11th Grade Assignment – Week #29

Individual Work

- From the *Logarithms* unit, do **Problem Sets #4 and #5.**

Group Assignment: *for Tuesday or Thursday.*

1. Cell Phone Decisions

A cell phone company offers three different plans to its customers, as follows:

- *The Minimum Plan*: For $40/month you get 450 minutes free (per month), and each additional minute is 45¢.
- *The Medium Plan*: For $60/month you get 900 minutes free (per month), and each additional minute is 40¢.
- *The Max Plan*: For $100/month you get 2000 minutes free (per month), and each additional minute is 25¢.

Graph each of the three plans in order to determine under what circumstances it would be best to choose each plan. The x-axis should be total minutes and the y-axis should be total cost. (Graph paper is on the next page.)

2. Baseball Cards

Dan is collecting baseball cards. There exists a total of 200 different cards. Every day, he goes to the store and buys one (random) baseball card – and he hopes that it's one he doesn't already have. What is the *expected* amount of time for Dan to collect all 200 cards? (Assume that he can't trade cards with friends.) (There are some hints found at the end of this document.)

Problem Set #4

Solve for x.

1) $7^{2x-3} = 16807^{5x+1}$
2) $7^{2x-3} = (\frac{1}{2})^{4x+2}$
3) $70e^{3-x} = 350$
4) $\log_4(16 \cdot 4^x) = 13$
5) $\ln x = 2$
6) $5^{4^{3^{2^x}}} = 1{,}234{,}567{,}890$
   (With problems like these, always begin from the top,
   e.g., $10^{3^2} = 10^9$, not 1000^2.)

Exponentiation

How did you solve problem #5? Recall that we can take the *log* of both sides of an equation. Similarly, we can raise both sides of an equation to the same base.

Example: Solve: $\ln x = -3$

Exponentiate both sides to get:

$$e^{\ln x} = e^{-3}$$
$$x = e^{-3} \approx 0.04979.$$

Solve for x.

7) $\log_4(12x) = 5$
8) $2\ln(3x) = 4$
9) $\ln(x-2) + \ln(2x-3) = 2\ln x$
10) $\log_4 x - \log_4(x-1) = \frac{1}{2}$

Word Problems

11) An amount of money is deposited into an account and quadruples in twenty years. If the interest is compounded monthly, what was the APR on the account during this time?
12) The half-life of uranium-235 is about 704 million years. This means that a mass of a quantity of uranium-235 will decrease to half of its initial mass in 704 million years and halve itself again in another 704 million years and so on. If the intial quantity of uranium-235 is 25 units. How many years will it take before there are 10 units of uranium-235?
13) Bob wishes to travel to Europe in three years and he needs $1800 in order to do so. If he has $1200 that he deposits into a CD where the interest is compounded monthly, what will his minimum APR need to be?
14) $15,000 is deposited into a bank account at 3.0092% APR where the interest is compounded quarterly. How long will it take the money to:
    a) Double
    b) Triple
    c) Reach $1,000,000
15) Five years ago, the population of a town was 12,800. Currently it is 15,300. If that rate continues, how long will it take the town's population to double from what it is today?
16) A new laptop computer costs $2000. In 2 years, the book value of the laptop is $1200. Find the value of the laptop in 5 years.

Problem Set #5

Solve for *x*.

1) $\log_4 16 = x$
2) $\log_x 8 = 3$
3) $\log_8 x = 512$
4) $\log_2(\frac{1}{8}) = x$
5) $\log_9 x = \frac{3}{2}$
6) $\log_9 x = \frac{2}{3}$
7) $\log_9 x = -\frac{3}{2}$
8) $\ln 4^x = 2$
9) $\ln(4e^x) = 2$
10) $\ln\left(\frac{4}{e^x}\right) = 2$
11) $\log_6(3) + \log_6 x = 4$
12) $5 + 7\log_2(3 + 2x) = 26$
13) $5^{x+1} = 125^{2x-4}$
14) $6^x = -6$
15) $6^{x+7} = 7^{6x-7}$
16) $e^{2x} + 5e^x + 6 = 0$
17) $\ln\sqrt{3-x} = 8$
18) $500 = 10000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$
19) $\ln(x) + \ln(5 - x) = \ln(x+3)$.
20) $\ln\left(\log_5\left(25 \cdot \frac{625^{e^x}}{125}\right)\right) = 8$

21) $7500 is deposited into an interest-bearing account at 7.3% APR compounded weekly. How long will it take the money to triple?

22) The world's population in 1980 was about 4.45 billion, and in 2000 it was about 6.08 billion. The population of the United States in 1980 was about 227 million, and in 2000 was about 283 million. Assuming a constant growth rate…
   a) What was the population of the U.S. in 1900?
   b) What was the population of the world in 1900?
   c) What percent of the world's population lived in the U.S. in 1900?
   d) What percent of the world's population lived in the U.S. in 2000?
   e) Assuming the growth rates stay the same, when will residents of the United States make-up less than 1% of the world's population.
   f) Is it valid to assume the growth rates will remain constant for the period of time found in e?

Hints for the "Baseball Cards" problem:

- The *Dartboard Principle* may be helpful.
- First try answering these questions:
   1) Question: What would be the *expected* number of flips of a coin in order to get both possibilities (heads and tails)?
   2) Question: What would be the *expected* number of roles of a die in order to get all six possible roles?
   3) Question: After having collected the 70th different baseball card (out of 200) what would be the *expected* number of purchases needed in order to get another different card?