11^{th} Grade Assignment – Week #31

Individual Work Do the following:

1) Convert from *polar* into *rectangular form*.
Remember: $\mathbf{r\,cis\,\theta = r\cos\theta + \textit{i}\,(r\sin\theta)}$

a) $6\cdot\text{cis}(60°)$ e) $2\cdot\text{cis}(70°)$

b) $\frac{\sqrt{2}}{2}\cdot\text{cis}\left(\frac{7\pi}{4}\right)$ f) $10\cdot\text{cis}\left(\frac{7\pi}{9}\right)$

c) $5\cdot\text{cis}\left(\frac{3\pi}{2}\right)$ g) $7.3\cdot\text{cis}(2.8)$

d) $\text{cis}\left(\frac{5\pi}{4}\right)$ h) $13\cdot\text{cis}(6.5)$

2) Convert from *rectangular* into *polar form*.

a) $-3+6i$ d) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

b) $5\sqrt{3}+5i$ e) $-2\sqrt{2}-2\sqrt{2}\,i$

c) $-7-3i$ f) $\sqrt{7}\,i$

3) Multiply

a) $[6\cdot\text{cis}(60°)]\,[7\cdot\text{cis}(40°)]$

b) $[4\cdot\text{cis}\left(\frac{2\pi}{3}\right)]\,[2\cdot\text{cis}\left(\frac{\pi}{3}\right)]$

c) $[5\cdot\text{cis}\left(\frac{5\pi}{3}\right)]\,[10\cdot\text{cis}\left(\frac{7\pi}{4}\right)]$

d) $[2\,\text{cis}(50°)]^4$

e) $(4+4i)^3$ (give answer in rect. form.)

f) $(1-\sqrt{3}\,i)^5$ (give answer in rect. form.)

Series Formulas as given in Lecture #1

$$e^x = \sum_{j=0}^{\infty}\frac{x^j}{j!} = \frac{1}{0!}+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots$$

$$\cos(x) = \sum_{j=0}^{\infty}(-1)^j\frac{x^{2j}}{(2j)!} = \frac{1}{0!}-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots$$

$$\sin(x) = \sum_{j=0}^{\infty}(-1)^j\frac{x^{2j+1}}{(2j+1)!} = \frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots$$

4) Use one of the Series Formulas (given above). First write out the first 5 terms, then put it into your calculator to get an approximation, and lastly check the answer for accuracy.

a) e^5

b) $\cos(120°)$
(Don't forget to change to radians!)

c) $\sin(270°)$

d) e

Group Assignment: *for Tuesday*

1) For each, change P into polar form, and then graph both P and Q.

a) $P = 3+3i$; $Q = P^2$

b) $P = 1+i$; $Q = P^4$

c) $P = \sqrt{3}+i$; $Q = P^3$

d) $P = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$; $Q = P^8$

2) For each of the below problems, give all of the requested roots, and then graph both X and all of its roots. (Hints for some of these can be found with #1.)

a) What are the two square roots of $X = 25$?
b) What are the two square roots of $X = 18i$?
c) What are the two square roots of $X = -9$?
d) What are the four 4th roots of $X = 16$?
e) What are the four 4th roots of $X = -4$?
f) What are the eight 8th roots of $X = 1$?
g) What are the three cube roots of $X = 8i$?

3) What properties are always followed with the graphs in #2?

4) What general method have you discovered for taking the roots of any number?

5) Our big hanging question!
What are the three cube roots of 1?

Group Assignment: *for Thursday*

Choose between doing #1 or #2.

1) **Taking Roots of Complex Numbers**
   a) What are the two square roots of $9i$?
   b) What are the two square roots of $-9i$?
   c) What are the two square roots of $-18 + 18\sqrt{3}\,i$?
   d) What are the four 4^{th} roots of $-8 - 8\sqrt{3}\,i$?
   e) What are the three cube roots of $-125i$?
   f) What are the three cube roots of $32\sqrt{2} - 32\sqrt{2}\,i$?
   g) What are the 8 eighth roots of $300 + 510\,i$?

2) **The Hat-Check Problem**
   This famous problem has been investigated by several mathematicians – first posed by Montmort in 1713, and later attributed to Euler. Euler managed to relate the solution to his famous constant, e.

   > *A group of (n) people go to lunch and afterwards pick up their hats at random. What is the probability that no one gets their own hat?*

   Determine solutions for various values of n.