

p) $\log_9 1$

q) $\log_4\left(\frac{1}{128}\right)$

r) $\log_5 1$

s) $\log_{27} 81$

t) $\log_{81}\left(\frac{1}{27}\right)$

2) Use one of the Laws of Logarithms into order to evaluate each logarithm. Do not use a calculator, but you may need to use the *Power and Base Tables*.

a) $\log_3(81\cdot 27)$

b) $\log_7\left(\frac{16807}{343}\right)$

c) $\log_6(7776^8)$

d) $\log_{64} 8$

e) $\log_{10}\left(\frac{1}{1000000}\right)$

f) $7^{\log_7 30}$

g) $\log_9(9^7)$

3) Expand each expression as much as possible.

a) $\log_2\left(\frac{x}{8y}\right)$

b) $\log_3\left(\frac{c^2}{81z}\right)$

c) $\log_{10}(100y^5)$

d) $\log_4\left(\frac{x^2z}{16y}\right)$

4) Condense each expression. (i.e., rewrite as one logarithm.)

a) $\log_5 4 + \log_5 a$

b) $\log_a 5 - \log_a x$

c) $3\log_{10} x + \log_{10} y$

d) $\log_2 x + 4\log_2 y - 1 - \log_2 z$

5) Use the Common Log Table to calculate each problem (without a calculator).

a) $\log 672$

b) $\log 78{,}300$

c) $\log 0.062$

d) $10^{3.8}$

e) $10^{2.84}$

f) $10^{-2.6}$

6) Use the *change of base formula* to calculate each problem. (Think about what the common base should be.)

a) $\log_8 16$

b) $\log_{32} 8$

c) $\log_{25}\left(\frac{1}{125}\right)$

7) First estimate the answer to one decimal place, then use your calculator to give an answer rounded to three significant figures.

a) $\log_5 160$

b) $\log_9 420$

c) $\log_8 5$

d) $\log_3 0.3$

e) $2^{4.83}$

f) $3^{-4.2}$

8) Solve for X. Use a calculator only when necessary.

a) $5^x = 100$

b) $30^x = 0.001$

c) $a^x = c$

d) $x^y = c$

e) $\log_6 x = 3$

f) $\log_8 40 = x$

g) $\log_x 300 = 2$

h) $6^{x-7} = 50$

i) $-7 + 4\log_2(4x-8) = 13$

— Logarithms – Part II —

Problem Set #7

Using Logarithms to make Calculations Easier

Before the advent of the modern calculator, logarithms were used to help make tedious calculations easier. For example, by using logarithm tables, long division problems could be reduced to subtraction, and taking the fifth root of a number could be done by just doing a simple division problem.

At first glance it may seem complicated, but once you got good at it, this method would save an engineer or scientist quite a bit of time. The final answer is a highly accurate approximation.

Here are a couple of examples:
(Underlined digits are a guess.)

Example: $768{,}000 \div 592.8$

$x = 768{,}000 \div 592.8$

$\log x = \log(768{,}000 \div 592.8)$

$\log x = \log(768{,}000) - \log(592.8)$

$= \log(7.68 \cdot 10^5) - \log(5.928 \cdot 10^2)$

$\log x \approx 5.8854 - 2.7729$

$\log x \approx 3.1125$

$x \approx \text{antilog}(3.1125)$

$x \approx 10^{3.1125}$

$x \approx 10^3 \cdot 10^{0.1125}$

$x \approx 1000 \cdot 1.29\underline{6}$

$\boxed{x \approx 1{,}29\underline{6}}$

Example: 38.7^7

$x = 38.7^7$

$\log x = \log(38.7^7)$

$\log x = 7 \cdot \log(38.7)$

$\log x = 7 \cdot \log(3.87 \cdot 10^1)$

$\log x = 7 \cdot [\log(3.87) + \log(10^1)]$

$\log x \approx 7 \cdot [0.5877 + 1]$

$\log x \approx 7 \cdot 1.5877$

$\log x \approx 11.1139$

$x \approx \text{antilog}(11.1139)$

$x \approx \underline{10^{11} \cdot 10^{0.1139}}$

$\boxed{x \approx 1.30 \cdot 10^{11}}$

Calculate by using the common log table with a method similar to the above examples.
NO CALCULATORS!

1) $39{,}200{,}000 \div 7320$
2) 4.38^6
3) $8349 \cdot 67.3$
4) $\sqrt[4]{83000}$
5) $834{,}100 \div 9.52$
6) 38.6^9
7) $425.2 \cdot 78390$
8) $\sqrt[3]{78400}$
9) $\sqrt[6]{32750}$
10) $\sqrt[10]{7000}$

Table of Common Logarithms

Note: Looking up row 73 and column 8 we read 8681, which means both $\log_{10} 7.38 \approx 0.8681$ and $10^{0.8681} \approx 7.38$

	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7127	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7521	7528	7536	7544	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8143	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8300	8306	8312	8319
68	8325	8332	8338	8344	8351	8357	8363	8370	8376	8382
69	8389	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8507
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8780	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9026
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9080
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9207	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9310	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9567	9571	9576	9581	9586
91	9590	9595	9600	9605	9610	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9676	9680
93	9685	9690	9694	9699	9704	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9764	9768	9773
95	9777	9782	9786	9791	9796	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9895	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9970	9974	9978	9983	9987	9991	9996