11th Grade Assignment – Week #21

Individual Work

- Take the test on *Possibility & Probability* found at the end of this document.
- Work on problems from **Problem Set #1** in the *Trigonometry – Part III* unit.

Group Assignment:

for Tuesday

- A Trigonometric Table. Fill out the table (on the right), while keeping these things in mind:
  - You should memorize the trig values for angles of 30°, 45°, and 60°, both in terms of exact (irrational) values and decimal approximations (e.g., $\cos(30°) = \sqrt{3}/2 \approx 0.866$).
  - However, for this table, only write the decimal approximations.
  - Be sure that you understand these identities:
    - $\sin(90° - \alpha) = \cos(\alpha)$
    - $\sin(180° - \alpha) = \sin(\alpha)$
    - $\cos(180° - \alpha) = -\cos(\alpha)$
    - $\tan(180° - \alpha) = -\tan(\alpha)$
  - Notice that equal increments of the angle measures do not correspond to equal increments of the trig values. **How does this different with sin, cos, and tan?**

A Trigonometric Table

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0			
15°	0.259		0.268
30°			
45°			
60°			
75°	0.966		3.73
90°			
105°			
120°			
135°			
150°			
165°			
180°			

  - How can this table help you to understand why the cos and tan of angles between 90° and 180° are negative?
- Discover the Law of Cosines by working through **Problem Set #2** in the *Trigonometry – Part III* unit.

for Thursday

- Finish **Problem Set #2** (from Tuesday, above) if you did not do so already.
- *Puzzle! Triangle & Square Pieces – Part I*
  Determine how you can cut an equilateral triangle into four pieces which can then be rearranged into a perfect square. Be very clear about exactly where and how the pieces must be cut.

Trigonometry – Part III

Problem Set #1

Some Trig Identities

- $sin(180° – \alpha) = sin(\alpha)$
- $sin(90° – \alpha) = cos(\alpha)$
- $sin^2\alpha + cos^2\alpha = 1$
- $\frac{a}{b} = \frac{\sin A}{\sin B}$ (Law of Sines!)
- $tan(\alpha) = \frac{sin(\alpha)}{cos(\alpha)}$
- $sin(½\alpha) = \sqrt{½ – ½\cos\alpha}$
- $sin(\beta – \alpha)$
  $= sin(\beta)cos(\alpha) – sin(\alpha)cos(\beta)$
- $cos(\alpha + \beta)$
  $= cos(\alpha)cos(\beta) – sin(\alpha)sin(\beta)$

And here are two new ones:

- $cos(180° – \alpha) = –cos(\alpha)$
- $tan(180° – \alpha) = –tan(\alpha)$

1) Given sin(10°) ≈ 0.1736, use the above formulas (but not the trig buttons on your calculator) to calculate the following. In each case, also state which formula you used.

a) *sin*(170°) d) *tan*(10°)

b) *cos*(80°) e) *cos*(170°)

c) *cos*(10°) f) *tan*(170°)

2) Without the use of a calculator, give an exact answer or an estimate.
(Do you still remember the *Basic Trig Facts* from Problem Set #1 of the *Trigonometry Part II* unit?)

a) *cos*(60°) l) *cos*(45°)

b) *sin*(0°) m) *cos*(90°)

c) *sin*(25°) n) *tan*(90°)

d) *cos*(30°) o) *sin*(60°)

e) *sin*(45°) p) *sin*(65°)

f) *tan*(30°) q) *sin*(90°)

g) *tan*(60°) r) *cos*(25°)

h) *cos*(0°) s) *tan*(45°)

i) *tan*(25°) t) *sin*(120°)

j) *sin*(30°) u) *cos*(120°)

k) *tan*(0°) v) *tan*(120°)

3) Without the use of a calculator, give an exact answer or an estimate. (Remember that sin^{-1} normally has two answers.)

a) $sin^{-1}(0.5)$ f) $cos^{-1}(0.25)$

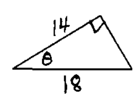
b) $cos^{-1}(\frac{\sqrt{2}}{2})$ g) $sin^{-1}(1)$

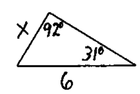
c) $sin^{-1}(0.8)$ h) $sin^{-1}(0)$

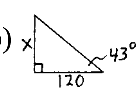
d) $tan^{-1}(0.5)$ i) $tan^{-1}(–\frac{\sqrt{3}}{3})$

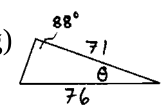
e) $sin^{-1}(\frac{\sqrt{2}}{2})$ j) $cos^{-1}(–0.5)$

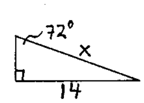
4) Find the variable indicated.

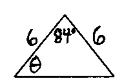
a) 

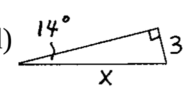
f) 

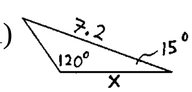
b) 

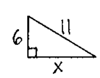
g) 

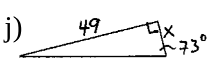
c) 

h) 

d) 

i) 

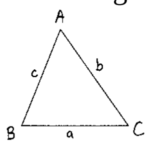
e) 

j) 

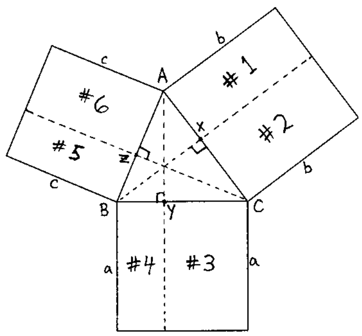
Problem Set #2

Two Proofs of *The Law of Cosines*

1) *A Proof using Squares.*
Fill in the steps as indicated.
Given acute triangle, ΔABC,



we attach squares to the sides of the triangle, draw altitudes to the sides of the triangle, and label everything as shown.



a) Explain why the length of AX is equal to c•cos(A).

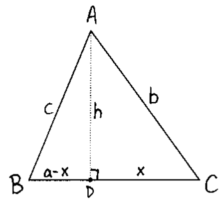
b) Similarly, find the lengths of CX, BY, CY, AZ, BZ.

c) Give expressions for the areas of the six rectangles, in terms of a, b, c, A, B, C. Which ones are equal?

d) Explain the final steps:

$$\#5 + \#6 + \#3 = \#1 + \#2 + \#4$$
$$c^2 + \#3 = b^2 + \#4$$
$$c^2 + \#3 + \#3 = b^2 + \#4 + \#3$$
$$c^2 + 2(\#3) = b^2 + a^2$$
$$\boxed{c^2 = a^2 + b^2 - 2ab \cdot \cos(C)}$$

2) *A Proof without Squares.*
We begin again with an acute triangle, ΔABC. We drop an altitude from the apex, thereby dividing the original triangle into two right triangles.



a) With ΔADC, use the Pythagorean Theorem to find an expression for h^2.

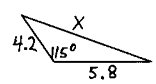
b) With ΔADB, use the Pythagorean Theorem to find an expression for h^2.

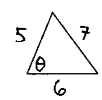
c) Use the above two answers to derive the *Law of Cosines*.

3) Each of the above two proofs only considered acute triangles. What would be different for the case of an obtuse triangle?

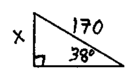
4) Which of the two proofs do you think is a better proof? Why?

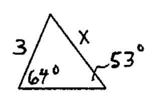
5) Use the *Law of Cosines* to find the indicated variable.

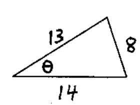
a) 

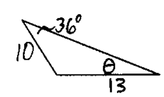
b) 

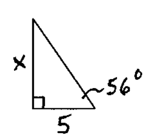
6) Find the variable indicated.

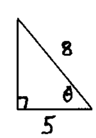
a) 

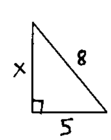
b) 

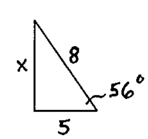
c) 

d) 

e) 

f) 

g) 

h) 

Permutations, Combinations & Probability Test

For each problem, write down at least what you put into your calculator.

All problems are worth four points.

1) How many license plates of 4 symbols can be made using 2 letters followed be 2 digits?

2) There are 13 differently colored crayons. How many different ways can you choose four of them?

3) Gail is buying a certain model bike. She has a choice of four different colors, two kinds of handlebars, two kinds of tires, and three different pedals. How many different kinds of bikes are possible?

4) Suppose that a club consists of 8 women and 6 men. In how many ways can a president and a secretary be chosen if the president is to be female and the secretary male?

5) Two random people are chosen from a group. What is the probability that both of them will have a birthday that falls on a weekend?

6) You can order a sandwich with cheese, onion, pickle, lettuce, tomato, or avocado. How many different sandwiches can you order that have three of those items?

7) In how many different ways can a^4b^6 be written without using exponents? (One way is aabbabbbba.)

8) A class of 10 will elect five people: a president, a secretary, and a social committee of three people. In how many ways can it be done?

9) Suppose you are getting dressed in a dark room. In your drawer, you have 4 red socks, 3 blue socks, and 2 brown socks. If you randomly select 2 socks, what is the probability that you will get 2 socks that match?

10) At a party, all 8 guests have their own jacket. At the end of the party, what is the probability of everyone grabbing a jacket randomly and getting their own jacket?

11) Two cards are drawn from a 52-card deck. Find the probability that…

a) You get a king and a queen.

b) Either both cards are kings or both are hearts.

12) If 24 pieces of sausage are randomly put onto a pizza that is sliced into 8 pieces (with none of the sausages getting cut), what is the probability that your slice will have 3 pieces of sausage?