11th Grade Assignment – Week #25

Group Assignment:

for Tuesday

- Do these problems from the *Cartesian Geometry – Part III* unit:
  - **Problem Set #3:** problems #3, 4, 8
  - **Problem Set #4:** problems #11-14
- **Isa's Birthday Puzzle**.

  Ning and John just met Isa. "When's your birthday?" they asked Isa.
  Isa thought a second and said, "I'm not going to tell you, but I'll give you some clues. One of these dates is my birthday." Then she wrote:
  May 15, May 16, May 19, June 17, June 18, July 14, July 16, August 14, August 15, August 17
  Then Isa whispered in Ning's ear the month – and only the month – of her birthday.
  To John, she whispered the day, and only the day.
  "Can you figure it out now?" she asked Ning.
  Ning said: "I don't know when your birthday is, but I know John doesn't know, either."
  John said: "I didn't know originally, but now I do."
  Ning said: "Well, now I know, too!"
  When is Isa's birthday?

for Thursday

- **The A-C Train Problem**. You are to build a train by linking together train cars that are either two units long (labeled A) or one unit long (labeled C). For example, the train C-A-A-C is six units long. There are, of course, other trains that are six units long, such as: A-C-A-C, or A-A-A or C-C-C-C-C-C.
  How many different trains are there that are 12 units long? 20 units long?
- Do **Problem Set #5:** problems #7, 11.
- If you have extra time, help each other out with some of the Individual Work (below).

Individual Work

- From *Cartesian Geometry – Part III* unit, do these problems (as much as you can, and as much is helpful for your learning):
- **Problem Set #3:** problems #1, 2, 3, 5, 6.
- **Problem Set #4:** problems #1, 2, 3, 15, 16, 17
- **Problem Set #5:** problems #2-6, 8-10.

Problem Set #3

1) Graph (on a separate sheet) each of the following.

a) $\frac{(x+7)^2}{5} + (y-3)^2 = 1$

b) $f(x) = x^2 + 10x + 20$

c) $9x^2 - 36x + 16y^2 + 96y + 36 = 0$

2) Give the domain and range of:

a) $f(x) = -3x^2 + 5$

b) $f(x) = 8 - \sqrt{x+3}$

3) Calculate the distance between the points (7,4) and (3,–4).

4) Derive a formula that determines the distance, d, between the two points (x_1,y_1) and (x_2,y_2).

5) Graph each equation on the graph below.

a) $y = x$

b) $y = x^2$

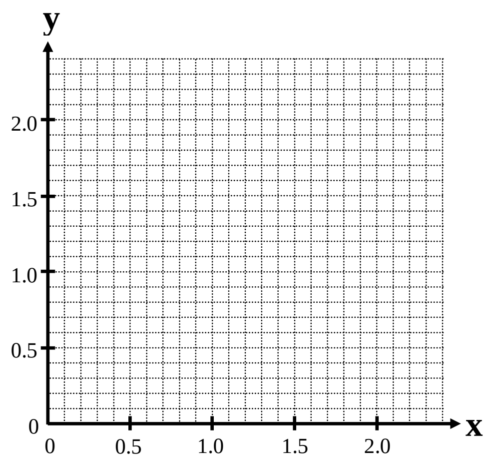
c) $y = x^3$

d) $y = x^4$

e) $y = x^{1/2}$

f) $y = x^{1/3}$

g) $y = x^{1/4}$



6) Give two-variable equations (using x and y) that express each of the two below sentences, then graph each of the two equations and find a common solution – the solution that satisfies both conditions.

The number of dimes and quarters is 12. The value of these coins is $2.25.

7) **The Trig Unit Circle**

On a separate sheet of blank paper, and done very neatly, draw a large circle, centered on the page, such that the diameter is about half of the width of the page. Draw the x and y axes through the center of the circle. The radius of this circle is 1.

a) What is the circle's circumference?

b) What is the equation of this circle?

Starting at (1,0) and moving in a counter-clockwise direction, write down the coordinates at that point on the circle you end up at if you travel a distance of…

c) π

d) $\pi/2$

e) $\pi/3$

f) $\pi/4$

g) $2\pi/3$

h) $4\pi/3$

i) $5\pi/4$

j) $\pi/6$

k) $5\pi/6$

l) $3\pi/2$

m) $7\pi/4$

n) $5\pi/3$

This circle you have created is very important! Keep it for future reference.

8) A cannonball is shot out of a cannon, and travels along the parabolic curve given as

$$y = -\frac{1}{100}(x-200)^2 + 400$$

where y is the height above the ground (in feet) and x is the horizontal distance from the cannon. The coordinates of the mouth of the cannon are (0,0).

a) Graph the path of the cannonball.

b) Give an approximation of the angle of inclination that the ball was shot.

c) How far away does the ball land from the cannon?

d) What are the coordinates of the ball when it is 300 feet above ground?

e) What are the coordinates of the ball when it is 200 feet above ground?

f) What is the maximum height that the cannonball reaches?

g) How far away from the cannon is the ball when it is at its maximum height?

Problem Set #4

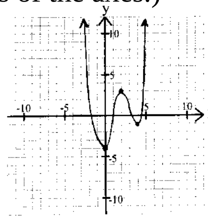
Graphing Functions

For each f(x), given further below in #1-3, graph:

a) f(x) d) 2•f(x)
b) f(x) + 4 e) –f(x)
c) f(x+4) f) f(–x)

This should result in six curves per graph. (Carefully choose the range of the values of the axes.)

1) $f(x) = x^2$
2) $f(x) = x^3$
3) f(x) as given here →.



Radian Measure

Look at the *Trig Unit Circle* that you made on the previous problem set. Imagine a vector with its tail at the origin and its tip starting at (1,0) and then moving counter-clockwise along the circle. We can describe where it ends up either by stating how far along the circle it has traveled (radian measure), or by stating the size of the angle that the vector has swept (degree measure).

4) A point travels $^1/_6$ of the way around the circle.
  a) How far did it travel?
  b) What angle did the vector sweep?
  c) What are the ending coordinates?

Can you see the connection this has to trigonometry? Using the above answers, we can see that:

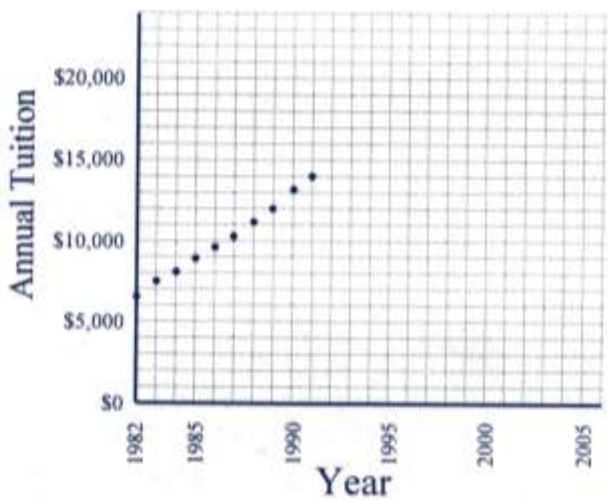
$\cos(60°) = ½$ and $\sin(60°) = \frac{\sqrt{3}}{2}$

Radian measure expresses the same thing as:

$\cos(^\pi/_3) = ½$ and $\sin(^\pi/_3) = \frac{\sqrt{3}}{2}$

Locate the point on the unit circle, convert to degrees, and then evaluate:

5) $\cos(^\pi/_4)$ 8) $\tan(^\pi/_4)$
6) $\sin(^\pi/_6)$ 9) $\sin(\pi)$
7) $\cos(^\pi/_2)$ 10) $\cos(^{3\pi}/_4)$



Extrapolating Data

The above graph represents the cost of annual tuition at a particular college between the years 1982 and 1991.

11) Draw a "best fit" straight line through the points, and then give an equation of that line.

12) Use your equation to predict what the tuition would be in each of the following years:
  a) 1988 c) 1970
  b) 2000 d) 2015

13) What are some of the limitations or weaknesses of the method you have used for predicting values into the future (or the past)?

14) What is the significance of the y-intercept of this line?

15) Graph the below equation and find its roots.
$y = –¼(x+7)^2 + 1$

16) Give two-variable equations (using x and y) that express each of the two given sentences. Then graph each of the two equations and find a common solution – the solution that satisfies both conditions.
  a) Together, a coffee and a donut cost $3.35. The donut costs 40¢ less than twice the price of the coffee.
  b) A rectangle has an area of 15. Its perimeter is 17.

17) Graph each of the two given equations and then find the exact common solution.
  a) $y = (x+3)^2 – 1$
  $y = –½(x+3)^2 + 5$
  b) $2y + 3x + 4 = 0$
  $8y^2 + x^2 – 48y + 8x + 80 = 0$

Problem Set #5

On the previous problem set we were introduced to *radian measure*, which is frequently used with trigonometric functions in the study of calculus. Our perspective of trigonometry is further changed as we think of the values of *cos*(θ) and *sin*(θ) as x and y coordinates on the *Trig Unit Circle*. More practice with these concepts should help to clarify things.

1) What advantages are there to thinking of trigonometric functions on the unit circle?

2) For each problem, locate the point on the unit circle, convert it to *degree measure*, and then evaluate it.

a) $\cos(\pi/6)$ e) $\sin(5\pi/6)$
b) $\sin(\pi/2)$ f) $\tan(3\pi/4)$
c) $\cos(\pi/2)$ g) $\sec(7\pi/6)$
d) $\tan(\pi/3)$ h) $\csc(5\pi/3)$

3) For each problem, locate the point on the unit circle, convert it to *radian measure*, and then evaluate it.

a) $\cos(45°)$ d) $\tan(135°)$
b) $\sin(120°)$ e) $\csc(270°)$
c) $\cos(0°)$ f) $\cot(240°)$

4) Let D represent the degree measure of a certain angle, and R represent the equivalent radian measure. For any angle, what is D:R?

5) Use the above ratio to convert radians to degrees.

a) $\pi/4$ d) 4π
b) $11\pi/6$ e) $3\pi/2$
c) $4\pi/5$ f) 2

6) Use the above ratio to convert degrees to radians.

a) 90° c) 216°
b) 150° d) 3600°

The Root-Factor Theorem states that if b is a root to a polynomial function, then (x–b) is a factor of the polynomial.

7) Find f(x) given that it is a third degree polynomial equa-tion with roots x = 0, 2, –3, and the coefficient of the x^3 term is 5.

8) Graph each function by identifying its roots, and plotting a few key points.

a) $f(x) = (x+4)(x+1)(x-2)$
b) $f(x) = -(x+4)(x+1)(x-2)$
c) $f(x) = -\frac{1}{2}(x-5)(x-1)$
d) $f(x) = (x+4)^2(x-2)$
(What does the exponent do?)
e) $f(x) = (x+3)(x+1)(x-1)(x-3)$
f) $f(x) = x^3 + 3x^2 - x - 3$
($x = -1$ is one root)

9) Where does the last function above intersect with $g(x) = 9x - 3$.

10) Graph each function.

a) $f(x) = x^3 - 4x$
b) $f(x) = -x^3 + 4x$
c) $f(x) = x^3 - 4x + 3$
d) $f(x) = (x+2)^3 - 4(x+2)$

11) Frank kicks a ball toward a 70-foot high fence. The path of the ball is given by

$$y = -\frac{1}{10}(x-30)^2 + 90,$$

where y is its height and x is its horizontal distance. Frank is 50 feet from the fence. Will the ball clear the fence?