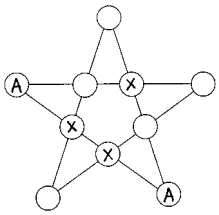
11th Grade Assignment – Week #14

Announcement regarding the upcoming Projective Geometry Main lesson

- It is a 3-week main lesson, running from Week #16 to Week #18.
- Each day you will watch about 40 minutes of recorded lectures, which will include instructions for various drawings.
- The only live lecture during those three weeks is the usual Friday tutorial (at the usual time).
- As opposed to the rest of your Math Academy experience this year, this main lesson does not have assignments designed for group work. However, if you wish to meet together to discuss some of the assignments, you are certainly welcome to do so.
- Part of the advantage with how this main lesson is set up is that you have added flexibility. For example, if you have a very busy day with other things, you can skip a day of doing projective geometry, and then catch up with your projective geometry work on another day.

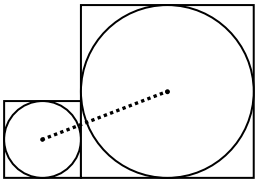
Group Assignment: *For Tuesday or Thursday*

1) **A's and X's**
The figure here must be filled in such that each row of four circles contains two A's and two X's. Which empty circle must be filled with an A?

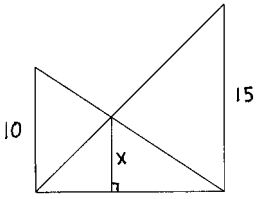


2) **Buckets**
You have two buckets. One measures exactly ten gallons, and the other measures exactly three gallons. You can use as much water as you need, but you may only use these two buckets. How can you measure out exactly one gallon of water? Find two different ways to solve this.

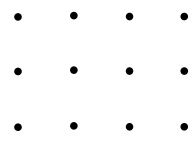
3) **Connected Circles**
The squares, shown here, have sides of length 14 and 34. What is the length of the line that joins the centers of the inscribed circles?



4) **Three Vertical Lines**
With the diagram shown here, the lines marked as 10, 15, and x are all parallel. Find x. (It is interesting to note that if the base of the figure were lengthened or shortened, the value of x would not change.)

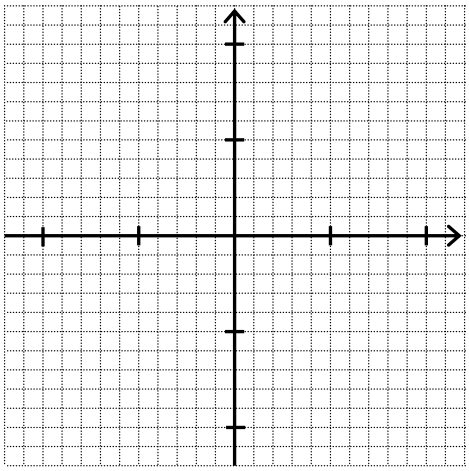
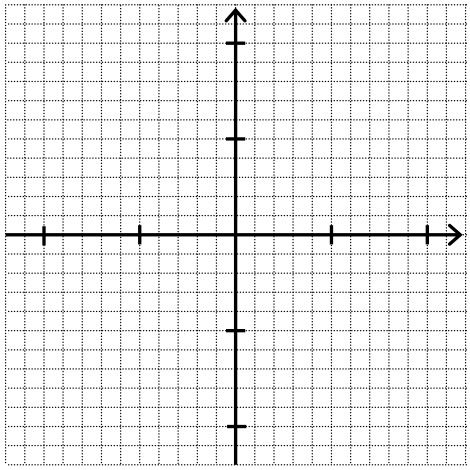
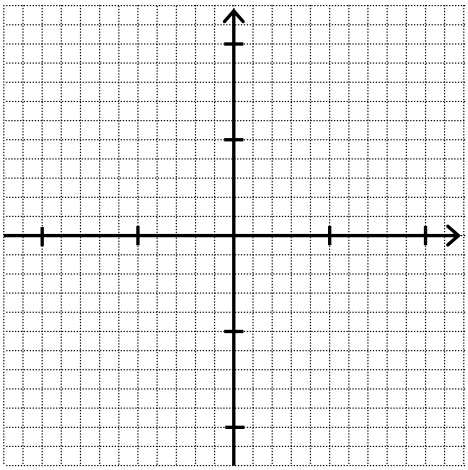
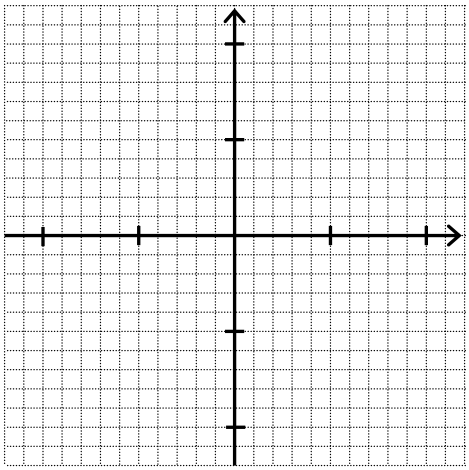
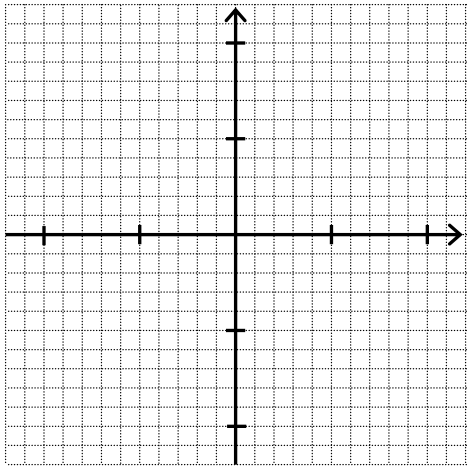
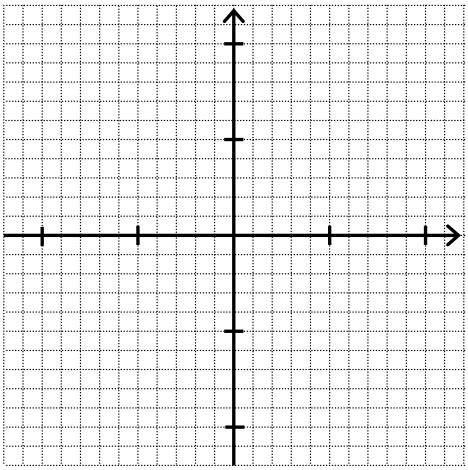
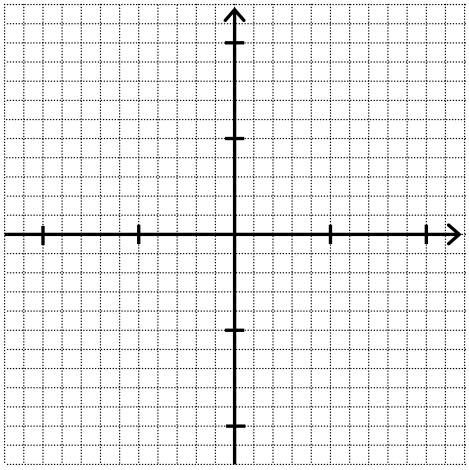
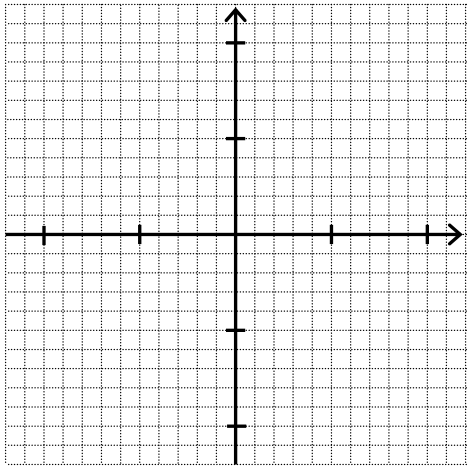
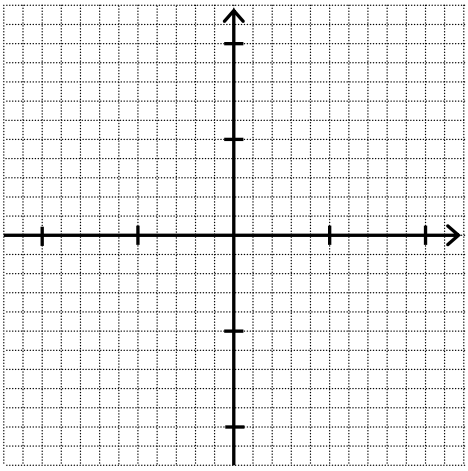


5) **Connecting Twelve Dots**
Without lifting your pencil off the page, and ending up back at the place where you started, draw five straight lines that pass through all 12 of the points in the 4-by-3 grid shown here.



Individual Work

- Carefully select problems that you need to work on from **Problem Set #4 and #5** (Cartesian Geometry – Part II).
- Any of the problems from the above Group Assignments that were not completed during your group meetings, can be worked on individually.

Problem Set #4

Graphing Circles

Graph each of the following:

1) $x^2 + y^2 = 16$
2) $(x+5)^2 + (y+3)^2 = 16$
3) With the previous problem, what does the 5, the 3, and the 16 tell us about the graph?

Graph each of the following:

4) $x^2 + y^2 = 9$
5) $x^2 + y^2 = 4$
6) $x^2 + y^2 = 7$
7) $(x+1)^2 + (y-4)^2 = 25$
8) $(x-3)^2 + y^2 = 1$
9) $(x-7)^2 + (y+2)^2 = 1$
10) $(x+6)^2 + (y+4)^2 = 5$
11) $y = (x-7)^2 - 4$
12) $y = -3(x+1)^2 + 7$
13) $y = -x^2 + 3$
14) $y = -(x+3)^2$
15) $y = \frac{1}{2}(x-2)^2 - 5$

Different Forms of an Equation

All of the equations above are given in *graphing form*, which is the easiest for graphing.

As an example let's take the following (which is a parabola on a Cartesian graph):

$y = (x-7)^2 - 4$.

If we multiply it out we get

$y = x^2 - 14x + 45$

We say that an equation like this is in *standard form*.

If both the x and y terms are squared then standard form is something like this:

$x^2 + 2x + y^2 - 8y + 8 = 0$

Rewrite each equation in standard form.

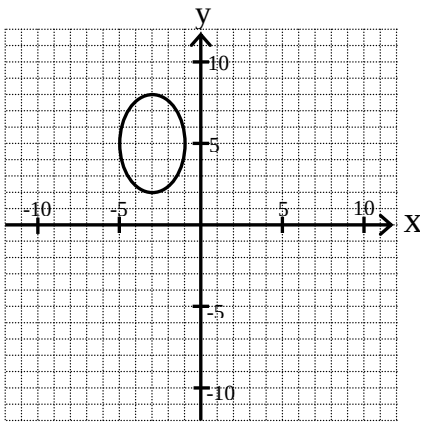
16) $y = 2(x+6)^2 - 3$
17) $(x+1)^2 + (y-4)^2 = 25$

Rewrite each equation in graphing form, and then graph it.

18) $y = x^2 + 10x + 21$
19) $y = -3x^2 - 24x - 45$
20) $y = \frac{1}{4}x^2 + 4x + 11$
21) $x^2 + 4x + y^2 - 6y - 12 = 0$
22) $x^2 - 10x + y^2 + 12y + 60 = 0$

Graphing an Ellipse

23) *Challenge!* Determine the equation of the ellipse given below, which has a center of (-3,5), a horizontal diameter of 4 and a vertical diameter of 6.



Problem Set #5

Graph each of the following:

1) $y = -x^2 + 3$

2) $x^2 + (y+2)^2 = 9$

3) $y = \frac{1}{3}(x-1)^2 - 4$

4) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

5) $(x+3)^2 + (y-5)^2 = 1$

6) $\frac{(x+3)^2}{16} + (y-5)^2 = 1$

7) $y = -2x^2 + 24x - 68$

8) $x = y^2 - 10y + 25$

9) $x^2 + 10x = 8y - y^2 - 37$

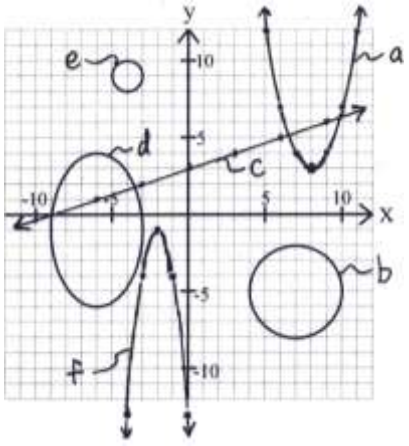
10) $9x^2 + 25y^2 - 54x - 144 = 0$

11) $x^2 + y^2 + 2x - 6y - 6 = 0$

12) $3y + 2x + 3 = 0$

13) $y = 2x^2 - 8x + 8$

14) Give the equation of each of the following graphs.



15) Find the (exact!) common solution to the graphs of a and c given above.

The Roots

The term *root* is often used in mathematics like the word *solution*. We will also use the term *root* here to mean where the graph of an equation crosses the x-axis.

16) Find the roots of the graph of each of these equations.
   a) $y = x^2 + 3x - 10$
   b) $2x + 3y = 18$
   c) $x^2 + (y+4)^2 = 25$

17) Give three solutions to the last of the above equations.

18) Graph each of the following:
   a) $y = -x^2 - 1$
   b) $y = -2(x+1)^2 + 3$
   c) $y = -(x-1)^2 + 2$

19) With the three above graphs, where exactly do…
   a) a and c intersect?
   b) a and b intersect?
   c) b and c intersect?

20) Using the equation $x^2 + y^2 - 6x + 4y + 4 = 0$
   a) Graph it.
   b) Where is $y = -5$?
   c) Where is $y = -4$?
   d) Find the roots of the graph.
   e) Why were the last two answers the same?

21) Hank took a test with 20 multiple choice questions. He received 1 point for each correct answer and lost ¼ point for each incorrect answer. If he answered each question, and got a raw score of 12½, how many did he get correct?