11^{th} Grade Assignment – Week #27

Notes:

- As usual, but perhaps especially for this *Logarithms* unit, you should use your calculator as little as possible.
- The *Power and Base Tables*, which are found at the end of this document, should be helpful for this *Logarithms* unit.
- **Important!**
  - *Common Log.* If the base of a log isn't shown, then the convention is that the base is 10.
    Example: $\log 1000$ really means $\log_{10} 1000$ (which is equal to 3).
  - Remember that e is approximately 2.71828182845904523536028747135 27.
    In this week's Lecture #1, I explain where e comes from.
  - The *Natural Log*, which is *log base e*, is very helpful in calculus and higher-level math. It is abbreviated as "ln". Therefore, whenever you see "ln", you should just think *log base e*
    Example: $\ln 50 \approx 3.912$ really means $\log_e 50 \approx 3.912$ and is the equivalent of $e^{3.912} \approx 50$.

Individual Work

- Take the *Cartesian Geometry – Part III* test, which is found at the end of this document.
- From the *Logarithms* unit, do **Problem Set #0**, which is actually Problem Set #4 from the 10^{th} Grade Workbook, and is also copied below.

Group Assignment:

for Tuesday

- Do these problems:

  1) $125^{1/3}$ 2) $125^{-1/3}$ 3) $125^{2/3}$ 4) $125^{-2/3}$ 5) $32^{2/5}$ 6) $32^{-4/5}$
  7) $\log_{16}\left(\frac{1}{16}\right)$ 8) $\log_{16} 256$ 9) $\log_{16} 1$ 10) $\log_{16} 2$

- From the *Logarithms III* unit, do **Problem Set #1.** Especially make sure everyone understands the *Laws of Logarithms*.

for Thursday

11) Proof of the *Continous Growth Formula.*

In the lectures this week, I have shown you these three formulas for growth:

Exponential Growth	*Compound Interest*	*Continuous Growth*
$P = P_0(1+r)^t$.	$P = P_0\left(1+\frac{r}{n}\right)^{nt}$	$P = P_0 e^{rt}$

The proof of the *Continous Growth Formula* is below. Go through the proof together, step by step, and make sure everyone understands it.

1. Given the *Compound Interest Formula* (above), and this definition of e: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$
2. Let $Q = {}^{n}/_{r}$, and $n = Q{\cdot}r$
3. $P = P_0\left(1+\frac{r}{n}\right)^{nt} \rightarrow P = P_0\left(1+\frac{1}{{}^{n}/_{r}}\right)^{nt} \rightarrow P = P_0\left(1+\frac{1}{Q}\right)^{Qrt} \rightarrow P = P_0\left[\left(1+\frac{1}{Q}\right)^{Q}\right]^{rt}$
4. Let n approach ∞. Therefore Q also approaches ∞.
5. The limit as Q approaches ∞ of $\left(1+\frac{1}{Q}\right)^{Q} = e$
6. Using substitution gives us $P = P_0 e^{rt}$ Q.E.D!

12) *The Dartboard Problem (Part I).*

A dartboard is divided into twelve equal-sized pie-wedged regions, one of which is the "target". Assume that each thrown dart has an equal chance of landing in any of the twelve regions. Darts will continue to be thrown until it hits the target. What is the probability that the number of throws needed to hit the target is:

a) Exactly 1 throw?
b) Exactly 2 throws?
c) Exactly 3 throws?
d) 3 throws or less?
e) More than 3 throws?
f) Exactly 20 throws?

g) A school bus has 15 passengers 20% of the time, 32 passengers 30% of the time, and 24 passengers 50% of the time. What is the overall average number of passengers?

Problem Set #0

1) Review. Calculate each.

a) $9^{5/2}$

b) 9^{2}

c) $9^{3/2}$

d) 9^{1}

e) $9^{1/2}$

f) 9^{0}

g) $9^{-1/2}$

h) 9^{-1}

i) $9^{-3/2}$

j) 9^{-2}

k) $9^{-5/2}$

l) $\log_{20} 400$

m) $\log_{20} 8000$

n) $\log_{25} 625$

o) $\log_{25} \left(\frac{1}{625}\right)$

p) $\log_{5} \left(\frac{1}{625}\right)$

q) $\log_{5} \left(\frac{1}{25}\right)$

r) $\log_{25} \left(\frac{1}{5}\right)$

s) $\log_{5} (-25)$

t) $\log_{7} \left(\frac{1}{7}\right)$

u) $\log_{27} 243$

v) $\log_{27} \left(\frac{1}{243}\right)$

The Laws of Logarithms

- $\log_b (M \cdot N) = \log_b M + \log_b N$
- $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$
- $\log_b N^k = k \cdot \log_b N$
- $\log_b \left(\frac{1}{N}\right) = -\log_b N$
- $\log_a b = \frac{1}{\log_b a}$
- $\log_b (b^k) = k$
- $b^{\log_b N} = N$
- Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

2) For each of the above laws, explain what it means or how it can be useful.

3) Use one of the Laws of Logarithms in order to evaluate each logarithm. Do not use a calculator, but you may need to use the *Power and Base Tables*.

a) $\log_2 (16 \cdot 32)$

b) $\log_4 \left(\frac{16384}{256}\right)$

c) $\log_5 (125^4)$

d) $\log_{125} 5$

e) $\log_3 \left(\frac{1}{27}\right)$

f) $\log_5 (5^8)$

g) $8^{\log_8 64}$

4) Use the *change of base formula*. (Think about what the common base should be.)

a) $\log_{27} 81$

b) $\log_8 4$

c) $\log_{16} \left(\frac{1}{8}\right)$

5) First estimate the answer to one decimal place, then use your calculator (and the *change of base formula*) to give an answer rounded to three significant figures.

a) $\log_2 15$

b) $\log_4 300$

c) $\log_3 2$

d) $\log_3 0.4$

e) $3^{5.23}$

f) $4^{-2.91}$

Logarithms – Part III

Problem Set #1

Review (from 10th grade)

Evaluate without a calculator.

1) $\log 100$
2) $\log \frac{1}{10}$
3) $\log_2 16$
4) $\log_2 8$
5) $\log_2 1$
6) $\log_2 ¼$
7) $\log_{43} 1$
8) $\log_{1,003} 1$
9) $\log_9 9^x$
10) $\ln 1$
11) $\ln e$
12) $\ln e^5$
13) $\ln e^x$
14) $\log_2 1024$
15) $\log_8 512$
16) $\log_8 \left(\frac{1}{512}\right)$
17) $\log_8 2$
18) $\log_8 ½$
19) $\log_8 0$
20) $\log_8 (-3)$

Properties of Logs.

21) $\log_b(xy)$ = _____.
22) $\log_b \frac{x}{y}$ =_____.
23) $\log_b a^x$ = _____.
24) $\log_b b^x$ = _____.
25) $b^{\log_b x}$ =_____.
26) State the change of base formula.

Evaluate by using the Properties of Logs.

27) $\log_4(64 \cdot 16)$
28) $\log_5 \left(\frac{625}{125}\right)$
29) $\log_8 64^5$
30) $\log_3 3^{12}$
31) $\log_6 6^{14}$
32) $11^{\log_{11} 8}$

Solve for x. Use a calculator only if necessary.

33) $5^x = 625$
34) $\log_5 625 = x$
35) $(½)^x = \frac{1}{64}$
36) $17^x = 1$
37) $2^x = ½$
38) $3^{x+1} = 3^{4x-1}$
39) $3^{x+1} = 27^{4x-1}$
40) $e^{\ln(x+4)} = 7$
41) $\log_7 \left(\frac{49}{7^x}\right) = 12$
42) $\ln(7e^x) = \log_3 81^x$
43) $\log_5 100 = x$
44) $5^x = 100$

Power and Base Tables

2nd Power

N	N^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

3rd Power

N	N^3
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

4th Power

N	N^4
1	1
2	16
3	81
4	256
5	625
6	1296
7	2401
8	4096
9	6561
10	10000

5th Power

N	N^5
1	1
2	32
3	243
4	1024
5	3125
6	7776
7	16807
8	32768
9	59049
10	100000

Base 2

N	2^N
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Base 3

N	3^N
1	3
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
9	19683
10	59049

Base 4

N	4^N
1	4
2	16
3	64
4	256
5	1024
6	4096
7	16384
8	65536

Base 5

N	5^N
1	5
2	25
3	125
4	625
5	3125
6	15625
7	78125

Base 6

N	6^N
1	6
2	36
3	216
4	1296
5	7776
6	46656

Base 7

N	7^N
1	7
2	49
3	343
4	2401
5	16807
6	117649

Base 8

N	8^N
1	8
2	64
3	512
4	4096
5	32768
6	262144

Base 9

N	9^N
1	9
2	81
3	729
4	6561
5	59049
6	531441

Cartesian Geometry III Test

Calculators should not be used on this test.

1) Calculate the distance between the points (2,–6) and (–5,–2). (4 points)

2) Find the domain and range of each function. (2 points each)

a) $f(x) = \frac{6}{x - 4}$

b) $h(z) = 3z + 2$

c) $g(y) = 2 + \sqrt{y + 6}$

3) Give an equation for which the graph of that equation is a hyperbola. (2 points)

4) Convert to degrees: (2 points each)

a) $\pi/3$

b) $11\pi/8$

5) Convert to radians: (2 points each)

a) 150°

b) 720°

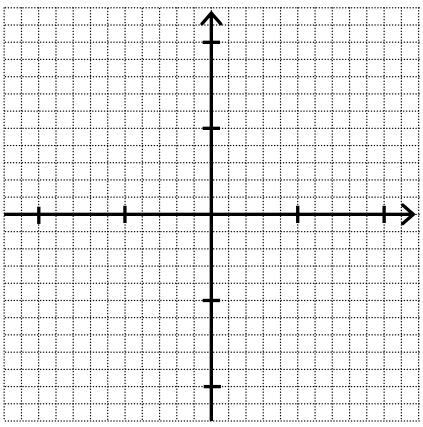
6) Evaluate (2 points each)

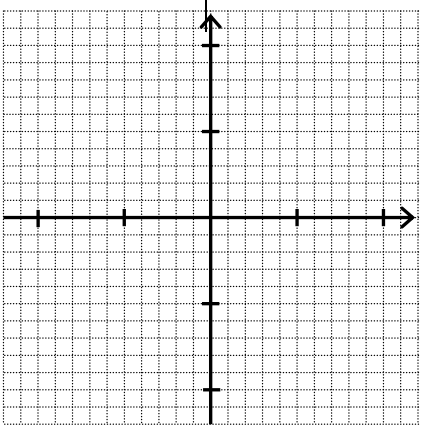
a) $\cos(4\pi/3)$ c) $\sec(\pi/6)$

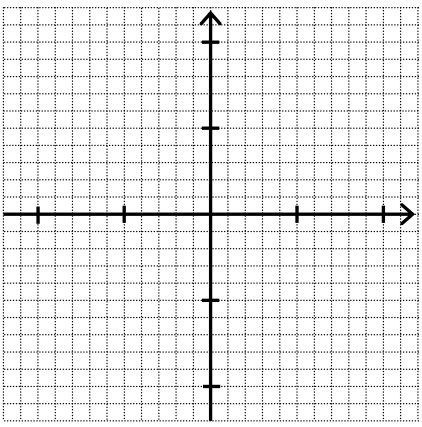
b) $\tan(3\pi/4)$ d) $\csc(5\pi/4)$

7) Give two-variable equations (using x and y) that express each of the two below sentences. Then graph each of the two equations and find a common solution – the solution that satisfies both conditions. (4 points)

"The sum of two numbers is 32. The larger number is 12 greater than twice the smaller."







8) A cannonball is shot out of a cannon, and travels along the parabolic curve given as

$y = -\frac{1}{100}(x-200)^2 + 400$

where y is the height above the ground (in feet) and x is the horizontal distance from the cannon. The coordinates of the mouth of the cannon is (0,0). What are the coordinates of the ball when it is 100 feet above ground? (2 points)

9) Given $f(x) = x^2 - 5$, graph each of the following. Be sure to label your graph! (4 points on the graph must be accurate.) (2 points each)

a) $f(x)$

b) $f(x) + 6$

c) $f(x+3)$

d) $f(-x)$

10) Graph each equation. Be sure to label your graph! (4 points each) (4 points on the graph must be accurate.)

a) $x^2 + y^2 + 6y = 0$

b) $f(x) = (x-5)(x-3)^2(x-1)$

c) $f(x) = x^3 + 4x^2 + 3x$