11th Grade Assignment – Week #19

Group Assignment: *For Tuesday or Thursday*

The unit *Possibility & Probability – Part II* is the main focus for these next two weeks. Many of these problems lend themselves to good problem-solving in groups. Carefully choose problems from **Problem Sets #1-3** to work on together. The rest of the problems you can save for individual work.

Individual Work

- Study the *Summary Sheet for Possibility and Probability* found at the end of this document, which summarizes what we have done thus far.
- Work on problems from **Problem Sets #1-3** of *Possibility & Probability – Part II*, focusing on the problems that you aren't doing in your group.

Problem Set #1 (Possibility & Probability– Part II)

Review (from 9th grade)

1) In how many ways…
   a) can the letters of the word ELDORA be rearranged?
   b) can the letters of the word MOUNTAIN be rearranged?

2) Harry's Restaurant offers 5 choices of salad, 12 choices for main dishes, and 5 different desserts. How many different 3-course meals can be ordered?

3) A license plate consists of 4 letters followed by 2 digits (e.g., XACC55). How many different plates are possible?

4) In how many ways can first, second, and third prizes be awarded in a race with 15 participants?

5) In how many ways can 10 people be put into a line?

6) Calculate
   (a) 6! (b) ${}_5P_5$ (c) ${}_5P_3$ (d) ${}_5P_1$
   (e) ${}_5C_5$ (f) ${}_5C_3$ (g) ${}_5C_1$ (h) ${}_5C_0$

7) If you have 5 pennies and 4 dimes in your pocket, and you reach in and pick 2 coins, what is the probability of getting…
   a) two pennies?
   b) two dimes?
   c) one of each?

8) One card is drawn from a 52-card deck. Find the probability that...
   a) The card is a jack.
   b) The card is a 6.
   c) The card is a club.
   d) The card is a jack or a 6.
   e) The card is a jack or a club.

9) Two cards are drawn from a 52-card deck. Find the probability that...
   a) The first card is a 5 and the second is a 6.
   b) One card is a 5 and the other is a 6.

10) There are 10 applicants for three different job positions at a department store. How many ways are there to fill the three positions?

11) In how many ways can 6 people be seated at a round table? (Moving each person one place to the right or left does not constitute a new arrangement.)

12) There are 6 multiple-choice questions on an exam, each with 4 possible answers. What is the probability of getting at least 5 answers correct, if you guess randomly?

Problem Set #2

1) In how many ways can the letters of the word …
   a) LAUGH be rearranged?
   b) BABBLE be rearranged?

2) There are 3 trails (A,B,C) on the north face of Mount Mary and 2 trails (X,Y) on the south face. How many routes are there going up the north face and down the south face?

3) How many 5-letter words can be made using A, B, C, D, and E if…
   a) each letter may only be used once?
   b) letters may repeat?

4) How many different…
   a) poker hands are possible? (Poker hands consist of 5 cards.)
   b) bridge hands are possible? (Bridge hands consist of 13 cards.)

5) In how many ways…
   a) can a committee of 3 be selected from a group of 8?
   b) can a committee of 5 be selected from a group of 8?

6) The names of 3 seniors, 4 juniors, and 5 sophomores are placed in a hat, and then two names are drawn. Find the probability that...
   a) the first name drawn is a junior and the second is a senior.
   b) both names drawn are sophomores.

7) In how many ways can a president, a vice-president, and a secretary be chosen from a group of eight people?

8) Two dice are rolled. Find the probability that…
   a) the sum of the numbers showing is 9.
   b) you get two 6's.
   c) you get at least one 6.

9) Two cards are drawn from a 52-card deck. Find the probability that...
   a) both cards are jacks.
   b) both cards are clubs.
   c) either both cards are jacks or both are clubs.

10) Use Pascal's triangle to expand…
   a) $(x+y)^4$
   b) $(x+2)^4$
   c) $(x-10)^4$

11) A coin is flipped four times. Find the probability of getting
   a) all heads.
   b) exactly 1 tail.
   c) exactly 2 heads.
   d) two or three heads.

12) All the girls in Emily's class play either basketball or volleyball. Ten of them play basketball, eight play volleyball, and four play both. How many girls are in the class?

Problem Set #3

1) How many ways can these letters be rearranged:
   a) RUNNER?
   b) ERROR?
   c) AAABBCCCCD?

2) In how many different ways can a 7-question multiple-choice test be answered if every question has A, B, C, or D as its options?

3) Fred needs to visit four cities. How many possible ways are there for the order in which to visit the cities?

4) How many ways can you choose 59 things out of 60 (without regard to order)?

5) An outing club has a membership of 4 women and 6 men. A social committee of 4 is to be formed. In how many ways can this be done if…
   a) there must be 2 women and 2 men on the committee?
   b) there must be at least 1 woman on the committee?
   c) all 4 must be of the same sex?

6) A railroad line has 8 stops. How many different one-way tickets are possible?

7) How many different ways are there to arrange 8 identical large chairs and 3 identical small chairs in a row?

8) What is the probability of randomly, but correctly, guessing the top three finishers in a 20-person race?

9) Two dice are rolled. Find the probability that...
   a) the sum of the numbers showing on the dice is 5.
   b) the sum of the numbers showing on the dice is 11.
   c) you get a 6 on exactly one die.
   d) you get a 6 on at least one die.

10) A bag has 2 red, 4 pink, and 6 blue marbles in it. Two marbles are drawn at random. Find the probability that...
   a) both are red.
   b) one is red and one is pink.
   c) neither is red.

11) Five coins are tossed. Find the probability of …
   a) getting all heads?
   b) getting one head?
   c) getting two heads?
   d) getting at least three heads?

12) What is the probability of drawing a red card or a 5 from a standard 52-card deck?

13) Ten identical coins are to be distributed randomly between four people. How many ways can this be done?

14) With a 13-card hand, what is the probability of getting…
   a) only red cards?
   b) no face cards (J, Q, K)?
   c) at least one face card?
   d) one card of each kind (one ace, one king, etc.)?
   e) exactly 11 diamonds?

Summary Sheet for
Possibility and Probability

The Fundamental Counting Principle (wardrobe problem)

The combined number of ways for two (independent) things to happen is the product of the number of ways for each of those things to happen separately.

Example: How many different outfits can Fred wear if he has 10 shirts and 8 pairs of pants?
Solution: For each of the 10 shirts he has 8 options for pants. Therefore, $10 \cdot 8 =$ **80**.

Permutations (prize problem)

The number of ways to select r out of n items, and put them into order is:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$ which is the first r items in n!

Example: How many ways can the top 5 prizes be given out if there are 12 participants?
Solution: ${}_{12}P_5 = \frac{12!}{7!} \rightarrow 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \rightarrow$ **95,040**

Combinations (committee problem)

The number of ways to choose a group of r out of n items is:

$$ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!\, r!} $$

Example: How many committees of 5 people can be chosen from a group of 12?
Solution: ${}_{12}C_5 = \frac{12!}{7!\, 5!} \rightarrow \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \rightarrow$ **792**

Distinguishable Arrangements (rearranging letters)

With a total of n items, of which a items are indistinguishable from each other, as are a further b items, etc., the number of distinguishable arrangements (or permutations) is:

$$ \frac{n!}{a!\, b!\, c!\, \ldots} $$

Example: How many ways are there to reorder the letters AAABBCCCCDEEE
Solution: $\frac{13!}{3!\, 2!\, 4!\, 3!} \rightarrow$ **3,603,600**

The Probability of an Event

The probability of an event successfully occurring, P(E)*, is equal to the number of possible (equally likely) success outcomes divided by the total number of possible (equally likely) outcomes.*

$$ P(E) = \frac{\text{number of successful outcomes}}{\text{number of total possible outcomes}} $$

Example: One card is drawn from a standard deck. Find the probability of getting a spade.
Solution: The probability is $\frac{13}{52}$ or **25%**.

Sets that are not Mutually Exclusive

If A and B share members, then the number of members that are either in A or B is A plus B minus the number of members in both A and B.

$$ A \cup B = A + B - (A \cap B) $$

Example: Everyone in a class is either a sophomore or a girl. 15 are sophomores and 12 are girls. If there are 8 students who are sophomore girls, how many are in the class?
Solution: $15 + 12 - 8 =$ **19**.

Summary Sheet for Possibility and Probability (continued)

Two Independent Events

If A and B are independent events, the probability that both A and B will occur is the product of probabilities of each occurring separately.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example: If you roll a die and flip a coin, what is the probability of getting a 2 and a head?
Solution: $\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

Statistical Probability (The *Law of Large Numbers*)

The more an event is repeated, the closer the average outcome gets to the expected (theoretical) outcome.

Example: If we flip a coin n times, we expect that the number of heads will get closer to 50% for larger values of n.

Example: Since the probability of rolling a sum of a 9 with two dice is 11.1%, we expect that if we roll two dice one thousand times, then we will get a 9 about 11% of the time.

The Probability of a Complement

If A is the complement of B, then the sum of their probabilities is equal to one, or 100%.

Example: What is the probability of flipping three coins and getting at least one head?
Solution: The complement (or opposite) of this is getting no heads, which has a probability of $1/8$. Therefore, the probability of getting at least one head is $1 - 1/8 = 7/8$.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Some properties of Pascal's Triangle:

- If we start counting with zero, then the number in the n^{th} row and r^{th} column is ${}_nC_r$.
- From the property that was used to generate the triangle, we have ${}_{n-1}C_r + {}_{n-1}C_{r-1} = {}_nC_r$
- The sum of the numbers in the n^{th} row is 2^n. In other words: $\sum_{r=1}^{n} {}_nC_r = 2^n$
- *The Binomial Theorem.* The n^{th} row is the set of coefficients in the expansion of the binomial $(x+y)^n$, or $(x+1)^n$.
- The sum of the numbers in each "shallow" diagonal is a Fibonacci number.