11th Grade Assignment – Week #22

Individual Work

- Work on problems from **Problem Set #3** in the *Trigonometry – Part III* unit.

Group Assignment:

for Tuesday

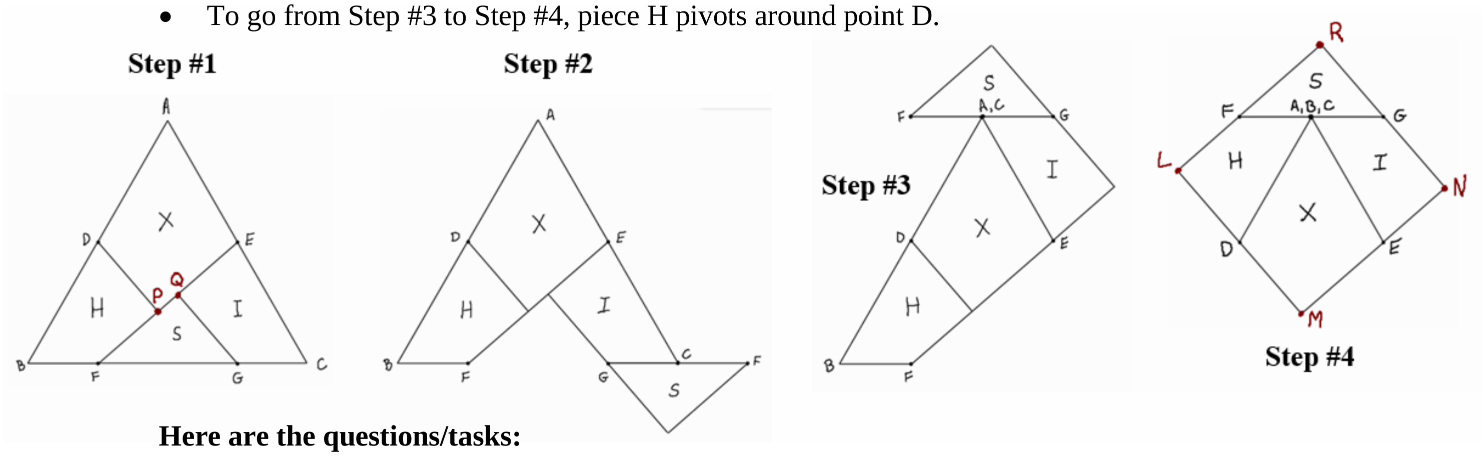
- Prove the Law of Tangents by working through **Problem Set #4** in the *Trigonometry – Part III* unit.

for Thursday

- *Puzzle! Triangle & Square Pieces – Part II*

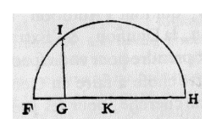
  This is a continuation of last week's puzzle problem. Below, you can see a four-step depiction of the transformation of an equilateral triangle. Note that in Step #1, D and E are midpoints of the sides of the triangle, F and G are located directly below D and E, and if we were to connect DEFG, we would get a rectangle. Also, FG is half of BC, and DP and GQ are of equal length and both are perpendicular to FE. **It is important that you can visualize the following progression:**

  - To go from Step #1 to Step #2, piece S pivots around point G.
  - To go from Step #2 to Step #3, piece S and I together pivot around point E.
  - To go from Step #3 to Step #4, piece H pivots around point D.



Here are the questions/tasks:

1) Find the area of an equilateral triangle that has edges of length 2.
2) Find the length of the side of a square that has the same area as the above triangle (in problem #1).
3) In the drawing from Step #4, find the length of LM and MN. What do you now know?
4) We will now alter the drawing in Step #1 in order to produce a perfect square in Step #4. To do this, we will simply start with point F in a slightly different location (not exactly directly below D). Does point F need to move to the left or the right?
5) Note: FG = ½ BC, FP = QE, and FE is the side of the final square, which (from problem #2, above) must be equal to $\sqrt[4]{3}$. We also know that the triangle (step #1) has an altitude of $\sqrt{3}$. Therefore, how can we use a compass and straightedge to construct the exact length of FE? (Hint: In our Descartes main lesson, we learned to take the square root of a line (GH in the drawing, above right), as long as we know "unity" (FG). The square root is then IG.)

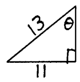


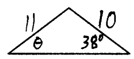
6) Once you have successfully determined a compass and straightedge method for the previous problem, you can then complete the slightly altered drawing in Step #1. Start with an equilateral triangle with each side equal to 2 decimeters (20 cm). Locate point F (see step #1 drawing) using Descartes's method, and then make DP and GQ perpendicular to FE.
7) Calculate all the angles in the Step #1 drawing you have just created.

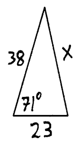
Problem Set #3

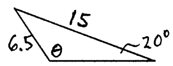
1) State the *Law of Sines* and the *Law of Cosines*. Under what circumstances do you use each one?

2) Find the variable indicated.

a) 

c) 

b) 

d) 

3) Regarding the last problem…

a) What makes it more difficult than the other problems?

b) In general, how can you tell by looking at a triangle that this may happen?

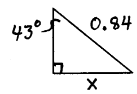
c) Make a sketch of another triangle that has the same initial measurements as the triangle in problem #2d.

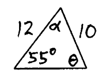
d) What are the two possible answers for θ?

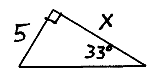
4) Without the use of a calculator, give an exact answer or an estimate.

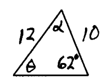
a) $cos(135°)$
b) $sin(135°)$
c) $tan(135°)$
d) $cos(150°)$
e) $sin(150°)$
f) $tan(150°)$
g) $tan^{-1}(2)$
h) $cos^{-1}(-0.5)$
i) $tan(38°)$
j) $sin(70°)$
k) $cos(70°)$
l) $sin(110°)$
m) $cos(110°)$
n) $cos^{-1}(0.866)$
o) $tan^{-1}(-\sqrt{3})$
p) $sin^{-1}(0.3)$

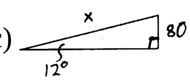
5) Find the variable indicated.

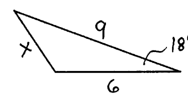
a) 

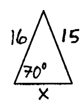
e) 

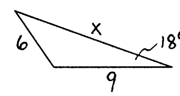
b) 

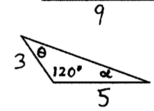
f) 

c) 

g) 

d) 

h) 

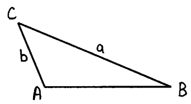
i) 

Problem Set #4

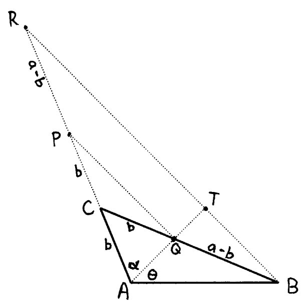
The Law of Tangents

While the *Law of Sines* and the *Law of Cosines* are very well known, the *Law of Tangents* is not well known, but is both useful and interesting.

To derive the *Law of Tangents*, we begin with any triangle, ΔABC.



We then extend side b by the length of side a, past point C up to point R. Points P and Q are found by measuring them at a distance of b away from point C. Lines are then joined as shown in the drawing below.



In order to derive the *Law of Tangents*, we must first answer the following questions:

1) How do we know that PQ is parallel to RT?

2) Find each of these angles in terms of ∠A and ∠B:
∠ACQ, ∠PQC, ∠PCQ, α, ∠CQA, ∠TQB, ∠AQP, ∠ATR, θ.

3) Which triangles in the above drawing are similar?

4) Let x = AT. Explain why TB = x•tan(θ).

5) Similarly, find RT.

6) Given all that has been said above (and hopefully it is all correct!) derive the *Law of Tangents*, which is:

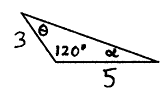
$$\frac{\tan[\frac{1}{2}(A-B)]}{\tan[\frac{1}{2}(A+B)]} = \frac{a-b}{a+b}$$

7) With the above formula…
a) What does ½(A+B) represent?
b) What does ½(A–B) represent?

Why is this formula useful?

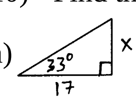
Usually when we are given two sides and an angle, and want to find a missing side, we simply use the *Law of Sines*. However, if the given angle is <u>in-between</u> the two given sides, the *Law of Sines* won't work. This is when we use the *Law of Tangents.*

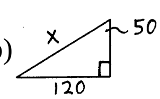
8) Use the *Law of Tangents* to solve this problem. (Hint: See the answers to the previous problem.) (Notice that it is the same as the last problem on the previous problem set.)

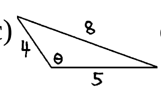


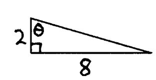
9) *Beware of SSA!* For each problem below make a quick, but fairly accurate sketch of the described triangle. (Recall that angle A is opposite side a, etc.) Comment on anything that is notable about each triangle.
a) ∠A = 15°; a = 8; b = 4
b) ∠A = 15°; a = 4; b = 8
c) ∠A = 70°; a = 4; b = 8

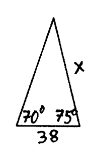
10) Find the variable indicated. (Beware of SSA!)

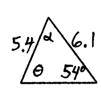
a) 

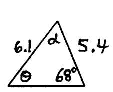
b) 

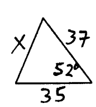
c) 

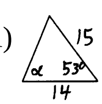
d) 

e) 

f) 

g) 

h) 

i) 

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