# $11^{th}$ Grade Assignment – Week #24

Group Assignment:

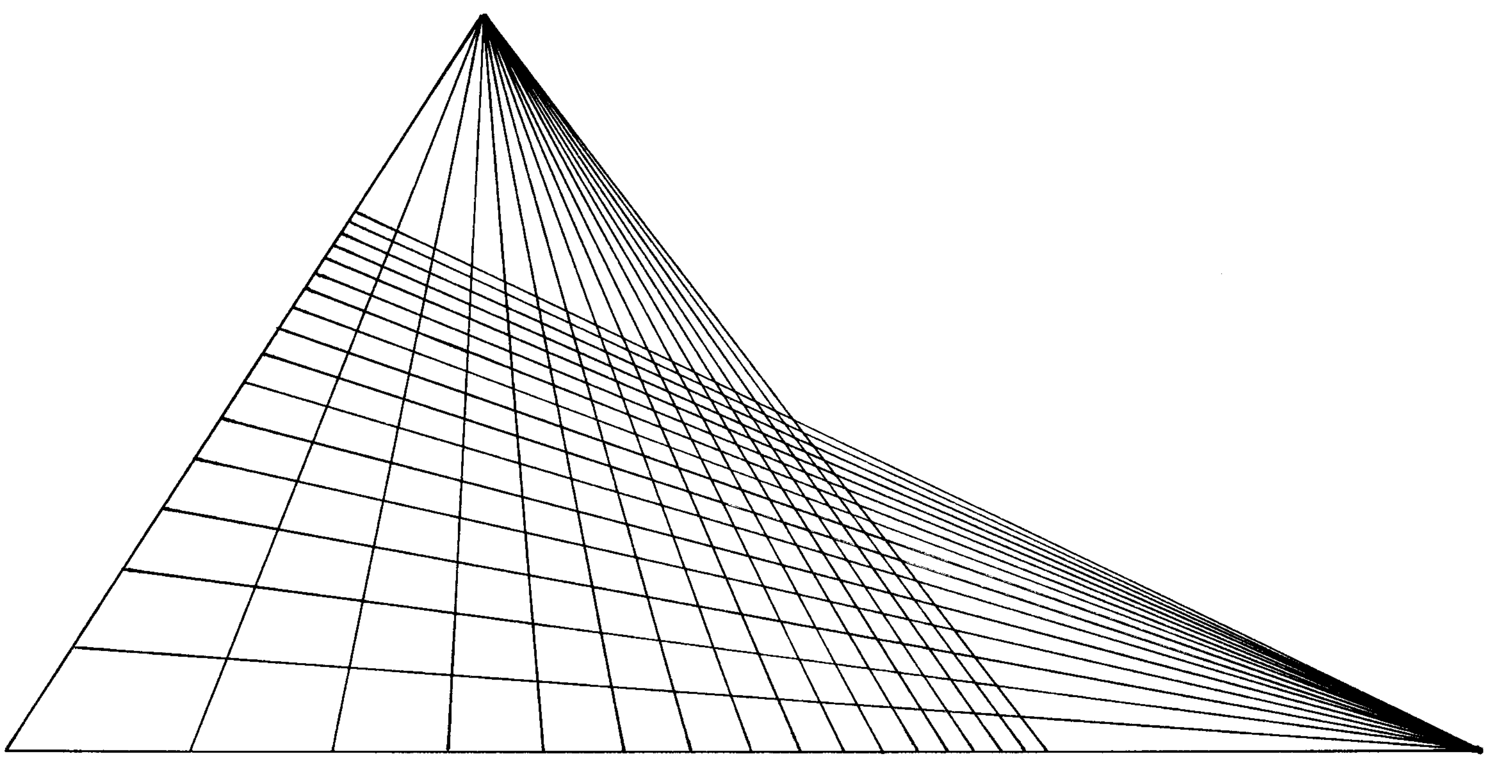
*for Tuesday*

- From the *Cartesian Geometry – Part III* unit, do **Problem Set #2**, problems #2, 7, 8, 9.
- **Using a Harmonic Net as a Coordinate System.**
  The following exercise is from the *Cartesian Geometry – Part II*, but I chose to delay it until we had taken our *Projective Geometry* course.

  The below drawing is created from a projective geometry *harmonic net*. It can be imagined as a checkerboard in two-point parallel perspective. If we draw the diagonals of the "squares", then we get two more points on the same line as the two vanishing points. These two new points are harmonic conjugates of the two original vanishing points.

  1) With the below drawing, choose one of the lower "horizontal" lines as the x-axis, and one of the center "vertical" lines as the y-axis. Plot points for $y = x^2$. Can you picture where the point would be for an infinitely large x? What is the resulting curve?
  2) Using the curve from the previous problem, imagine "nailing down" two points (e.g., (1,1) and (-2,4)) and then sliding the top vanishing point infinitely far upwards, and the right vanishing point infinitely far to the right. This makes it so that the horizontal lines are evenly spaced, and the vertical lines are evenly spaced. What does the curve look like now?
  3) Considering the drawing from problem #1, what would need to be changed in order that the resulting curve would be a hyperbola?

  Unless otherwise stated, we normally graph equations using the standard Cartesian coordinate system. But keep in mind that any equation can instead be graphed on a different coordinate system, resulting in a completely different curve. (Do you remember briefly doing *Polar Coordinates* in the *Cartesian Geometry – Part II* unit?)



## Individual Work

- **Before Thursday's group meeting**, make your own *Trig Unit Circle*.
  Here are the instructions:
  - For this exercise, you will use the *Trig Unit Circle* given at the end of this document. You will need to do this very neatly, because you will use this in the future as a reference. You may wish to create this circle from scratch yourself on a clean piece of stronger paper.
  - The *Unit Circle* has these properties: the radius is 1, the diameter is 2, and the circumference is $2\pi$.
  - Remember that the vertex of the angle is always at the origin (0,0), and the angle is measured counterclockwise, using the x-axis from the origin to the point (1,0).
  - There are 16 key points on the circle, which are defined by these angles:
    0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, 210°, 225°, 240°, 270°, 300°, 315°, 330°.
  - At each of these locations on the circle, you need to write the **Radian Measure** (explained in detail in a moment), the **Angle Measure**, and the **Coordinate**. Note that the first two locations are done for you.
  - The *Radian Measure* is simply the distance, measured along the circumference of the circle, from the starting point (1,0) to the point in question. Can you see why the radian measure for 30° is $^{\pi}/_{6}$? (It's because we have traveled $^{1}/_{12}$ of the distance around the circle, and $^{1}/_{12}$ of the circumference of the circle (which is $2\pi$) is $^{\pi}/_{6}$.)
- **Take the trigonometry test** which is at the end of this document.
- From *Cartesian Geometry – Part III* unit, do **Problem Set #1** (all of the problems), and **Problem Set #2**, problems #1, 3, 4, 5, 6.

## Group Assignment:

*for Thursday*

- Check with one another that your *Trig Unit Circle* is correct.
- Finish any of the problems from Tuesday that you still need to work on.
- **Let's make a Deal!** On the game show "Let's Make a Deal", the contestant was always given the choice of three doors (A, B or C). Hidden behind one of the doors was a new car, and behind each of the other two doors was a joke prize (e.g., a goat). The contestant would state which door he chooses, hoping behind that door there would be a new car. But Monty Hall, the game show host, who knows what is behind each door, would *always* then open one of the other doors – a door that doesn't have the car behind it. Then Monty would offer the contestant the chance to switch his choice to the other remaining door. The question before us is: should he switch, and, if he does, how does that change the probability of his winning a car?

# Cartesian Geometry – Part III

## Problem Set #1

1) Find each of the following given that:

$f(x) = x^2 - 4$
$g(x) = \sqrt{x+5}$
$h(x) = 3x + 5$

| | | | |
|---|---|---|---|
| a) | $f(5)$ | g) | $f(f(3))$ |
| b) | $g(20)$ | h) | $f(g(7))$ |
| c) | $h(⅔)$ | i) | $f(h(-2))$ |
| d) | $f(-3)$ | j) | $f(h(x))$ |
| e) | $g(-3)$ | k) | $h(f(x))$ |
| f) | $g(x^2-7)$ | l) | $h(f(g(x)))$ |

2) Graph the three functions, f(x), g(x), and h(x) as defined above.

### Domain and Range

In higher level mathematics, it is important to have a good understanding of the terminology and language used with functions. In the beginning such language may seem awkward. We will now introduce the idea of *domain* and *range*.

*Domain* is the set of real numbers that can be put into a given function, and *range* is the set of real numbers that can possibly come out of the function.

3) For each of the following functions, explain why the domain and range is as stated.

a) $f(x) = x^2 - 4$
The domain is x can be any real number.
The range is $f(x) \geq -4$

b) $g(x) = \sqrt{x+5}$
The domain is $x \geq -5$
The range is $g(x) \geq 0$

c) $h(x) = 3x + 5$
The domain is x can be any real number.
The range is f(x) can be any real number.

4) Graph each of the following.

a) $x^2 + y^2 = 6$

b) $x = -\frac{1}{2}y^2$

c) $y = 3x^2$

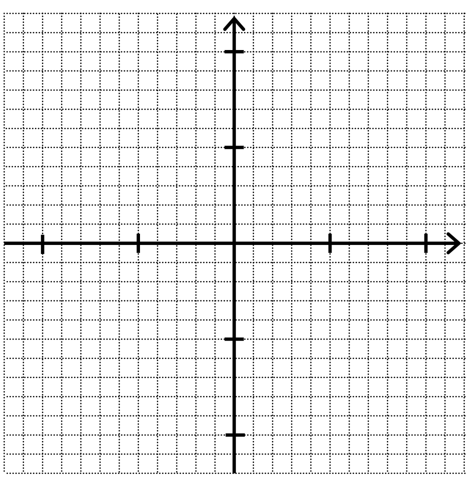
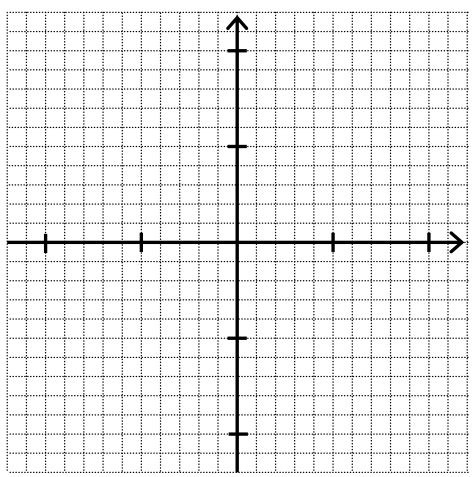
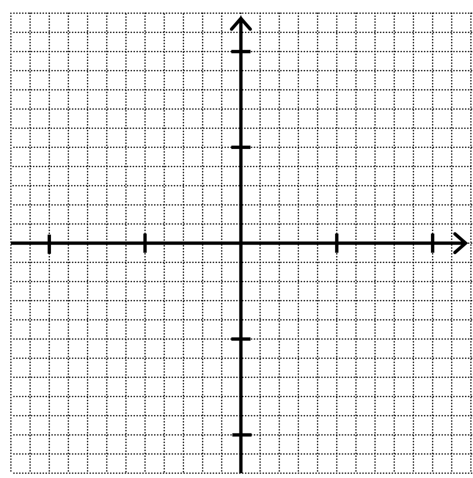
d) $\frac{(x+3)^2}{36} + \frac{(y-1)^2}{16} = 1$

e) $y = -3x^2 - 24x - 42$

f) $25x^2 + 10y^2 - 250 = 0$

5) Graph each of the two below equations and then find the exact common solution.

$x^2 + (y-3)^2 = 25$
$y - 3x = -5$

# Problem Set #2

1) Calculate the distance between the points (3,5) and (–4,2).

2) Give the domain and range of each of the six trigonometric functions.

3) Graph each of the following.

a) $\frac{x^2}{4} + \frac{(y+5)^2}{9} = 1$

b) $f(x) = x^2 + 10x + 18$

c) $f(x) = \frac{2}{3}x + 5$

d) $16x^2 + 7y^2 - 42y - 49 = 0$

e) $x^2 + y^2 + 6y = 0$

4) Graph each of the two given equations and then find the exact common solution.

a) $2x - 3y = -21$
$2x + y = -1$

b) $y = 2x^2 - 12x + 15$
$x - y = 3$

5) Find the domain and range.

a) $f(x) = \frac{1}{x^2 - 9}$

b) $g(y) = 4 + \sqrt{5-y}$

6) Graph $x \cdot y = 6$.

7) We know that the graph of the equation $x^2 + y^2 = 9$ (on a Cartesian coordinate system) is a circle. Let's see what happens if we change it slightly. Graph each equation:

a) $x^2 - y^2 = 9$

b) $-x^2 + y^2 = 9$

c) $x^2 + y^2 = -9$

## Curve Shape

8) Graph each of the following by only showing the points that fall in the first quadrant (i.e., where the values for x and y are both positive). Use the graph given below.

a) $x + y = 1$

b) $x^2 + y^2 = 1$

c) $x^3 + y^3 = 1$

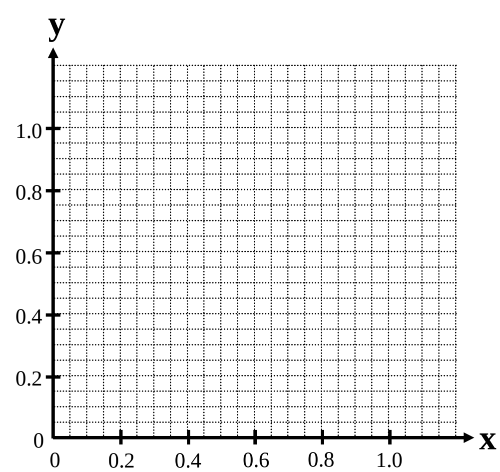
d) $x^4 + y^4 = 1$

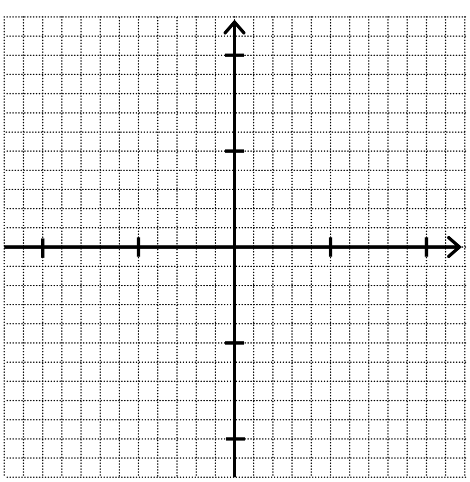
e) $x^{1/2} + y^{1/2} = 1$

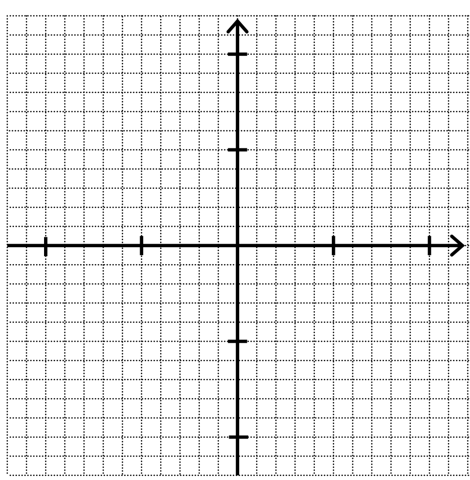
f) $x^{1/3} + y^{1/3} = 1$

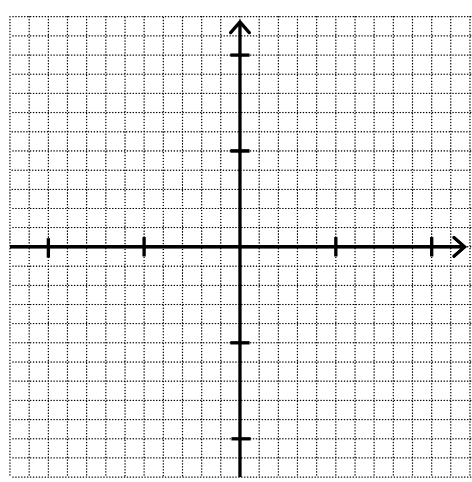
g) $x^{1/4} + y^{1/4} = 1$

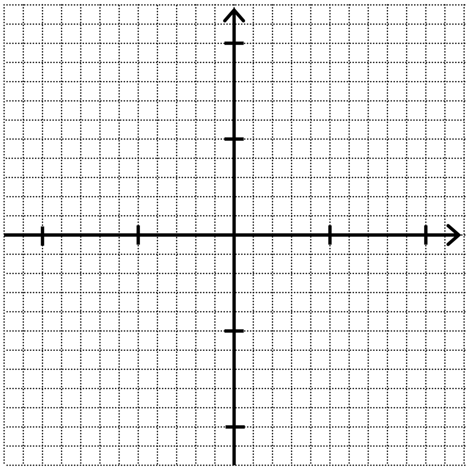
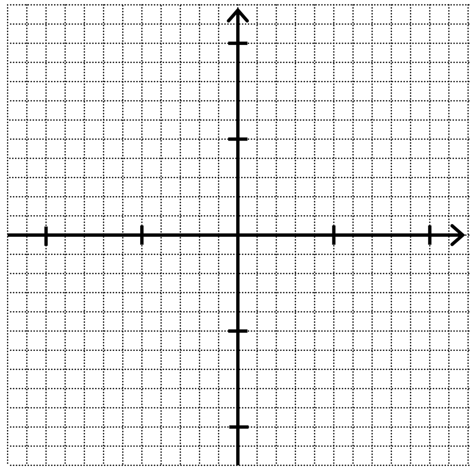
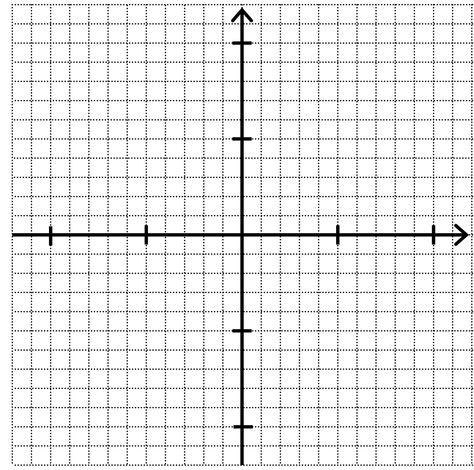
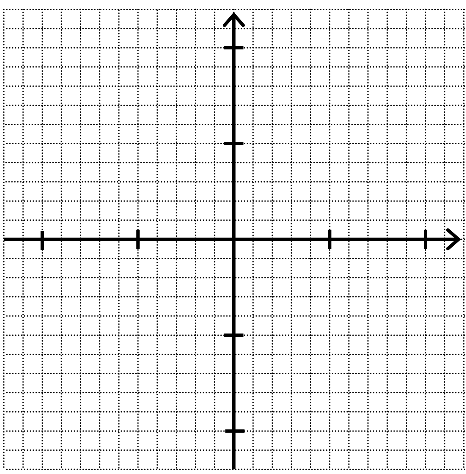
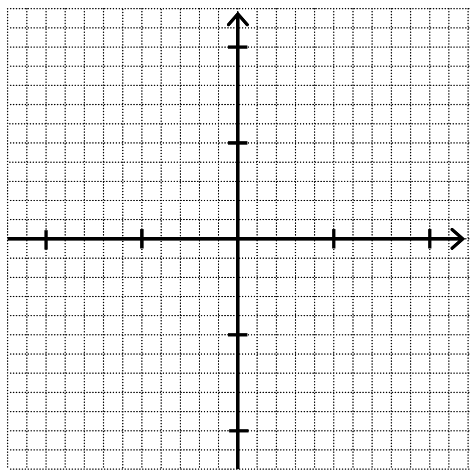
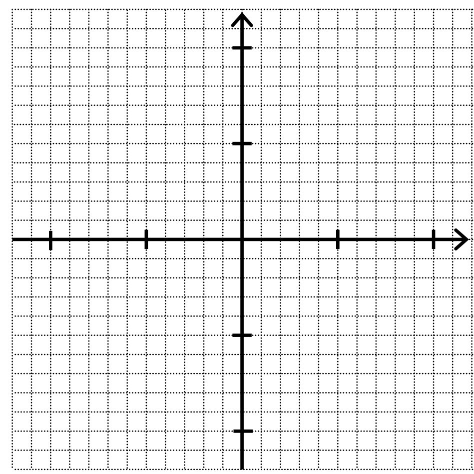
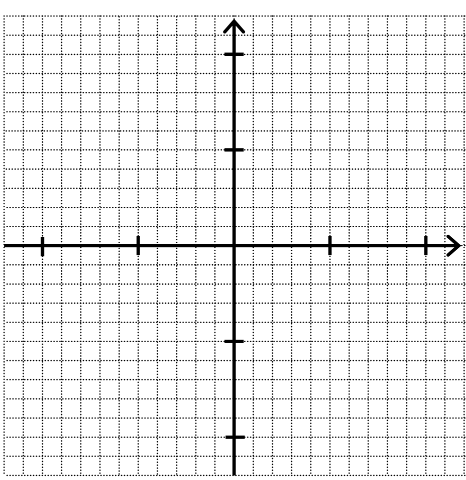
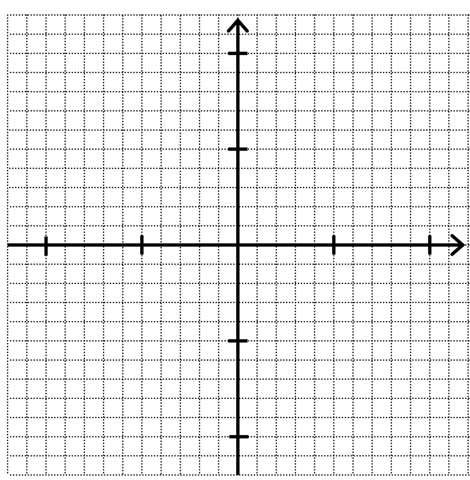
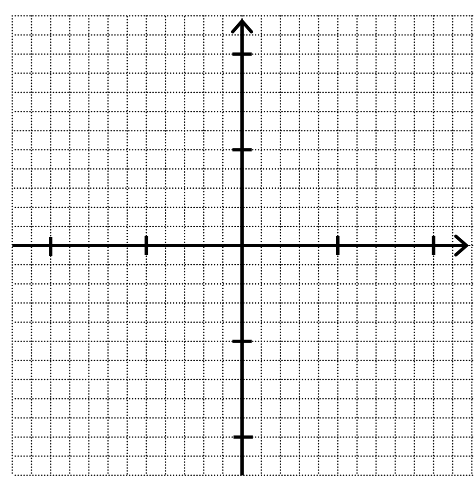
9) The general form of the above equations is $x^n + y^n = 1$. What can be said about how the value of n affects the curve?







# Trig III Test

The Three Laws:

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Law of Cosines :

$c^2 = a^2 + b^2 - 2ab\cdot\cos(C)$

Law of Tangents:

$\frac{\tan[\frac{1}{2}(A-B)]}{\tan[\frac{1}{2}(A+B)]} = \frac{a-b}{a+b}$

Give an answer as accurately as possible
**(1 point each.) You may not use a calculator!**

1) $\sin(150°)$

2) $\cos(135°)$

3) $\tan(120°)$

4) $\csc(15°)$

5) $\sec(120°)$

6) $\sin^{-1}(\frac{\sqrt{3}}{2})$

7) $\cot^{-1}(\sqrt{3})$

8) $\sec^{-1}(-\sqrt{2})$

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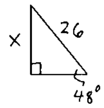
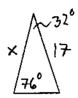
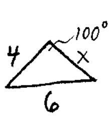
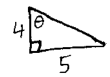
**You may use a calculator, if you wish, on the rest of the test.**

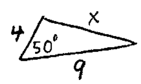
9) Find the variable indicated. **(4 points each.)**

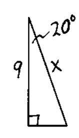
a) 


b) 


c) 

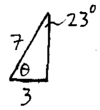

d) 


e) 


f) 

10) Find the variable indicated. **(4 points each.)**

a)

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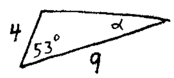
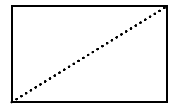



11) The *Golden Rectangle* is a special rectangle where the ratio of the base to the height is $\Phi$:1, where $\Phi \approx 1.618$. What is the angle that is formed by the base and a diagonal?



12) A plane is about to land on an 8-km long runway. At a given moment, the plane is 5km from one end of the runway and 4km from the other end. How far above the ground is it?

13) Explain why the following identity is true, or how it can be shown/proved to be true.
$tan^2\alpha + 1 = sec^2\alpha$

# The Trig Unit Circle