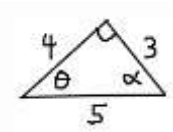
11th Grade Assignment – Week #23

Group Assignment:

for Tuesday

1) With the triangle shown on the right, calculate the values of the two angles, then fill out the table.



	α =	θ =
sin		
cos		
tan		
csc		
sec		
cot		

- Do **Problem Set #5**. Do the problems in this order, and get as far as you can: #1, 5, 8, 4, 6, 3, 7, 2.

for Thursday

- From **Problem Set #6**, do problems #1-3, 32, 31, 30

Individual Work

- Overall, you task is to prepare for the trigonometry unit test which will be included in next week's assignment.
- Do the rest of the problems from **Problem Set #5 and #6** that your group did not complete.

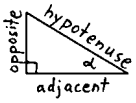
Problem Set #5

The Six Trig Functions

We are now quite familiar with the three most important trig functions: *sine, cosine and tangent*. The last three trig functions are the reciprocals of the first three:

- The reciprocal of *cosine* is *secant* (*sec*).
- The reciprocal of *sine* is *cosecant* (*csc*).
- The reciprocal of *tangent* is *cotangent* (*cot*).

Therefore, the six trigonometric functions are as follows:

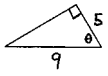


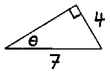
- $cos(\alpha) = \frac{adj}{hyp}$; $sec(\alpha) = \frac{hyp}{adj}$;
- $sin(\alpha) = \frac{opp}{hyp}$; $csc(\alpha) = \frac{hyp}{opp}$;
- $\tan(\alpha) = \frac{opp}{adj}$; $cot(\alpha) = \frac{adj}{opp}$;

1) Without the use of a calculator, give an exact answer or an estimate.

a) *cos*(30°) g) *cos*(20°)
b) *sec*(30°) h) *sec*(20°)
c) *sin*(45°) i) *sin*(16°)
d) *csc*(45°) j) *csc*(16°)
e) *tan*(60°) k) *tan*(72°)
f) *cot*(60°) l) *cot*(72°)

2) Find the variable indicated. (Beware of SSA!) Also, for any triangle with no solution, explain why it is impossible.

a) 

b) 

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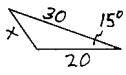
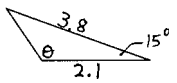
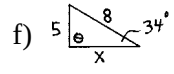
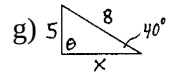
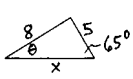
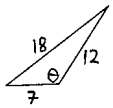
d) 

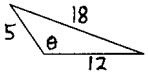
e) 

f) 

g) 

h) 

i) 

j) 

3) With a sextant, the captain of a ship measures that the angle between the lines of sight of the two lighthouses is 32.7°. The captain also knows that the ship is 4.8km from one lighthouse and 3.1km from the other lighthouse. How far apart are the two lighthouses?

4) Fill in the below table (with exact values).

Table of the Six Trig Functions

Values of θ

	0°	30°	45°	60°	90°	120°	135°	150°	180°
cos(θ)									
sin(θ)									
tan(θ)									
sec(θ)									
csc(θ)									
cot(θ)									

5) *Pythagorean Trigonometric Identities.* Starting with the basic identity

$sin^2\theta + cos^2\theta = 1$

derive a new identity by…

a) Dividing all the terms by $cos^2\theta$.

b) Dividing all the terms by $sin^2\theta$.

6) Without using the trig buttons on your calculator, calculate the value of the other five trig functions of θ given that…

a) For $\theta = 17°$ $sin(17°) \approx 0.2924$

b) For $\theta = 53°$ $cos(53°) \approx 0.6018$

c) For $\theta = 39°$ $tan(39°) \approx 0.8098$

7) Find the area and the lengths of the three altitudes of a triangle that has sides of length 5, 6, 7.

8) A baseball diamond is a square with 90-foot long sides. The pitcher's mound is 60½ feet from home plate (in a line with second base). How far is it from the pitcher's mound to first base?

Problem Set #6

Trigonometric Identities

1) All of the identities that we have encountered so far are listed below. Explain what each one means and how it can be useful.

Pythagorean Identities:

$sin^2\alpha + cos^2\alpha = 1$

$tan^2\alpha + 1 = sec^2\alpha$

$1 + cot^2\alpha = csc^2\alpha$

Supplementary Identities:

$sin(180°-\alpha) = sin(\alpha)$

$cos(180°-\alpha) = -cos(\alpha)$

$tan(180°-\alpha) = -tan(\alpha)$

Complementary Identities:

$sin(90°-\alpha) = cos(\alpha)$

$cos(90°-\alpha) = sin(\alpha)$

$tan(90°-\alpha) = cot(\alpha)$

The Three Laws:

Law of Sines: $\frac{a}{b} = \frac{sin\,A}{sin\,B}$

Law of Cosines :

$c^2 = a^2 + b^2 - 2ab\cdot cos(C)$

Law of Tangents:

$\frac{\tan[\frac{1}{2}(A-B)]}{\tan[\frac{1}{2}(A+B)]} = \frac{a-b}{a+b}$

Trig Function Relationships:

$sec(\alpha) = \frac{1}{cos(\alpha)}$; $csc(\alpha) = \frac{1}{sin(\alpha)}$

$cot(\alpha) = \frac{1}{tan(\alpha)}$; $tan(\alpha) = \frac{sin(\alpha)}{cos(\alpha)}$

Ptolemy's Formulas (From last year):

$sin(\frac{1}{2}\alpha) = \sqrt{\frac{1}{2} - \frac{1}{2}cos\alpha}$

$sin(\beta-\alpha) = sin(\beta)cos(\alpha) - sin(\alpha)cos(\beta)$

$cos(\alpha+\beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$

2) Given the trig facts you have memorized, plus the additional three facts below, calculate each of the below values.
(Hint: You only need your calculator once! No trig buttons!)

$\sec(80°) \approx 5.759$; $\sin(40°) \approx 0.6428$; $\cos(20°) \approx 0.9397$

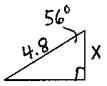
a) $\cot(60°)$ b) $\sec(60°)$ c) $\cos(160°)$ d) $\sin(140°)$

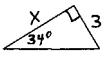
e) $\csc(140°)$ f) $\cos(50°)$ g) $\sec(100°)$ h) $\tan(120°)$

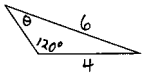
3) Knowing the *sine* and *cosine* of 45° and 30°, use one of Ptolemy's formulas, and leave answers in radical form.

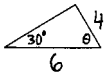
a) $\sin(15°)$ b) $\sin(22\frac{1}{2}°)$ c) $\cos(75°)$

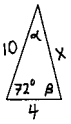
4) Find the variable indicated.

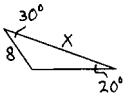
a) 

b) 

c) 

d) 

e) 

f) 

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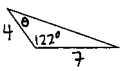
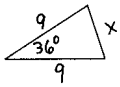
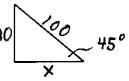
h) 

i) 

j) 

Without the use of a calculator, give an exact answer or an estimate.

5) $\sin(60°)$
6) $\cos(45°)$
7) $\tan(55°)$
8) $\sin(150°)$
9) $\cos(170°)$
10) $\tan(135°)$
11) $\sec(30°)$
12) $\csc(135°)$
13) $\cot(120°)$
14) $\cos(80°)$
15) $\sec(80°)$
16) $\sin(140°)$
17) $\csc(140°)$
18) $\sin^{-1}(\frac{\sqrt{3}}{2})$
19) $\cos^{-1}(\frac{\sqrt{3}}{2})$
20) $\sec^{-1}(2)$
21) $\tan^{-1}(\frac{\sqrt{3}}{3})$
22) $\cot^{-1}(\frac{\sqrt{3}}{3})$
23) $\csc^{-1}(0.5)$
24) $\sin^{-1}(0.2)$
25) $\csc^{-1}(2)$
26) $\tan^{-1}(-3)$
27) $\cot^{-1}(-3)$
28) $\sec^{-1}(-\sqrt{2})$

29) Find all of the angles of a triangle that has sides of length 6, 8 and 11.

30) Two ships leave a port at noon, one heading due north at 15 mph, and the other heading at a bearing of 57° west of north at 13mph. How far apart are the ships at 2:30pm?

31) Find the perimeter of a regular 12-gon inscribed in a circle with a diameter of 1. The 12-gon takes up what percentage of the circle's area?

32) A person is standing 800 feet (on level ground) from where a balloon is released. The balloon rises straight upwards at a constant speed. At a certain moment, the person sees the balloon at a 23.6° angle of elevation. Exactly one minute later he sees it at an angle of elevation of 54.1°. How far did the balloon rise during that minute, and what is the balloon's speed?