11^{th} Grade Assignment – Week #26

Individual Work

- From *Cartesian Geometry – Part III* unit, do as much as you can from **Problem Set #6**. Don't do problem #6, as we will save these graphs for later.
- Begin to prepare for the *Cartesian Geometry – Part III* test (which will be part of next week's assignment) by studying Problem Sets #5 and #6.
- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment:

for Tuesday

- **The A-B-C Train Problem**. (Note: answers are given on the next page.)
  You are to build a train by linking together train cars that can be one of three types: two units long and labeled A, two units long and labeled B, or one unit long and labeled C. For example, the train A-C-A-A is seven units long, and so is the train B-C-A-B.

  Answer the following:

  1) Determine the sequence that arises out of the A-B-C train problem. It may be helpful to create a table where the left column (0, 1, 2, 3, 4…) is the length of the train and the right column is the number of possible trains of that length.
  2) Give a recursive formula for the above A-B-C train sequence. (Later, we will derive a general formula for this A-B-C train sequence.)

- **Weighted Average**.
  3) What is Kate's test average if she scored a 68 on her midterm test (worth 35%) and a 90 on her final exam (worth 65%)?
  4) What is Frank's test average if he scored 85, 68, and 90 on his three tests, and they are weighted in a 2:3:6 ratio?
  5) Li's farm has only goats and sheep in a ratio (goats to sheep) of 2 : 5. If each sheep weighs 80kg and each goat weighs 68kg, what is the average weight of an animal on Li's farm?

- **Series practice.** (Do these problems only if you have time.) Expand and then simplify:

  6) $\sum_{i=0}^{6} 3i$ 7) $\sum_{i=1}^{4} 35+5i$ 8) $\sum_{i=8}^{11} i^2$ 9) $\sum_{i=0}^{3} x^i$

  Make sure everyone in the group understands the *Power Series Formula*, which states:

  $$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

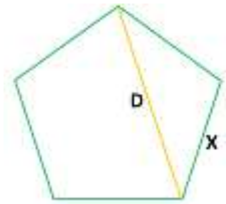
  Use the *Power Series Formula* to evaluate or simplify each one:

  10) $(1+4+4^2+4^3+4^4+4^5+4^6)$ 11) $(1+3+3^2+\ldots+3^{18})$

  12) $\sum_{i=0}^{6} 4^i$ 13) $\sum_{i=0}^{18} 3^i$ 14) $\sum_{i=0}^{n} 3^i$

- **Extra Challenge Problems!**
  15) Find the recursive formula and general formula for this sequence: 7, 17, 47, 137
  16) Find the *General General Formula* for any "combined" sequence, which has the recursive formula $x_n = a \cdot x_{n-1} + b$. You should test your formula by showing that it works for the above sequence.

for Thursday



- **The wonders of Φ.**
  Phi (Φ) arises in mathematics is several places. For starters, it is the ratio of the diagonal to the side of a pentagon. (Somewhat similarly, recall that π is the ratio of the circumference to the diameter of a circle.) From the drawing here you can set x=1, and then derive the ratio $(D-1):1 = 1:D$.

  17) Solve the ratio $(D-1):1 = 1:D$.
  You should get two answers (using the quadratic formula). The positive answer is the value of Φ, and the negative answer we will call $\hat{\Phi}$ ("phi hat").

  18) Here is the Fibonacci sequence:
  1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597
  What new sequence do you get by dividing pairs of numbers (larger by the smaller) in the Fibonacci sequence, starting at the beginning? (Give these numbers rounded to four decimal places.) What do you notice?

  19) Given $\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618034$ and $\hat{\Phi} = \frac{1-\sqrt{5}}{2} \approx -0.618034$, simplify each in terms of Φ and $\hat{\Phi}$:

  a) Φ^2 b) $\hat{\Phi}^2$ c) $\Phi \cdot \hat{\Phi}$ d) $1-\Phi$ e) $\Phi - 1$ f) $1-\hat{\Phi}$ g) $\Phi + \hat{\Phi}$

  h) $\Phi - \hat{\Phi}$ (Give this answer in square root form.)

- **If you still have time…**
  - Try to figure out how I got the ratio $(D-1):1 = 1:D$ from the above pentagon.
  - Work on the "Extra Challenge Problems" from Tuesday's group assignment.
  - Try to take the final step for our big goal: to find the general formula for the Fibonacci sequence. (Lecture #2 left off with this as a question.)

Answers:

1) x_n = 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365, 2731
2) $x_n = x_{n-1} + 2x_{n-2}$
3) $0.35 \cdot 68 + 0.65 \cdot 90 = 82.3$
4) $(2/11)(85) + (3/11)(68) + (6/11)(90) \approx 83.1$
5) $(5/7)(80) + (2/7)(68) \approx 76.57$kg
6) 63
7) 190
8) 376
9) $x^3 + x^2 + x + 1$
10) $\frac{4^7 - 1}{4-1} = 5461$
11) $\frac{3^{19} - 1}{3-1} = 581{,}130{,}733$
12) Same as #10
13) Same as #11
14) $\frac{3^{n+1} - 1}{2}$
15) recursive: $x_n = 3x_{n-1} - 4$; general: $x_n = 5 \cdot 3^n + 2$
16) $x_n = \frac{(ax_0 - x_0 + b)a^n - b}{a-1}$
17) $\Phi = \frac{1+\sqrt{5}}{2} \approx 1.61803398874989485$
$\hat{\Phi} = \frac{1-\sqrt{5}}{2} \approx -0.61803398874989485$
18) 1, 2, 1.5, $1.\overline{6}$, 1.6, 1.625… It gets closer and closer to Φ.
19) a) $\Phi + 1$ b) $\hat{\Phi} + 1$ c) -1 d) $\hat{\Phi}$ e) $-\hat{\Phi}$
f) Φ g) 1 h) $\sqrt{5}$

— Cartesian Geometry – Part III —

Problem Set #6

1) Give the domain and range of:

a) $f(x) = \frac{5}{x}$

b) $f(x) = x^2 - 7$

c) $f(x) = \sqrt{x-5} + 10$

2) Convert to degrees:

a) $\pi/3$ d) $11\pi/8$

b) $7\pi/4$ e) 1

c) $2\pi/3$ f) 3.7

3) Convert to radians:

a) 90° c) 180°

b) 330° d) 220°

4) Evaluate

a) $\cos(2\pi/3)$ d) $\sec(2\pi/3)$

b) $\sin(7\pi/4)$ e) $\csc(7\pi/4)$

c) $\tan(5\pi/6)$ f) $\cot(5\pi/6)$

5) Graph each equation.

a) $x = 4(y-3)^2 + 2$

b) $x^2 + (y+2)^2 = 9$

c) $f(x) = -x^2 + 12x - 36$

d) $x^2 - 2x + y^2 + 14y + 48 = 0$

e) $f(x) = (x-5)(x-3)(x-1)$

f) $f(x) = (x-5)(x-3)^2(x-1)$

g) $f(x) = (x-5)(x-3)^2(x-1)^2$

h) $f(x) = x^3 + 4x^2 + 3x$

i) $f(x) = x^3 - 2x^2 - 19x + 20$

j) $f(x) = x^2 - 7x + 6$

k) $f(x) = x^3 - 7x + 6$

l) $f(x) = x^3 + 5x^2$

m) $f(x) = x^3 - 4x$

n) $f(x) = \frac{1}{10}x^3 - 10x$

o) $f(x) = \frac{1}{10}x^3 + 10x$

p) $f(x) = x^4 - 4x^2$