11th Grade Assignment – Week #12

Group Assignment: *For Tuesday and Thursday*

- *Card Trick!*
  (Note: Those of you who were in my 8th grade Math Academy will likely remember this card trick. If so, then don't spoil the secret of the trick to the others in the group. But, don't worry, here in 11th grade, we will take this trick further.)

  Here is what you need to do:
- Everyone in the group will need their own deck of cards (52 cards).
- Figure out a way so that you can all watch the 8-minute video together, which is titled "JYMA - G11 - W12 - Tuesday Group Assignment". You should find it along with the lectures for this quarter. Watch it together by having someone share their screen, but be sure to click the boxes "Share Sound", and "Optimize for Video Clip" (which are found at the bottom left of the screen in share screen dialogue box).
- Here are the questions for you to work on:
  1) How was it that I was able to predict the card? Can you now predict the card as I did? You should practice!
  2) Give a mathematical explanation for how the trick works.
  3) Derive a single formula that allows you to do this trick given different values for the following:
     - The number of piles, **P**, you keep (which was 3).
     - The number, **M**, you count up to when making the piles (which was 10). (Note: if you count up to 6, cards 7 and higher have a value of 6. If you count up to 14, then a Jack has a value of 11, a Queen has a value of 12, and a King has a value of 13.)
     - The number of cards, **D**, that you look at in the beginning (which was 26).

     If you are successful, then this formula will allow you to calculate the "magic card", **C**, in terms of P, M, and D. For example, you could then perform the trick by first counting out 29 cards (D=29), keeping 5 piles at the end (P=5), and counting up to 7 with each pile (M=7).
  4) What are the limitations of the above formula?

Individual Work

- Carefully select problems that are helpful for you from **Problem Set #6** (from *Complex Numbers, Part I*).
- Take the **Complex Numbers Test**, found at the end of this document.

Problem Set #6

Simplify.

1) $(4+3i)(7-2i)$
2) $(4+3i)(4-3i)$
3) $(x-5)^2$
4) $(5-i)^2$
5) $(7-2i)^2$
6) $(x+6)^3$
7) $(6+i)^3$
8) $[x+(2-i)][x+(2+i)]$
9) $[x-(3+5i)][x-(3-5i)]$
10) $\frac{3i}{5+7i}$
11) $\frac{12-i}{5+2i}$
12) $\frac{6+i}{4-i}$
13) $\frac{20-15i}{4-3i}$
14) $\frac{4+17i}{2+i}$

Solve each equation using the *quadratic formula*.

15) $x^2+3x-5 = 0$
16) $x^2-6x+34 = 0$
17) $x^2-4x-6 = 0$
18) $x^2-5x+7 = 0$

"Complex" Factoring.

19) x^2-20
20) x^2+20
21) x^4-9
22) x^2-4x-6
23) $x^2-8x+17$

24) Find each of the following given that:
$f(x) = x^2+5x+3$
$g(x) = \sqrt{x-10}$
$h(x) = 4x-2$

a)	$f(6)$	h)	$h(f(-3))$
b)	$g(14)$	i)	$f(g(x))$
c)	$g(20)$	j)	$f(h(x))$
d)	$g(6)$	k)	$g(h(x))$
e)	$h(x^3)$	l)	$h(f(x))$
f)	$f(3x)$	m)	$f(h(g(11)))$
g)	$f(h(0))$	n)	$f(g(h(-1)))$

A Hanging Question

On the previous problem set we investigated several "interesting questions". The most difficult question was the last one: "*What are the three cube roots of 1*?" Our goal will be to answer this question and to find a more systematic approach to answering these kinds of questions. But this will have to wait until our second unit on complex numbers. There is much more about graphing, trigonometry and complex numbers that needs to be learned in order to understand how to solve this problem.

As always, you can be comforted with the fact that many great mathematicians had many tough questions in their minds for many years – some of which they eventually found answers to (and how rewarding that was!) and others they were never able to answer.

Complex Numbers, Part I
Test

Problems are worth 4 points each.

Simplify

1) $(6-2i)(3+i)$

2) $(12-5i)^2$

3) $\frac{8+2i}{1+i}$

4) $[x+(3+7i)][x+(3-7i)]$

5) $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^4$

Find each value, given these function definitions:

$f(x) = 2x - 5$

$g(x) = x^2 + 4$

6) f(3)

7) g(f(4))

8) f(g(x))

Solve. (Complex numbers are allowed).

9) $3x^2 + 12 = 0$

10) $x^4 + 4x^2 - 45 = 0$

11) $(x+3)^2 = 3x + 4$

"Complex" Factor (Each one is possible!)

12) $x^2 - 3$

13) $x^2 + 17$

14) $x^2 - 10x + 61$